Neutrino-electron processes in a dense plasma with the presence of an external magnetic field.

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Abstract

The neutrino-electron processes in a dense magnetized plasma are investigated under conditions of a supernova envelope. It is shown that under conditions considered the total set of neutrino-electron processes reduces to the neutrino-electron scattering with the conservation of the Landau level number. The probability and the neutrino integral action on plasma are calculated.

1 Introduction.

Neutrino physics under extreme conditions of density and/or temperature and in the presence of a strong magnetic field is a vigorously growing line of investigations. Dense plasma and magnetic field playing the role of an active medium can essentially modify the neutrino properties in comparison with a vacuum one. By this means, the studies of neutrino involving processes in medium has a fundamental value for theoretical aspect of elementary particles physics. On the other hand, the neutrino processes and the neutrino-electron processes in extremal active medium in particular, are of principle interest for astrophysics, where a dense matter, strong magnetic field and a large

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number of neutrino can take place simultaneously (a process of a coalescence of neutron star, an explosion of a supernova) [1].

In this study we investigate the neutrino – electron processes in a dense magnetized plasma. We consider the physical situation when plasma component dominates over the magnetic field as an "active medium", so the electron chemical potential, μ, is the dominating physical parameter: [1]

$$\mu^2 > 2eB \gg T^2, E^2 \gg m_e^2, \tag{1}$$

where T is the plasma temperature, E is the typical neutrino energy, m_e is the electron mass. Under the conditions (1) plasma electrons occupy the excited Landau levels. As one can see the electron mass is the smallest parameter of our task, so we will neglect its wherever this creates no complications. While the conditions (1) corresponds to the relatively weak magnetic field, the magnetic field strength is assumed as a large parameter enough, so that the following condition is satisfied:

$$eB \gg \mu E. \tag{2}$$

In the present astrophysical view, such extreme conditions (1), (2) could be realized, as an example, in a supernova envelope, where the electron chemical potential is assumed to be $\mu \sim 15$ MeV, plasma temperature $T \sim 3$ MeV. The magnetic field could be generated up to $10^{15} - 10^{16}$ G. [2–4].

It can be shown that the most part of the neutrino-electron processes is suppressed under conditions considered. Really, the processes with the positron in an initial states and relatively low energy electron in final state, ($\nu\bar{\nu} \leftrightarrow e^- e^+$, $\nu \leftrightarrow \nu e^- e^+$, $e^+\nu \rightarrow \nu e^+$, $e^+ \leftrightarrow e^+\bar{\nu}\nu$), are suppressed by the statistical factors in the degenerate plasma. As for the remained processes with the electron both in the initial and in the final states, ($e^- \leftrightarrow \nu\bar{\nu}e^-$, $\nu e^- \rightarrow \nu e^-$), one can show that the processes with the conservation Landau level number are only possible under conditions (1), (2). Actually, using the energy and momentum conservation low, one can obtain the following relation for the difference between the initial and final Landau level number:

$$n - n' = \frac{q_3^2 - q_0^2 + 2p_z q_z - 2q_0\varepsilon_n}{2eB},$$

where $\varepsilon_n \simeq \sqrt{p_z^2 + 2eBn}$ is the energy of an ultrarelativistic plasma electron occupying the n-th Landau level, p_z is the z - component of its momentum,

[1] We use natural units in which $c = \hbar = 1$, $e > 0$ is the elementary charge.

$q_\alpha = k_\alpha \pm k'_\alpha$, k_α and k'_α are the four-vectors of energy-momentum of the initial and final neutrino correspondingly. Plus in 4-vector q_α corresponds to the process of a synchrotron radiation (absorbtion) of neutrino pair and minus corresponds to the process of the neutrino-electron scattering.

With the possible astrophysical applications in mind, we consider that the the electron energy ε_n and the component of the momenta p_z are of the order of μ, while the components of four-vector q_α are of the order of neutrino temperature $T_\nu \sim T \ll \sqrt{eB}$. Under conditions (2) we have:

$$n - n' \sim O\left(\frac{\mu T}{eB}\right).$$

Consequently, if the magnetic field being relatively weak (1), is simultaneously strong enough (2), then $n = n'$. By this means, the processes of the synchrotron radiation and absorption of a neutrino pair are forbidden by the energy and momentum conservation. Thus from total set of processes involving neutrino, electrons and positrons the processes of the neutrino-electron scattering when both initial and final electrons occupy the same Landau level only can be possible under the conditions considered.

The numerical calculations of the differential cross-section of this process in the limit of a weak magnetic field ($eB < \mu E$) were performed previously [5]. The purpose of our work is to calculate analytically not only the probability of this process, but also the volume density of the neutrino energy and momentum losses under the conditions (1),(2).

2 Probability of the neutrino-electron scattering with the conservation of the Landau level number.

We start from the effective local Lagrangian of the neutrino – electron interaction in the framework of the Standard Model:

$$L_{eff} = \frac{G_F}{\sqrt{2}}[\overline{e}\gamma_\alpha(c_v - c_a\gamma_5)e]\, j^\alpha, \tag{3}$$

where $j^\alpha = [\overline{\nu}\gamma_\alpha(1-\gamma_5)\nu]$ is the current of massless left neutrinos, $c_v = \pm 1/2 + 2sin^2\theta_W$, $c_a = \pm 1/2$. Here upper signs correspond to the electron neutrino ($\nu = \nu_e$) when both Z and W boson exchange takes part in a

process. The lower signs correspond to μ and τ neutrino ($\nu = \nu_\mu, \nu_\tau$), when the Z boson exchange is only presented in the Lagrangian (3).

The amplitude of the neutrino-electron scattering process can be obtained immediately from the Lagrangian (3) by means of substitution of the solutions of the Dirac equation in a magnetic field (see, for example [6]).

After the summation over the spin states of both the initial and the final electrons which occupy the same excited Landau levels with number n, the S-matrix element squared is:

$$\sum_{s,s'} \mid S \mid^2 \simeq 32 G_F^2 (g_v^2 + g_a^2) \pi^3 \mathcal{T} eBn \frac{[2(k\tilde{\varphi}q)^2 + q_\parallel^2 (kk')]}{EE'\varepsilon_n \varepsilon'_n L_y L_z V^2 (-q_\parallel^2)} \delta^3(\,). \qquad (4)$$

Here $\mathcal{T}$ is the total interaction time, $\delta^3(\,) = \delta(\varepsilon_n - \varepsilon'_n + q_0)\delta(p_z - p'_z + q_z)\delta(p_y - p'_y + q_y)$ corresponds to the energy and momentum conservation law in the presence of an external magnetic field directed along the z axis in the gauge $A^\mu = (0, 0, Bx, 0)$, $k^\alpha = (E, \vec{k})$, $k'^\alpha = (E', \vec{k}')$ are the 4-vectors of energy-momentum of the initial and the final neutrino correspondingly, $q^\alpha = k^\alpha - k'^\alpha$, $q_\parallel^2 = q\tilde{\Lambda}q = q_0^2 - q_3^2$, $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ and $\tilde{\varphi}_{\alpha\beta} = \epsilon_{\alpha\beta\mu\nu}\varphi_{\mu\nu}/2$ are the dimensionless magnetic field tensor and the dual tensor, $\tilde{\Lambda}_{\alpha\beta} = (\tilde{\varphi}\tilde{\varphi})_{\alpha\beta} = \tilde{\varphi}_{\alpha\gamma}\tilde{\varphi}_{\gamma\beta}$, $V = L_x L_y L_z$ is the normalization volume.

The probability of the neutrino scattering on plasma electrons per unit time has a physical meaning only being integrated over both the final and the initial electron states as well:

$$W_{(\nu e^- \to \nu e^-)} = \sum_{n=0}^{n_{max}} \frac{1}{\mathcal{T}} \int \sum_{s,s'} \mid S \mid^2 \; dn_{e^-} \, dn'_{e^-} \, \frac{d^3k'}{(2\pi)^3} \, V \, (1 - f(E')). \qquad (5)$$

Here n_{max} corresponds to the maximal possible Landau level number, which is defined as the integer part of the ratio $\mu^2/2eB > 1$, $f(E')$ is a distribution function of final neutrinos, $f(E') = [e^{(E'-\mu_\nu)/T_\nu} + 1]^{-1}$, E' is the final neutrino energy, μ_ν and T_ν are the effective chemical potential and the spectral temperature of the neutrino gas correspondingly. In a general case the neutrino spectral temperature T_ν can differ from the plasma temperature T (we do not assume the equilibrium between neutrino gas and plasma). The number of plasma electrons in the presence of a magnetic field directed along the z axis is estimated by the following way:

$$dn_{e^-} = \frac{dp_y \, dp_z}{(2\pi)^2} \, L_y \, L_z \, f(\varepsilon_n), \quad f(\varepsilon_n) = [e^{(\varepsilon_n - \mu)/T} + 1]^{-1},$$

where p_y is the generalized momentum which defines the position of the center of a Gaussian packet along the x axis, $x_0 = -p_y/eB$. The result of our calculations of the probability for the neutrino-electron scattering can be presented in the following form:

$$\begin{aligned} W_{(\nu e^- \to \nu e^-)} &= \frac{G_F^2\,(c_v^2 + c_a^2)\,eB\,T^2\,E}{4\,\pi^3} \sum_{n=0}^{n_{max}} \frac{1}{z^2} \\ &\times \int_{-a}^{b} d\xi\,\Phi(\xi)\Big\{(1+z^2)(1+u^2) - 4uz + \\ &+ \frac{\xi}{zr\tau}(z^2-1)(z-u)\Big\} \quad + \quad (u \to -u), \end{aligned} \tag{6}$$

where $z = \sqrt{1 - 2eBn/\mu^2}$, $\Phi(\xi) = \xi[(e^{\xi}-1)(e^{\eta_\nu - r - \xi/\tau}+1)]^{-1}$, $a = r\tau z(1+u)/(1+z)$ and $b = r\tau z(1-u)/(1-z)$, $r = E/T_\nu$, $\tau = T_\nu/T$, $\eta_\nu = \mu_\nu/T_\nu$, $u = cos\theta$, θ is the angle between the initial neutrino momentum $\vec{k}$ and the magnetic field direction. As one can see, the probability (6) does not depend on the value of the magnetic field strength, but is not isotropic. The dependence on the angle θ manifests this anisotropy of the neutrino-electron process in the presence of a magnetic field.

In the limit of a very dense plasma ($\mu^2 \gg eB$), when a great number of Landau levels are occupied by plasma electrons, the summation over n can be replaced by integration over z:

$$\sum_{n=0}^{[\mu^2/2eB]} F(z) \simeq \frac{\mu^2}{eB} \int_0^1 F(z)\,z\,dz.$$

In this case the contribution from the lowest Landau levels turns out to be negligibly small, so the main contribution into the probability arises from the highest Landau levels.

3 Integral neutrino action on a magnetized plasma.

The general expression for the volume density of neutrino energy and momentum losses in a medium due to the neutrino scattering on plasma

electrons is following:

$$(\dot{\varepsilon}, \vec{\mathcal{F}}) = \frac{1}{(2\pi)^3} \int \frac{(q_0,\, \vec{q})\, d^3k}{e^{(E-\mu_\nu)/T_\nu} + 1}\, dW_{(\nu e^- \to \nu e^-)}. \tag{7}$$

The zeroth component, $\dot{\varepsilon}$, determines the neutrino energy loss in unit volume per unit time. In general, a neutrino propagating through plasma can lose energy as well as gain one. So, we will understand the "loss" of energy in the algebraic sense.

The vector $\vec{\mathcal{F}}$ in Eq.(7) is associated with the volume density of the neutrino momentum loss in unit time, and therefore it defines the neutrino force acting on plasma. Because of the isotropy of plasma without a magnetic field, in the presence of a magnetic field one would expect to obtain the neutrino force action along the magnetic field only. However, as it was shown above, the probability of the neutrino-electron scattering (6) is the symmetric functions with respect to the substitution $u \to -u$ (or $\theta \to \pi - \theta$). This means that the neutrino scattering on excited electrons does not give a contribution into the neutrino force acting on plasma along the magnetic field. Thus, under the conditions (1), (2) there is no neutrino force action on plasma at all. Therefore, this force is caused by a contribution of neutrino interactions with ground Landau level electrons only. The investigation of the neutrino-electron processes in a strong magnetic field limit, when electrons and positrons occupied the lowest Landau level was performed in [7]. In particular, in this paper it was shown that the neutrino force action on plasma along the magnetic field turns out to be of the same order and, what is essential, of the same sign as the one caused by the β-processes [8]. Because of this the result obtained in [7] has a more wide area of application. It can be used in the case of relatively weak magnetic field as well.

For the neutrino energy loss in unit volume per unit time in the limit of a very dense plasma we obtained the following result:

$$\dot{\varepsilon}_B = \frac{G_F^2(c_v^2 + c_a^2)}{\pi^3}\, \mu^2\, T^4\, n_\nu\, J_B(\tau), \tag{8}$$

$$\begin{aligned} J_B(\tau) \quad &= \quad \frac{\tau^4}{2} \int\limits_0^1 \frac{dz}{z^2} \int\limits_0^\infty dy\, y^2\, [y(1-z^2) + 4\, z\, (1+z^2)\,] \\ &\times \frac{1 - e^{y(1-\tau)}}{1 - e^{-y\tau}}\; e^{-y(1+z)/2z}, \end{aligned} \tag{9}$$

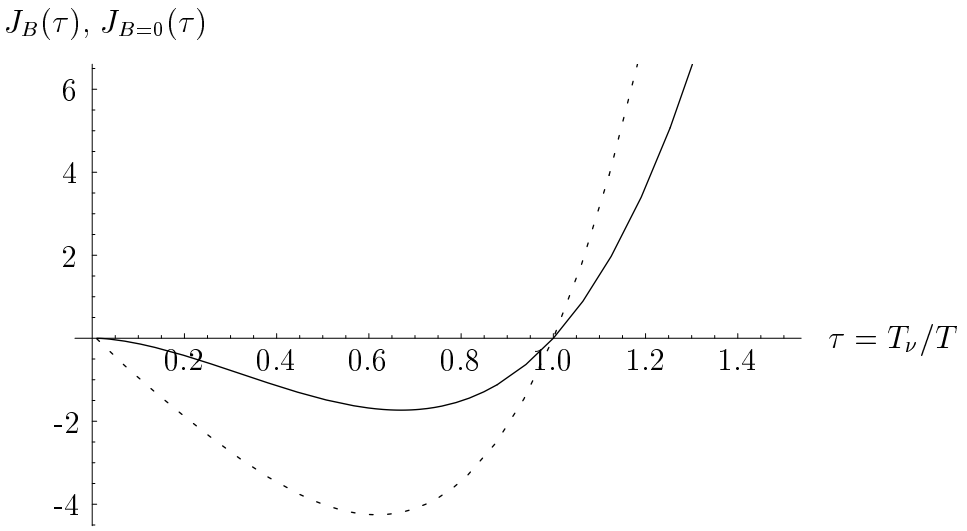

Figure 1: The functions $J_B(\tau)$ (solid line) and $J_{B=0}(\tau)$ (dashed line) versus the relative spectral neutrino temperature.

where n_ν is the concentration of initial neutrinos, the parameter τ has a meaning of a relative neutrino spectral temperature, $\tau = T_\nu/T$. It is interesting to compare this result with the one in a non-magnetized plasma which can be presented in a similar form:

$$\dot{\varepsilon}_{B=0} = \frac{G_F^2(c_v^2 + c_a^2)}{\pi^3}\,\mu^2\,T^4\,n_\nu\,J_{B=0}(\tau), \tag{10}$$

$$J_{B=0}(\tau) = 4\tau^4 \int\limits_0^\infty d\xi\,\xi^2\,\frac{e^{\xi(\tau-1)} - 1}{e^{\xi\tau} - 1}. \tag{11}$$

The functions $J_B(\tau)$ and $J_{B=0}(\tau)$ define the dependence of the neutrino energy losses on the relative neutrino spectral temperature in a magnetized plasma and in a plasma without field correspondingly.

The graphs of the functions (9) and (11) are presented in Fig.1. As one would expect, at neutrino spectral temperature smaller than the plasma one ($\tau < 1$) the values of the functions $J_B(\tau)$ and $J_{B=0}(\tau)$ are negative. It implies that neutrino propagating via medium captures energy from the plasma. At the T_ν more then T ($\tau > 1$), neutrino gives up energy to the plasma. At the point $\tau = 1$ there is a thermal equilibrium when there is no energy exchange between neutrino and electron-positron plasma. It can be

seen that the neutrino energy loss in a magnetized plasma is less than the one in non-magnetized plasma. By this means, under the conditions (1), (2) the magnetized plasma becomes more transparent for neutrinos than plasma without field.

4 Conclusions.

In this paper we have presented the results of our investigations of the neutrino-electron processes in a dense magnetized plasma. We have considered the physical situation, which could be realized in a supernova envelope, when from both components of the active medium, a magnetic field and plasma, the plasma component dominates. At the same time, a magnetic field was assumed to be not too small ($\mu^2 > 2eB \gg \mu E$). The probability and the volume density of the neutrino energy-momentum losses have been calculated. The main conclusions are following:

- If the magnetic field strength being relatively weak, $eB < \mu^2$, is simultaneously strong enough, $eB \gg \mu T$, the most part of the neutrino-electron processes are suppressed. The process of the neutrino-electron scattering when both initial and final electrons occupy the same Landau level number is only possible.

- The neutrino scattering on excited electrons does not give a contribution into the neutrino force acting on plasma. This force is caused by the neutrino-electron processes when plasma electrons occupy the lowest Landau level only. Thus the result for this force, obtained in paper [7] has a more general application. It may be used even in the limit of dense plasma when chemical potential is considerably greater than the magnetic field strength ($\mu^2 \gg eB$).

- The combine effect of plasma and strong magnetic field leads to a decrease of the neutrino energy loss in comparison to the one in a pure plasma. Therefore, the complex medium, plasma + strong magnetic field, is more transparent for neutrinos than non-magnetized plasma.

One would believe that the result obtained will be useful for the detail analysis of astrophysical cataclysms like supernova explosions.

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