$B \to \rho\gamma$ Decay in the Large Energy Effective Theory

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1 Introduction

There is considerable theoretical interest in the decays $B \to K^*\gamma$ and $B \to \rho\gamma$ as these processes are under intensive experimental investigations at CLEO, and lately at the B-factories at KEK and SLAC.

The present measurements of the branching ratios of $B \to K^*\gamma$ decays from the CLEO [1], BABAR [2], and BELLE [3] collaborations yield the following charge-conjugated world averages [4]:

$$\begin{aligned} \mathcal{B}_{\rm exp}(B^\pm \to K^{*\pm}\gamma) &= (3.8 \pm 0.5) \times 10^{-5}, \\ \mathcal{B}_{\rm exp}(B^0 \to K^{*0}\gamma) &= (4.3 \pm 0.4) \times 10^{-5}. \end{aligned} \tag{1}$$

The Cabibbo-Kobayashi-Maskawa (CKM) disfavored $B \to \rho\gamma$ decays have not yet been observed. The current best limits are from the BABAR collaboration. One has (at 90% C.L.) [5]:

$$\begin{aligned} \mathcal{B}_{\rm exp}(B^\pm \to \rho^\pm\gamma) &< 2.3 \times 10^{-6}, \\ \mathcal{B}_{\rm exp}(B^0 \to \rho^0\gamma) &< 1.4 \times 10^{-6}. \end{aligned} \tag{2}$$

Combined using the isospin symmetry, they yield an improved upper limit on the ratio of the branching ratios [5]:

$$R_{\rm exp}(\rho\gamma/K^*\gamma) = \frac{\mathcal{B}_{\rm exp}(B \to \rho\gamma)}{\mathcal{B}_{\rm exp}(B \to K^*\gamma)} < 0.05. \tag{3}$$

Measurement of this ratio will provide an independent determination of the CKM matrix element ratio $|V_{td}/V_{ts}|$. It has been argued in the literature that a combination of the Heavy Quark Effective Theory and the Large Energy Effective Theory (HQET/LEET) provides a sound theoretical basis to calculate the branching ratios.

In this paper, predictions for the branching ratios of the $B \to \rho\gamma$ decays are reviewed which have been calculated in the HQET/LEET framework. The uncertainty connected with the theoretical input (in particular, form factors) is considerably reduced in the ratio of the branching ratios:

$$R_{\rm th}(\rho\gamma/K^*\gamma) = \frac{\mathcal{B}_{\rm th}(B \to \rho\gamma)}{\mathcal{B}_{\rm th}(B \to K^*\gamma)}. \tag{4}$$

Experimental values of the $B \to K^*\gamma$ branching ratios in combination with the theoretical estimate of the ratio above allow to obtain predictions for the $B \to \rho\gamma$ decays with reduced uncertainty. The isospin-violating ratio and the direct CP-asymmetry in the decays $B \to \rho\gamma$ are also briefly discussed. More details one can find in Ref. [6].

2 $B \to \rho\gamma$ decay width in NLO

The effective Hamiltonian for the $B \to \rho\gamma$ decay (equivalently $b \to d\gamma$ process) at the scale $\mu = O(m_b)$, where m_b is the b-quark mass, is as follows:

$$\begin{aligned} \mathcal{H}_{\rm eff} &= \frac{G_F}{\sqrt{2}} \Big\{ V_{ub} V_{ud}^* \left[C_1(\mu)\, \mathcal{O}_1^{(u)}(\mu) + C_2(\mu)\, \mathcal{O}_2^{(u)}(\mu) \right] \\ &+ V_{cb} V_{cd}^* \left[C_1(\mu)\, \mathcal{O}_1^{(c)}(\mu) + C_2(\mu)\, \mathcal{O}_2^{(c)}(\mu) \right] \\ &- V_{tb} V_{td}^* \left[C_7^{\rm eff}(\mu)\, \mathcal{O}_7(\mu) + C_8^{\rm eff}(\mu)\, \mathcal{O}_8(\mu) \right] + \ldots \Big\}, \end{aligned} \tag{5}$$

where the set of operators is ($q = u, c$):

$$\mathcal{O}_1^{(q)} = (\bar{d}_\alpha \gamma_\mu (1 - \gamma_5) q_\beta)\, (\bar{q}_\beta \gamma^\mu (1 - \gamma_5) b_\alpha), \tag{6}$$

$$\mathcal{O}_2^{(q)} = (\bar{d}_\alpha \gamma_\mu (1 - \gamma_5) q_\alpha)\, (\bar{q}_\beta \gamma^\mu (1 - \gamma_5) b_\beta), \tag{7}$$

$$\mathcal{O}_7 = \frac{e m_b}{8\pi^2}\, (\bar{d}_\alpha \sigma^{\mu\nu} (1 + \gamma_5) b_\alpha)\, F_{\mu\nu}, \tag{8}$$

$$\mathcal{O}_8 = \frac{g_s m_b}{8\pi^2}\, (\bar{d}_\alpha \sigma^{\mu\nu} (1 + \gamma_5) T^A_{\alpha\beta} b_\beta)\, G^A_{\mu\nu}. \tag{9}$$

The strong and electroweak four-quark penguin operators are assumed to be present in the effective Hamiltonian (5) and denoted by ellipses there.

They are not taken into account due to the small values of the corresponding Wilson coefficients.

The effective Hamiltonian (5) sandwiched between the B- and ρ-meson states can be expressed in terms of matrix elements of bilinear quark currents defining a heavy-light transition. These matrix elements are dominated by strong interactions at small momentum transfer and cannot be calculated perturbatively. The general decomposition of the matrix elements on all possible Lorentz structures admits seven scalar functions (form factors): $V^{(\rho)}$, $A_i^{(\rho)}$ $(i = 0, 1, 2)$, and $T_i^{(\rho)}$ $(i = 1, 2, 3)$ of the momentum squared $q^2 = (p_B - p_\rho)^2$ transferred from the heavy meson to the light one [7]:

$$\langle \rho(p_\rho, \varepsilon^*)|\bar{d}\,\gamma^\mu b|\bar{B}(p_B)\rangle = \frac{2i\,V^{(\rho)}(q^2)}{m_B + m_\rho}\,\mathrm{eps}(\mu, \varepsilon^*, p_\rho, p_B), \tag{10}$$

$$\begin{aligned}\langle \rho(p_\rho, \varepsilon^*)|\bar{d}\,\gamma^\mu\gamma_5 q_\nu b|\bar{B}(p_B)\rangle = {}& 2m_\rho\, A_0^{(\rho)}(q^2)\,\frac{(\varepsilon^* q)}{q^2}\, q^\mu \\ &+ A_1^{(\rho)}(q^2)\,(m_B + m_\rho)\left[\varepsilon^{*\mu} - \frac{(\varepsilon^* q)}{q^2}\, q^\mu\right] \\ &- A_2^{(\rho)}(q^2)\,\frac{(\varepsilon^* q)}{m_B + m_\rho}\,\left[(p_B + p_\rho)^\mu - \frac{(m_B^2 - m_\rho^2)}{q^2} q^\mu\right],\end{aligned} \tag{11}$$

$$\langle \rho(p_\rho, \varepsilon^*)|\bar{d}\,\sigma^{\mu\nu} q_\nu b|\bar{B}(p_B)\rangle = 2\,T_1^{(\rho)}(q^2)\,\mathrm{eps}(\mu, \varepsilon^*, p_\rho, p_B), \tag{12}$$

$$\begin{aligned}\langle \rho(p_\rho, \varepsilon^*)|\bar{d}\,\sigma^{\mu\nu}\gamma_5 q_\nu b|\bar{B}(p_B)\rangle = {}& \\ &-i\,T_2^{(\rho)}(q^2)\,[(m_B^2 - m_\rho^2)\,\varepsilon^{*\mu} - (\varepsilon^* q)\,(p_B + p_\rho)^\mu] \\ &-i\,T_3^{(\rho)}(q^2)\,(\varepsilon^* q)\,\left[q^\mu - \frac{q^2}{m_B^2 - m_\rho^2}\,(p_B + p_\rho)^\mu\right],\end{aligned} \tag{13}$$

where $\mathrm{eps}(\mu, \varepsilon^*, p_\rho, p_B) = \varepsilon^{\mu\nu\alpha\beta}\varepsilon^*_\nu p_{\rho\alpha} p_{B\beta}$. The heavy quark symmetry and the behavior of the final meson in the large energy limit (the large recoil limit) allow to reduce the number of independent form factors to two only: $\xi_\perp^{(\rho)}(q^2)$ and $\xi_\parallel^{(\rho)}(q^2)$. The $B \to \rho\gamma$ decay amplitude is proportional to one of them – $\xi_\perp^{(\rho)}(q^2)$, which is related to the form factors introduced above for the case $q^2 = 0$ as follows (terms of order m_ρ^2/m_B^2 are neglected):

$$\frac{m_B}{m_B + m_\rho}\, V^{(\rho)}(0) = \frac{m_B + m_\rho}{m_B}\, A_1^{(\rho)}(0) = T_1^{(\rho)}(0) = T_2^{(\rho)}(0) = \xi_\perp^{(\rho)}(0). \tag{14}$$

These relations among the form factors in the symmetry limit are broken by perturbative QCD radiative corrections arising from the vertex renormalization and the hard spectator interactions. To incorporate both types of QCD corrections, a tentative factorization formula for the heavy-light form factors at large recoil and at leading order in the inverse heavy meson mass was introduced [7]:

$$F_k^{(\rho)} = C_{\perp k} \xi_\perp^{(\rho)} + \Phi_B \otimes T_k \otimes \Phi_\rho, \tag{15}$$

where $F_k^{(\rho)}$ is any of the four form factors in the $B \to \rho$ transitions related by Eq. (14), $C_{\perp k} = C_{\perp k}^{(0)}[1 + O(\alpha_s)]$ are the renormalization coefficients, T_k is a hard-scattering kernel calculated in $O(\alpha_s)$, Φ_B and Φ_ρ are the light-cone distribution amplitudes of the B- and ρ-mesons convoluted with the kernel T_k.

In the leading order the electromagnetic penguin operator $\mathcal{O}_7$ contributes in the $B \to \rho\gamma$ decay amplitude at the tree level. Taking into account the definitions of the $B \to \rho$ transition form factors in the tensor (12) and the axial-tensor (13) currents and the symmetry relation $T_1^{(\rho)}(0) = T_2^{(\rho)}(0)$, the amplitude has the form:

$$\begin{aligned} M^{(0)} &= -\frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* \frac{e \bar{m}_b(\mu)}{4\pi^2} C_7^{(0)\mathrm{eff}}(\mu)\, T_1^{(\rho)}(0) \\ &\times \left[(Pq)(e^* \varepsilon^*) - (e^* P)(\varepsilon^* q) + i\, \mathrm{eps}(e^*, \varepsilon^*, P, q) \right], \end{aligned} \tag{16}$$

where $q = p_B - p_\rho$ and e^* are the photon four-momentum and polarization vector, respectively, and $P = p_B + p_\rho$.

The branching ratio can be easily obtained and results in the form:

$$\mathcal{B}_{\rm th}^{\rm LO}(B \to \rho\gamma) = \tau_B \, \frac{G_F^2 \alpha |V_{tb} V_{td}^*|^2 m_B^3}{32\pi^4} \left[1 - \frac{m_\rho^2}{m_B^2} \right]^3 \bar{m}_b^2(\mu)\, |C_7^{(0)\mathrm{eff}}(\mu)|^2\, |T_1^{(\rho)}(0)|^2. \tag{17}$$

It is natural to assume the μ-dependence of the form factor, $T_1^{(\rho)}(0,\mu)$, for compensating the dependence on the scale μ originated by the b-quark mass, $\bar{m}_b(\mu)$, and the Wilson coefficient, $C_7^{(0)\mathrm{eff}}(\mu)$, in the branching ratio.

The branching ratio of the $B \to K^*\gamma$ decays can be easily obtained from Eq. (17) by replacing: $V_{td} \to V_{ts}$, $m_\rho \to m_{K^*}$, and $T_1^{(\rho)}(0) \to T_1^{(K^*)}(0)$, which yield the ratio of branching ratios (4) as:

$$R_{\rm th}^{(0)}(\rho\gamma/K^*\gamma) = S_\rho \left| \frac{V_{td}}{V_{ts}} \right|^2 \left[\frac{m_B^2 - m_\rho^2}{m_B^2 - m_{K^*}^2} \right]^3 \left| \frac{T_1^{(\rho)}(0,\mu)}{T_1^{(K^*)}(0,\mu)} \right|^2, \tag{18}$$

where $S_\rho = 1$ for the charged ρ-meson and $S_\rho = 1/2$ for the neutral one.

There is also contribution from the annihilation diagrams to the $B \to \rho\gamma$ decay width. This additional contribution modifies the equation above as follows:

$$R_{\rm th}(\rho\gamma/K^*\gamma) = R_{\rm th}^{(0)}(\rho\gamma/K^*\gamma)\,[1 + \Delta R(\rho/K^*)]\,. \tag{19}$$

The radiation from quarks inside such a meson is compensated by the diagram with the photon emitted from the vertex (for a recent review of this topic see Ref. [8]). Only one annihilation diagram with the photon emitted from the spectator quark in the B-meson is numerically important and its strength can be parameterized by the dimensionless factor ε_A:

$$\Delta R(\rho/K^*) = \lambda_u\,\varepsilon_A, \tag{20}$$

$$\lambda_u = \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} = -\left|\frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}\right| {\rm e}^{i\alpha} = F_1 + iF_2, \tag{21}$$

where α is one of the angle from the unitarity triangle. In the neutral B-meson decays the parameter ε_A is numerically small and can be neglected at the accuracy accepted. For the charged B-meson decays the LCSR value $\varepsilon_A = 0.3 \pm 0.1$ [9] is used in the analysis.

There is also QCD corrections [of order $O(\alpha_s)$] which are called further as the next-to-leading order (NLO) ones. The total NLO correction to the $B \to \rho\gamma$ decay width consists of:

• <u>b-quark mass $\bar{m}_b(\mu)$.</u> In the modified minimal subtraction scheme at the renormalization scale μ it can be connected with the b-quark pole mass, $m_{b,{\rm pole}}$, by the relation:

$$\bar{m}_b(\mu) = m_{b,{\rm pole}} \left[1 + \frac{\alpha_s(\mu)}{4\pi}\, C_F \left(3 \ln \frac{m_{b,{\rm pole}}^2}{\mu^2} - 4\right)\right]. \tag{22}$$

• <u>Wilson coefficient $C_7^{\rm eff}(\mu)$.</u>

$$C_7^{\rm eff}(\mu) = C_7^{(0){\rm eff}}(\mu) + \frac{\alpha_s(\mu)}{4\pi}\, C_7^{(1){\rm eff}}(\mu). \tag{23}$$

The explicit expressions for the Wilson coefficient can be found in Ref. [10].

• <u>The factorizable NLO corrections to the form factors.</u> These corrections are described by the diagrams with the $\mathcal{O}_7$-operator (8). They can be divided into the vertex and hard-spectator corrections [7]:

$$T_1^{(\rho)}(0,\mu) = \xi_\perp^{(\rho)}(0) \left[1 + \frac{\alpha_s(\mu)}{4\pi}\, C_F \left(\ln \frac{m_{b,{\rm pole}}^2}{\mu^2} - 1\right) + \frac{\alpha_s(\mu_{\rm sp})}{4\pi}\, C_F\, \frac{\Delta F_\perp(\mu_{\rm sp})}{2\xi_\perp^{(\rho)}(0)}\right], \tag{24}$$

where $\xi_\perp^{(\rho)}(0)$ is the value of the $T_1^{(\rho)}(0)$ form factor in the HQET/LEET limit, $\mu_{\rm sp} = \sqrt{\mu\Lambda_{\rm H}}$ ($\Lambda_{\rm H} \simeq 0.5$ GeV) is the typical scale of the gluon virtuality in the hard-spectator corrections, and $\Delta F_\perp^{(\rho)}$ is the dimensionless quantity which defines the strength of the hard-spectator corrections:

$$\Delta F_\perp^{(\rho)}(\mu_{\rm sp}) = \frac{8\pi^2 f_B f_\perp^{(\rho)}(\mu_{\rm sp})}{3m_B\lambda_{B,+}} < \bar{u}^{-1} >_\perp^{(\rho)} (\mu_{\rm sp}) \simeq 1.64. \qquad (25)$$

The estimation was done on the scale of the spectator corrections $\mu_{\rm sp} = 1.52$ GeV [6].

• The nonfactorizable NLO corrections. They are also of two types: the vertex and the hard-spectator corrections. The nonfactorizable vertex corrections can be taken from inclusive $B \to X_d\gamma$ decay [11]. The nonfactorizable hard-spectator ones were recently calculated by several groups [6, 12, 13].

The total contribution to the form factor originated by the hard-spectator corrections is [6]:

$$\begin{aligned} \Delta_{\rm sp} T_1^{(\rho)}(0,\mu) \ \simeq\ & \frac{\alpha_s(\mu)}{4\pi}\, C_F\, \frac{\Delta F_\perp^{(\rho)}(\mu)}{2} \left[1 + \frac{C_8^{(0)\rm eff}(\mu)}{3C_7^{(0)\rm eff}(\mu)} \right. \\ & \left. + \ \frac{C_2^{(0)}(\mu)}{3C_7^{(0)\rm eff}(\mu)} \left(1 + \frac{V_{cd}^* V_{cb}}{V_{td}^* V_{tb}}\, \frac{h^{(\rho)}(z,\mu)}{\langle \bar{u}^{-1}\rangle_\perp^{(\rho)}(\mu)} \right) \right], \end{aligned} \qquad (26)$$

where $h^{(\rho)}(z,\mu)$ is the complex function of the quark mass ratio $z = m_c^2/m_b^2$ originated by the c-quark loop which analytic expression can be found in Ref. [6].

The NLO corrections discussed above modify the $B \to \rho\gamma$ branching ratio and the result for the charged-conjugate averaged branching ratio can be written in the form:

$$\begin{aligned} \bar{\mathcal{B}}_{\rm th}(B^\pm \to \rho^\pm\gamma) = \tau_{B^+}\, & \frac{G_F^2 \alpha |V_{tb}V_{td}^*|^2}{32\pi^4}\, m_{b,\rm pole}^2\, m_B^3 \left[1 - \frac{m_\rho^2}{m_B^2} \right]^3 \left[\xi_\perp^{(\rho)}(0) \right]^2 \qquad (27) \\ \times \Big\{ (C_7^{(0)\rm eff} + A_R^{(1)t})^2 & + (F_1^2 + F_2^2)\,(A_R^u + L_R^u)^2 + 2F_1\,[C_7^{(0)\rm eff}(A_R^u + L_R^u) + A_R^{(1)t} L_R^u] \Big\}, \end{aligned}$$

where $L_R^u = \epsilon_A\, C_7^{(0)\rm eff}$. The NLO amplitude $A^{(1)t}(\mu)$ of the decay presented here can be decomposed in three contributing parts [6]:

$$A^{(1)t}(\mu) = A_{C_7}^{(1)}(\mu) + A_{\rm ver}^{(1)}(\mu) + A_{\rm sp}^{(1)\rho}(\mu_{\rm sp}), \qquad (28)$$

where the correction due to the b-quark mass is included in the $A^{(1)}_{\rm ver}(\mu)$ part. The amplitude $A^{(1)K^*}(\mu)$ for the $B \to K^*\gamma$ decay can be written in a similar form and differs from $A^{(1)t}$ by the hard-spectator part $A^{(1)K^*}_{\rm sp}(\mu)$ only [6]. Note that the u-quark contribution $A^u(\mu)$ from the penguin diagrams, which also involves the contribution of hard-spectator corrections, can not be ignored in the $B \to \rho\gamma$ decay.

Using the presentation (27) of the branching ratio, the dynamical function $\Delta R(\rho/K^*)$, defined by Eq. (19), in the NLO and with the annihilation contribution taken into account can be written:

$$\Delta R(\rho/K^*) = \left[2\epsilon_A\, F_1 + \epsilon_A^2 (F_1^2 + F_2^2)\right] \left(1 - \frac{2A^{(1)K^*}}{C_7^{(0)\rm eff}}\right) - \frac{2A^{(1)K^*}}{C_7^{(0)\rm eff}} \qquad (29)$$
$$+ \frac{2}{C_7^{(0)\rm eff}}\, {\rm Re}\left[A^{(1)\rho}_{\rm sp} - A^{(1)K^*}_{\rm sp} + F_1 (A^u + \epsilon_A A^{(1)t}) + \epsilon_A (F_1^2 + F_2^2) A^u\right].$$

3 Phenomenology of $B \to \rho\gamma$ Decays

$B \to \rho\gamma$ **Branching Ratios.** For numerical predictions of the $B \to \rho\gamma$ branching ratios it is better to use ratio of the $B \to \rho\gamma$ and $B \to K^*\gamma$ decay widths (4) and then connect it with the experimentally measured values of $B \to K^*\gamma$ branching ratios (1).

To do this, let us start with the discussion of form factors. $SU_F(3)$-breaking effects in the QCD form factors $T_1^{(K^*)}(0)$ and $T_1^{(\rho)}(0)$ have been evaluated within the QCD sum-rules [9]. These can be taken to hold also for the ratio of the HQET form factors. Thus, we take $\zeta = \xi_\perp^{(\rho)}(0)/\xi_\perp^{(K^*)}(0) \simeq 0.76 \pm 0.06$. As it was pointed out in Ref. [14], the error here is not on ζ by itself, but rather on the deviation of ζ from its $SU_F(3)$-symmetry limit, i. e. $1 - \zeta$.

The main uncertainties in the dynamical functions $\Delta R(\rho/K^*)$ come from the uncertainties in the CKM angle α and the nonperturbative parameters $\xi_\perp^{(\rho)}(0)$ and $\xi_\perp^{(K^*)}(0)$. Taking into account various parametric uncertainties, it is found that the dynamical functions $\Delta R(\rho/K^*)$ are constrained in the range [6]:

$$\left|\Delta R(\rho^\pm/K^{*\pm})\right| \le 0.25, \qquad \left|\Delta R(\rho^0/K^{*0})\right| \le 0.13, \qquad (30)$$

with the central values $\Delta R(\rho^\pm/K^{*\pm}) \simeq \Delta R(\rho^0/K^{*0}) \simeq 0$. This quantifies the statement that the ratio $R_{\rm th}(\rho\gamma/K^*\gamma)$ is stable against $O(\alpha_s)$ and $1/m_B$-corrections.

Taking into account the ratio of the CKM matrix elements: $|V_{td}/V_{ts}| = 0.194 \pm 0.029$, the branching ratios can be estimated as [6]

$$\bar{\mathcal{B}}_{\rm th}(B^\pm \to \rho^\pm\gamma) = (0.90 \pm 0.33[\mathrm{th}] \pm 0.10[\mathrm{exp}]) \times 10^{-6},$$
$$\bar{\mathcal{B}}_{\rm th}(B^0 \to \rho^0\gamma) = (0.49 \pm 0.18[\mathrm{th}] \pm 0.04[\mathrm{exp}]) \times 10^{-6},$$

where the SM favoured range $77^\circ \leq \alpha \leq 113^\circ$ was used. In the above estimates, the first error is due to the uncertainties of the theory and the second is from the experimental data on the $B \to K^*\gamma$ branching ratios. The recent experimental upper limits on these decays by the BABAR collaboration (2) are approximately a factor three above the predicted ones. We expect that the BABAR and BELLE experiments will soon reach the SM sensitivity in these decays.

Isospin-Violating Ratios. The numerical analysis is presented for the charge-conjugate averaged of the isospin-violating ratios in the $B \to \rho\gamma$ decays:

$$\Delta = \frac{1}{4}\left[\frac{\Gamma(B^- \to \rho^-\gamma)}{\Gamma(\bar{B}^0 \to \rho^0\gamma)} + \frac{\Gamma(B^+ \to \rho^+\gamma)}{\Gamma(B^0 \to \rho^0\gamma)}\right] - 1. \tag{31}$$

The dependence on the unitarity triangle angle α is presented in Fig. 1. The charge-conjugate average Δ for the $B \to \rho\gamma$ decays is found to be likewise stable against the NLO and $1/m_B$-corrections [6]. In the expected range of the CKM parameters, this quantity is inside the interval $|\Delta| \leq 10\%$.

Direct CP-Asymmetry. The direct CP-asymmetry in the $B^\pm \to \rho^\pm\gamma$ decay rates is defined as follows:

$$\mathcal{A}_{\rm CP}(\rho^\pm\gamma) = \frac{\mathcal{B}(B^- \to \rho^-\gamma) - \mathcal{B}(B^+ \to \rho^+\gamma)}{\mathcal{B}(B^- \to \rho^-\gamma) + \mathcal{B}(B^+ \to \rho^+\gamma)}. \tag{32}$$

The CP-asymmetry $\mathcal{A}_{\rm CP}(\rho^\pm\gamma)$ receives contributions from the hard-spectator corrections which tend to decrease its value estimated from the vertex corrections alone. The dependencies of the CP-asymmetry on the angle α and on the quark mass ratio $\sqrt{z} = m_c/m_b$ are presented in Fig. 2. The Standard Model estimates show that the direct CP-asymmetry is definitely positive and for $0.2 \lesssim \sqrt{z} \lesssim 0.3$ is inside the interval: $5\% < \mathcal{A}_{\rm CP}(\rho^\pm\gamma) < 15\%$.

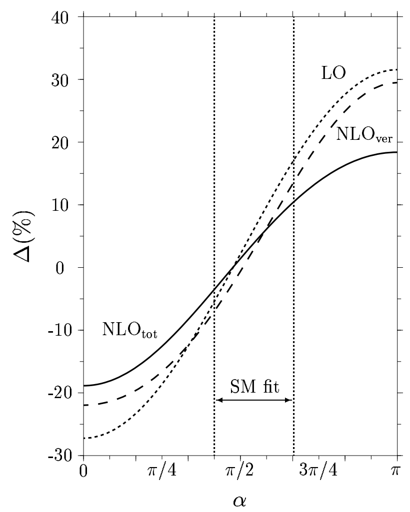

Figure 1: The charge-conjugate averaged ratio Δ for $B \to \rho\gamma$ decays as a function of the unitarity triangle angle α in the leading order (dotted curve), next-to-leading order without (dashed curve) and with (solid curve) hard-spectator corrections. The $\pm 1\sigma$ allowed band of α from the SM unitarity fits is also indicated.

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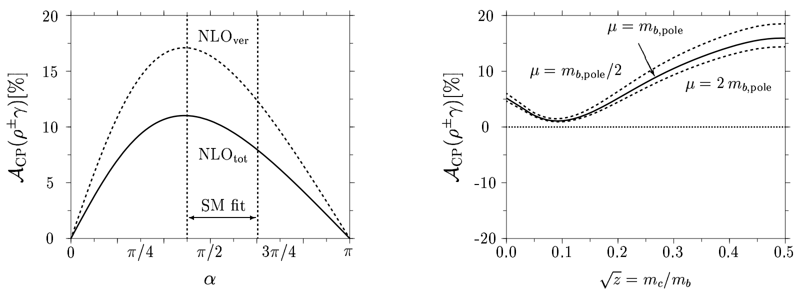

Figure 2: Left figure: Direct CP-asymmetry in the decays $B^\pm \to \rho^\pm\gamma$ as a function of the unitarity triangle angle α without (dotted curves) and with (solid curves) the hard-spectator corrections. The $\pm 1\sigma$ allowed band of α from the SM unitarity fits is also indicated. Right figure: Direct CP-asymmetry in the decays $B^\pm \to \rho^\pm\gamma$ as a function of the quark mass ratio $\sqrt{z} = m_c/m_b$; the scale dependence of the asymmetry is shown in the interval: $m_{b,\text{pole}}/2 \le \mu \le 2m_{b,\text{pole}}$.

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