Correspondence of States and Observables for BRST-BFV, Dirac and Refined Algebraic Quantizations of Constrained Systems

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Abstract

Correspondence between BRST-BFV, Dirac and refined algebraic approaches to quantize constrained systems is analyzed. Refined algebraic quantization approach based on modifying an inner product rather than on imposing the constraints is generalized to the case of nontrivial structure functions. The results are illustrated for the simple example and for the semiclassical theory.

1. Different approaches have been developed to quantizing constrained systems. The most serious difficulty for all of them is to introduce an inner product.

Let us start from the simplest example. Consider the system with one degree of freedom. States are specified then by the wave functions $\Psi(x)$, $x \in \mathbf{R}$. Let $\Lambda = p$ be a constraint.

The Dirac approach [1] tells us that physical states should obey an additional condition $\hat{\Lambda}\Psi \equiv -i\frac{\partial\Psi}{\partial x} = 0$, so that $\Psi(x) = \Psi_0 = const$. One notices that the inner product of the form $\int dx|\Psi(x)|^2 = |\Psi_0|^2 \cdot \infty$ diverges.

To define nevertheless the physical inner product, one usually [2] imposes additional gauge conditions of the type $X = 0$ such that $\{X;\Lambda\} \neq 0$ classically. For the case $X = x$, one should consider the wave functions on the "gauge surface" $x = 0$ only, so that the inner product will be $|\Psi(0)|^2$. Generally, the gauge fixing approach may depend on choice of the additional conditions and lead to the problem of Gribov copies [3].

However, there is an additional quantization approach ("refined algebraic quantization" [4,5]) without difficulties with inner products. Instead of imposing constraints on physical states, one modifies the inner product. For

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example $\Lambda = p$, the modification is

$$< \Phi, \Phi >= \int dx \Phi^*(x) \left(2\pi\delta \left(-i\frac{\partial}{\partial x} \right) \Phi \right)(x). \qquad (1)$$

Wave functions Φ ("auxiliary states") may be arbitrary. Since

$$2\pi\delta(-i\frac{\partial}{\partial x})\Phi(x) = \int d\alpha e^{i\alpha(-i\frac{\partial}{\partial x})}\Phi(x) = \int d\alpha \Phi(x+\alpha) = \int d\alpha \Phi(\alpha),$$

an explicit form of the inner product (1) is $< \Phi, \Phi >= |\int d\alpha \Phi(\alpha)|^2$. One can notice that wave functions $\Phi(x)$ such that $\int dx \Phi(x) = 0$ are of zero norm. They are equivalent then to zero. Two auxiliary states Φ_1 and Φ_2 are called equivalent iff $\int dx \Phi_1(x) = \int dx \Phi_2(x)$. Equivalence classes may be viewed as physical states; they are specified by numbers $\int dx \Phi(x)$. Auxiliary states related by formula $\Phi \to \Phi - i\frac{\partial X}{\partial x}$ specify the same physical state.

Thus, in the Dirac approach the wave function Ψ is defined uniquely but must satisfy the constraint $\hat{\Lambda}\Psi = 0$, while in the refined algebraic quantization the wave function Φ may be arbitrary but is defined up to a gauge transformation $\Phi \to \Phi + \hat{\Lambda}X$.

The correspondence of Dirac and refined algebraic states Ψ and Φ is given by the relation $\Psi(x) = 2\pi\delta(-i\frac{\partial}{\partial x})\Phi(x)$. Explicitly, $\Psi(x) = \int d\alpha \Phi(\alpha)$, so that the Dirac condition $\frac{\partial \Psi}{\partial x} = 0$ is satisfied. The prescription $||\Psi||^2 = |\Psi(0)|^2$ is also obtained.

2. The inner product of the type (1) may be written for the more general cases as well:

$$< \Phi, \Phi >= (\Phi, \ \Phi) \qquad (2)$$

where the operator should satisfy the following properties:

$$^+ = \ ; \qquad \geq 0; \qquad \hat{\Lambda}_a = 0. \qquad (3)$$

Then two auxiliary states Φ are called equivalent iff their difference is of zero norm; equivalence classes are viewed as physical states. For example, states $\Phi = \hat{\Lambda}_a X^a$ are obviously equivalent to zero; therefore, there is a gauge freedom: two auxiliary states

$$\Phi \to \Phi + \hat{\Lambda}_a X^a \qquad (4)$$

correspond to the same physical state. The correspondence between Ψ- and Φ-states is

$$\Psi = \quad \Phi. \tag{5}$$

The Dirac wave function Ψ satisfies then the constraint condition

$$\hat{\Lambda}_a^+ \Psi = 0. \tag{6}$$

3. For the closed-algebra case, the operator has been constructed explicitly. For the abelian group [4] $[\hat{\Lambda}_a; \hat{\Lambda}_b] = 0$; $\hat{\Lambda}_a = \hat{\Lambda}_a^+$, $a, b = \overline{1, M}$ with the continuous spectrum of $\hat{\Lambda}_a$ one sets

$$\begin{array}{c} = \prod_a 2\pi\delta(\hat{\Lambda}_a); \\ < \Phi, \Phi >= (\Phi, \prod_a 2\pi\delta(\hat{\Lambda}_a)\Phi). \end{array} \tag{7}$$

For the nonabelian case, the constraints may have nontrivial imaginary parts [6]

$$\hat{\Lambda}_a = \check{\Lambda}_a - \frac{i}{2} \; {}^{\prime b}_{ab}; \qquad [\check{\Lambda}_a; \check{\Lambda}_b] = i \; {}^{\prime c}_{ab} \check{\Lambda}_c; \qquad \check{\Lambda}_a^+ = \check{\Lambda}_a. \tag{8}$$

The operator is constructed as follows. Let L_a be generators of the Lie algebra, $[L_a; L_b] = i \; {}^{\prime c}_{ab} L_c$. Then the mapping $L_a \mapsto \check{\Lambda}_a$ is a representation of the Lie algebra. Consider the corresponding representation of the Lie group $\check{T} : \exp(i\mu^a L_a) \mapsto \exp(i\mu^a \check{\Lambda}_a)$. By $Ad(L_a)$ we denote the adjoint representation of the Lie algebra, $(Ad(L_a)\rho)^c = i \; {}^{\prime c}_{ab} \rho^b$, while $Ad\{g\}$ is an adjoint representation of the group: $(Ad\{g\}\rho)^c = (\exp A)^c_b \rho^b$ with $A^c_b = -\mu^a \; {}^{\prime c}_{ab}$, $g = \exp(i\mu^a L_a)$. Then [5]

$$= \int d_R g (det Ad\{g\})^{-1/2} \check{T}(g). \tag{9}$$

To introduce operator in general case, one should investigate the relationship between Dirac, refined algebraic and BRST-BFV approaches (see [7] for more details).

4. To develop the BRST-BFV approach [8], it is necessary to introduce additional degrees of freedom: Lagrange multipliers and momenta λ^a, π_a, $a = \overline{1, M}$, ghosts and antighosts $C^a, \overline{C}_a$, canonically conjugated momenta $\overline{\Pi}_a, \Pi^a$, $a = \overline{1, M}$. The nontrivial (anti)commutation relations are: $[\lambda^a, \pi_b] = i\delta^a_b$, $[C^a, \overline{\Pi}_b]_+ = \delta^a_b$, $[\overline{C}_a, \Pi^b]_+ = \delta^b_a$. Operators $\overline{C}_a$ and Π^b are anti-Hermitian,

others are Hermitian. The main object of the BRST-BFV method is the B-charge Ω. For the closed-algebra case, it has the form

$$\Omega = C^a \check{\Lambda}_a - \frac{i}{2} \overset{\checkmark}{U}{}^a_{bc} \overline{\Pi}_a C^b C^c - \frac{i}{2} \overset{\checkmark}{U}{}^a_{ba} C^b - i\pi_a \Pi^a. \tag{10}$$

It is formally Hermitian and nilpotent,

$$\Omega^+ = \Omega; \qquad \Omega^2 = 0. \tag{11}$$

For the open-algebra case with nontrivial structure functions, the B-charge is looked for in the following form:

$$\Omega = -i\pi_a \Pi^a + C^a \hat{\Lambda}_a + ... + \Omega^{n b_1 ... b_{n-1}}_{a_1 ... a_n} \overline{\Pi}_{b_1} ... \overline{\Pi}_{b_{n-1}} C^{a_1} ... C^{a_n} + ... \tag{12}$$

Features of Dirac and refined algebraic quantizations are presented in the BRST-BFV approach. Analogously to the Dirac case, physical states Υ are not arbitrary but should satisfy the BRST-BFV condition

$$\Omega \Upsilon = 0, \tag{13}$$

Similarly to the refined algebraic approach, the gauge freedom is also allowed, the gauge transformation between equivalent states is

$$\Upsilon \to \Upsilon + \Omega X, \tag{14}$$

States Υ and $e^{[\Omega, \rho]_+} \Upsilon$ are then also equivalent.

Another requirement is that physical states should be of zero ghost number, $N = \Pi^a \overline{C}_a - \overline{\Pi}_a C^a$, so that $N\Upsilon = 0$.

5. The most nontrivial problem is to introduce an inner product for the BRST-BFV formalism. Consider the Schrodinger representation for the BFV wave function Υ, $\Upsilon = \Upsilon(q, \lambda, \Pi, \overline{\Pi})$, q and λ are Bose variables, Π and $\overline{\Pi}$ are Grassmannian. The operators are rewritten then as $C^a = \frac{\partial}{\partial \overline{\Pi}_a}$; $\overline{C}_a = \frac{\partial}{\partial \Pi^a}$; $\pi_a = -i\frac{\partial}{\partial \lambda^a}$; $p_i = -i\frac{\partial}{\partial q^i}$, the left derivatives are considered here. The inner product is indefinite. Formally, it is as follows [9]

$$(\Upsilon_1, \Upsilon_2) = \int dq \prod_{a=1}^{M} d\mu^a d\overline{\Pi}_a d\Pi^a (\Upsilon_1(q, i\mu, \Pi, \overline{\Pi}))^* \Upsilon_2(q, -i\mu, \Pi, \overline{\Pi}). \tag{15}$$

The integration and conjugation rules are $(\overline{\Pi}_{a_1} ... \overline{\Pi}_{a_l} \Pi^{b_1} ... \Pi^{b_s})^* = (-1)^s \Pi^{b_s} ... \Pi^{b_1} \overline{\Pi}_{a_l} ... \overline{\Pi}_{a_1}$, $\int d\overline{\Pi}_a \overline{\Pi}_a = 1$, $\int d\Pi^a \Pi^a = 1$. The inner product

space (15) requires additional investigation, since indefinite inner product spaces are specified not only by indefinite inner product but also by Hilbert topology: a class of allowed BFV wave functions Υ should be specified.

The inner product (15) seems to be divergent, but this is not the case. Namely, a typical BFV wave function is as follows. Consider our simplest example, $\Lambda = -i\partial/\partial q$, $\Omega = -i\frac{\partial}{\partial q}\frac{\partial}{\partial\overline{\Pi}} - \frac{\partial}{\partial\lambda}\Pi$. Let the dependence of the wave function on Fermi variables be Gaussian, $\Upsilon(q,\lambda,\Pi,\overline{\Pi}) = \exp[-\alpha\overline{\Pi}\Pi]\Upsilon_0(q,\lambda)$. Then the B-condition (13) reads $(i\alpha\frac{\partial}{\partial q} - \frac{\partial}{\partial\lambda})\Upsilon_0(q,\lambda) = 0$, so that $\Upsilon_0(q,\lambda) = \Phi(q+i\alpha\lambda)$ and

$$\Upsilon(q,\lambda,\Pi,\overline{\Pi}) = \exp[-\alpha\overline{\Pi}\Pi]\Phi(q+i\alpha\lambda). \tag{16}$$

The inner product (15) is taken then to the form $\int dqd\mu d\overline{\Pi}d\Pi\exp[-2\alpha\overline{\Pi}\Pi]\Phi_1^*(q-\alpha\mu)\Phi_2(q+\alpha\mu)$. Integration over Grassmannian variables gives us factor 2α, while after substitution $q-\alpha\mu = q_1$, $q+\alpha\mu = q_2$ one finds $(\Upsilon_1,\Upsilon_2) =< \Phi_1,\Phi_2 >= \int dq_1\Phi_1^*(q_1)\int dq_2\Phi_2(q_2)$. This is in agreement with the refined algebraic quantization. We see that Φ-states and B-states correspond as

$$\Phi(q) = \Upsilon(q,0,0,0) \tag{17}$$

It follows also from eq.(16) that the Dirac state $\Psi(q) = \int dx\Phi(x)$ is related to the B-state as follows, $\Psi(q) = \int d\mu d\overline{\Pi}d\Pi\Upsilon(q,-i\mu,\Pi,\overline{\Pi})$.

6. The considered prescriptions for Φ, Ψ are valid for the general closed-algebra case as well.

For the abelian algebra, it is possible [10] to take any B-state by transformations (14) to the gauge

$$A_a^-\Upsilon \equiv \frac{1}{\sqrt{2}}[\pi_a - i\alpha_a^b\hat{\Lambda}_b]\Upsilon = 0. \tag{18}$$

Combining relations (18) and (13), one finds $[\frac{\partial}{\partial\overline{\Pi}_a} + \alpha_b^a\Pi^b]\Upsilon = 0$, so that

$$\Upsilon(q,\lambda,\Pi,\overline{\Pi}) = \exp[-\overline{\Pi}_a\alpha_b^a\Pi^b]\exp[-\lambda^a\alpha_a^b\hat{\Lambda}_b]\Phi(q) \tag{19}$$

Calculating the integral (15), one finds $(\Upsilon,\Upsilon) = (\Phi,\prod_{a=1}^M 2\pi\delta(\hat{\Lambda}_a)\Phi)$. Thus, one should set Φ to be of the form (17), provided that gauge condition (18) is satisfied.

For the state $\tilde{\Upsilon} = \Upsilon + \Omega X$, with $X = X_{00}(q,\lambda) + X^a_{01}(q,\lambda)\overline{\Pi}_a + X_{10,a}(q,\lambda)\Pi^a + ...$, the function $\tilde{\Phi}(q) = \tilde{\Upsilon}(q,0,0,0)$ is related to $\Phi(q)$ as

$$\tilde{\Phi}(q) = \Phi(q) + \hat{\Lambda}_a X^a_{01}(q,0). \tag{20}$$

Therefore, $\tilde{\Phi}$ and Φ are gauge-equivalent (eq.(4)), so that formula (17) is valid for the states $\Upsilon + \Omega X$ as well up to a gauge transformation.

The Dirac wave function is given by the relation

$$\Psi(q) = \int \prod_a d\mu^a d\overline{\Pi}_a d\Pi^a \Upsilon(q, -i\mu, \Pi, \overline{\Pi}). \tag{21}$$

For the states of the form (19), relation (5) is checked by the direct calculation. For states $\Upsilon = \Omega X$, the integrand in eq.(21) is a full derivative, so that $\Psi = 0$ and gauge transformations do not influence on the Dirac wave function.

7. For the closed-algebra case, one can use the Marnelius gauge [11]

$$C^a\Upsilon = 0, \qquad \pi_a\Upsilon = 0 \tag{22}$$

Since the ghost number of Υ is zero, condition (22) means that

$$\Upsilon = \Phi(q) \tag{23}$$

The inner product (15) has the form $0 \cdot \infty$. It is ill-defined and requires renormalization then. This is the expression

$$(\Upsilon, e^{t[\Omega,\rho]_+}\Upsilon) \tag{24}$$

which is formally equal to (Υ, Υ). The gauge fermion ρ is chosen to be $\rho = -\lambda^a\overline{\Pi}_a$.

For states (23), the inner product is calculated as follows. One notices $[\Omega,\rho]_+ = -\lambda^a\check{\Lambda}_a + \frac{i}{2}\lambda^a\,{}^b_{ab} - i\lambda^a\overline{\Pi}_b C^c\,{}^b_{ac} - \overline{\Pi}_a\Pi^a$ and reproduces eq.(7) for the abelian case. For the general case, one finds

$$(e^{t[\Omega,\rho]_+}\Upsilon)(q,\lambda,\Pi,\overline{\Pi}) = e^{-t\lambda^a\hat{\Lambda}_a}\Phi(q)e^{\overline{\Pi}_a B^a{}_b(\lambda,t)\Pi^b}$$

with $\hat{\Lambda}_a$ of the form (8) and $B(\lambda,t) = -\int_0^t d\tau Ad\{\exp(-\tau\lambda^a L_a\}$. Formula (9) is indeed reproduced for (Υ,Υ). We see that relation (17) is valid for the

nonabelian case for the Marnelius gauge (22), since relation (20) is satisfied for nonabelian groups. Formula (17) is checked for arbitrary gauge then.

It follows from eq.(5) that the Dirac wave function has the form (21) for $\Upsilon = e^{t[\Omega;\rho]_+}\Phi$. Since for $\Upsilon = \Omega X$ the integral (21) vanishes as a full derivative, formula (21) is obtained for any gauge.

8. To find [7] a form of the operator entering to the inner product (2) for the nontrivial structure functions case, let us postulate the correspondence (17). It is well-defined, since equivalent B-states give us equivalent Φ-states.

Then for the Marnelius gauge $\Upsilon = \Phi(q)$ one has $[\Omega,\rho]_+ = -\overline{\Pi}_a\Pi^a - \lambda^a\hat{\Omega}_a$ with

$$\hat{\Omega}_a = \Omega_a(\overline{\Pi}, C) = [\overline{\Pi}_a, \Omega]_+ = \hat{\Lambda}_a + ... + n\Omega^{nb_1...b_{n-1}}_{a_1...a_{n-1}a}\overline{\Pi}_{b_1}...\overline{\Pi}_{b_{n-1}}C^{a_1}...C^{a_{n-1}} + ...$$

and $(\Phi, e^{t[\Omega,\rho]_+}\Phi) = \int dq\Phi^*(q)\prod_{a=1}^M d\mu^a d\overline{\Pi}_a d\Pi^a e^{-t\overline{\Pi}_a\Pi^a + it\mu_a\hat{\Omega}_a}\Phi(q)$ with $\hat{\Omega}_a = \Omega_a(\overline{\Pi}, \partial/\partial\overline{\Pi})$. Therefore,

$$= \int \prod_{a=1}^M d\mu^a d\overline{\Pi}_a d\Pi^a e^{-\overline{\Pi}_a\Pi^a + i\mu_a\hat{\Omega}_a(\overline{\Pi},\partial/\partial\overline{\Pi})}1. \qquad (25)$$

By the direct calculations, one checks that $^+ = $. Proof of property ≥ 0 is an open problem.

To justify property $\hat{\Lambda}_a = 0$, one writes

$$\hat{\Lambda}_b Y^b(q) = \int \prod_{a=1}^M d\mu^a d\overline{\Pi}_a d\Pi^a \exp[\Pi^a\overline{\Pi}_a + i\mu_a\hat{\Omega}_a]\Omega\overline{\Pi}_b Y^b(q). \qquad (26)$$

Making use of formula $e^{\Pi^a\overline{\Pi}_a + i\mu_a\hat{\Omega}_a}\Omega = \Omega^+ e^{\Pi^a\overline{\Pi}_a + i\mu_a\hat{\Omega}_a}$ being a corollary of the relation $\Omega^+[\Omega,\rho]_+ = [\Omega,\rho]_+\Omega$, one notices that integral (26) vanishes as a full derivative.

The Dirac wave function is of the form (21) for $\Upsilon = e^{t[\Omega;\rho]_+}\Phi$; equivalent B-states give the same Dirac states according to eq.(21). Thus, eq.(21) is valid for $\Upsilon \sim \Phi$.

9. Let us consider the properties of quantum observables in different quantization approaches.

In the BRST-BFV approach, observables are viewed as series

$$H_B = H + ... + H^{nb_1...b_n}_{a_1...a_n}\overline{\Pi}_{b_1}...\overline{\Pi}_{b_n}C^{a_1}...C^{a_n} + ... \qquad (27)$$

The operator coefficient functions $H^{nb_1\ldots b_n}_{a_1\ldots a_n}(\hat{p},\hat{q})$ are chosen in such a way that

$$H_B^+ = H_B, \qquad [\Omega, H_B] = 0. \tag{28}$$

These properties provide that physical states (13) are taken by the operator H to physical, while equivalent states are taken to equivalent; the inner product is conserved under evolution.

One has: $\Omega H_B = C^c\hat{\Lambda}_c H + \hat{\Lambda}_b H_a^{1b} C^a + ...,$, $H_B\Omega = HC^c\hat{\Lambda}_c + ...$, where ... are terms with ghost momenta. Therefore, H should obey the property $[H;\hat{\Lambda}_a] = \hat{\Lambda}_b H_a^{1b}$ for some operators H_a^{1b}.

Since $(H_B\Upsilon)(q,0,0,0) = H\Upsilon(q,0,0,0)$, it is the operator H that corresponds to the B-observable (27) in the refined algebraic quantization approach. An important feature of the physical observable is that the corresponding evolution operator e^{-iHt} should be unitary with respect to the inner product (9). This means that $(e^{-iHt})^+\ e^{-iHt} =$ or

$$H^+ \ = \ H. \tag{29}$$

Check of property (29) is the following. One writes

$$(\ H - H^+\ )\Phi(q) = \int \prod_{a=1}^{M} d\mu_a d\overline{\Pi}_a d\Pi^a (e^{[\Omega;\rho]_+}H_B - H_B^+ e^{[\Omega;\rho]_+})\Phi(q).$$

Since $H_B^+ e^{[\Omega;\rho]_+} - e^{[\Omega;\rho]_+}H_B = [\Omega; \int_0^1 d\tau e^{\tau[\Omega,\rho]_+}[H_B;\rho]e^{(1-\tau)[\Omega,\rho]_+}]_+$, one justifies eq.(29).

Let Φ be an auxiliary state corresponding to the Dirac state $\Psi = \ \Phi$. The observable H takes it to $H\Phi$. This corresponds to the Dirac state

$$H\Phi = H^+\ \Phi = H^+\Psi.$$

Therefore, it is the operator H^+ that corresponds to the observable H in the Dirac approach, while $\exp(-iH^+t)$ is an evolution operator.

10. There are some examples of explicit calculations of the inner product (2) for the nontrivial structure functions case. Let $q = (q^1, q^2, q^3)$, $\Lambda_1 = a(a^2, a^3)p_1$; $\Lambda_2 = p_2$. Classically, $\{\Lambda_1;\Lambda_2\} = \partial_2 \log a(q^2,q^3)\Lambda_1$, so that the structure functions are indeed nontrivial. Making use of the form of the B-charge

$$\Omega = -i\pi_1\Pi^1 - i\pi_2\Pi^2 + \hat{p}_1 a C^1 + (\hat{p}_2 - i\overline{\Pi}_1\partial_2 \log a C^1 + \frac{i}{2}\partial_2 \log a)C^2,$$

one finds [7] $(\Phi,\ \Phi) = \int dx_3 |\int dx_1 dx_2 \frac{\Phi(x_1,x_2,x_3)}{\sqrt{a(x_2,x_3)}}|^2$, and $\Psi(x_1,x_2,x_3) = \frac{1}{\sqrt{a(x_2,x_3)}} \int dy_1 dy_2 \frac{\Phi(y_1,y_2,x_3)}{\sqrt{a(y_2,x_3)}}$. The Dirac wave function indeed satisfies the conditions $\hat{\Lambda}_1^+\Psi = 0$, $\hat{\Lambda}_2^+\Psi = 0$.

11. Another group of examples is based on the semiclassical approximation (see [12] for details). Let $\hat{\Lambda}_a$ depend on the small parameter h as

$$\hat{\Lambda}_a = \frac{1}{h}\Lambda_a(\sqrt{h}\hat{p}, \sqrt{h}\hat{q}) + \Lambda_a^{(1)}(\sqrt{h}\hat{p}, \sqrt{h}\hat{q}) + ...$$

Consider the wave packet (Maslov complex-WKB [13]) states

$$\Phi(q) = e^{\frac{i}{h}S} e^{\frac{i}{h}P(q\sqrt{h}-Q)}\ \text{'}(q - \frac{Q}{\sqrt{h}})$$

specified by classical variables $X = (S \in \mathbf{R}, P \in \mathbf{R}^n, Q \in \mathbf{R}^n)$ and quantum function $\text{'} \in S(\mathbf{R}^n)$. It happens that $(\Phi,\ \Phi)$ is not exponentially small only if classical constraints vanish,

$$\Lambda_a(P,Q) = 0$$

Under this condition,

$$< \Phi, \Phi > \simeq h^{M/2}|c|^2(\ \text{'}, \prod_a 2\pi\delta(\frac{\partial\Lambda_a}{\partial Q}\xi + \frac{\partial\Lambda_a}{\partial P}\frac{1}{i}\frac{\partial}{\partial\xi})\ \text{'}),$$

provided that action of the gauge group is nontrivial, without stationary subgroups. The constraints are linearized in the semiclassical approximation then.

One can also consider gauge transformations of the semiclassical states of two types:

(a) "small gauge transformations" without changing classical state : $X \mapsto X$, $\text{'} \mapsto \text{'} + (\frac{\partial\Lambda_a}{\partial Q}\xi + \frac{\partial\Lambda_a}{\partial P}\frac{1}{i}\frac{\partial}{\partial\xi})\chi^a$;

(b) "large gauge transformations": $\Phi \mapsto e^{-i\tau\mu^a\hat{\Lambda}_a}\Phi$; $X \mapsto \lambda_{\mu\tau}X$, $\text{'} \mapsto V_{\mu\tau}(\lambda_{\mu\tau}X \leftarrow X)\ \text{'}$.

Semiclassical gauge transformations should be unitary (particularly, take zero-norm states to zero-norm states) and satisfy the Batalin quasigroup property [14]: classically, $\lambda_{\mu_1}\lambda_{\mu_2}X = \lambda_{\mu_3}X$ for some $\mu_3 = \mu_3(\mu_1,\mu_2,X)$. Semiclassically [12],

$$V_{\mu_1}(\lambda_{\mu_1}\lambda_{\mu_2}X \leftarrow X)V_{\mu_2}(\lambda_{\mu_2}X \leftarrow X) = V_{\mu_3}(\lambda_{\mu_3}X \leftarrow X).$$

One can also investigate semiclassical observables and evolution [12].

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