Wave-function and CKM renormalization

D. Espriu*
Departament d'Estructura i Constituents de la Matèria
and
CER for Astrophysics, Particle Physics and Cosmology
Universitat de Barcelona, Diagonal 647, Barcelona E-08028 Spain

Abstract

In this presentation we clarify some aspects of the LSZ formalism and wave function renormalization for unstable particles in the presence of electroweak interactions when mixing and CP violation are considered. We also analyze the renormalization of the CKM mixing matrix which is closely related to wave function renormalization. The effects due to the electroweak radiative corrections that are described in this work are small, but they will need to be considered when the precision in the measurement of the charged current sector couplings reaches the 1% level. The work presented here is done in collaboration with Julian Manzano and Pere Talavera.

UB-ECM-PF 02/25
September 2002

*espriu@ecm.ub.es

1 Introduction

In this conference two recent determinations of the unitarity triangle angle β have been presented[1]. BaBar finds $\sin(2\beta) = 0.75 \pm 0.09 \pm 0.04$, while Belle measures $\sin(2\beta) = 0.82 \pm 0.12 \pm 0.05$. The first error is statistical and it is rapidly decreasing, while the second one is systematical and it will eventually limit the experimental determination of this fundamental quantity. It is hoped that after 2007, LHCb[2] will be able to reduce the overall uncertainty to less than 1% .

Given this state of affairs, it is clear that in a not too distant future a day will come when our experimental colleagues, when reporting on their high precision measurement of the Kobayashi-Maskawa matrix elements, will have to tell us in which renormalization scheme the corresponding mixing angles and CP violating phase(s) have been measured, exactly as they do so when reporting on the measured value of α_s.

It is essential to have the renormalization mechanism well under control to be able to separate the radiative corrections due to electroweak physics from those due to new physics. Most likely the latter —if they exist at all— are of the same size or even smaller than the former, exactly as a similar analysis in the neutral current sector has shown in recent years. Effective lagrangian techniques are important in this context[3].

In the neutral sector it is already totally mandatory to include electroweak radiative corrections to bring theory and experiment into agreement. Tree level results are incompatible with experiment by many standard deviations [4]. In a few years electroweak radiative corrections will be required in the studies analysing the "unitarity" of the CKM matrix too [1]. Corrections are of several types. With an on-shell[5] scheme in mind, we need counter terms for the electric charge, Weinberg angle and wave-function renormalization (wfr.) for the W gauge boson. We shall also require wfr. for the external fermions and counter terms for the entries of the CKM matrix. These are in fact related in a way that will be described below [6]. Finally one needs to compute the 1PI vertex parts of the different processes.

Several proposals have been put forward in the literature to define appropriate counter terms both for the external legs and for the CKM matrix elements. The original prescription for wfr. diagonalizing the on-shell prop-

[1]The CKM matrix is certainly unitary, but the physical observables that at tree level coincide with these matrix elements certainly do not necessarily fulfil a unitarity constraint once quantum corrections are switched on.See e.g. [3] for a discussion on this point

agator was introduced in [7]. In [8] the wfr. "satisfying" the conditions of [7] were derived. However since [8] does not take care about the branch cuts present in the self-energies those results must be considered only consistent up to absorptive terms. Later it was realized [9] that the on-shell conditions defined in [7] where inconsistent and in fact impossible to satisfy for a minimal set of renormalization constants due to the imaginary branch cuts present in the self-energies. The author of [9] circumvented this problem by introducing a prescription that *de facto* eliminates such branch cuts, but at the price of not diagonalizing the propagators in flavour space.

As we shall see later, some Ward identities based on the $\mathrm{SU}(2)_L$ gauge symmetry relate wfr. and counter terms for the CKM matrix elements [6]. So two of the necessary ingredients to renormalize the charged current vertices are actually related. In [10] it was seen that if the prescription of [8] was used in the counter terms for the CKM matrix elements, the results were in violation of gauge invariance. An additional condition for the gauge invariance of the physical amplitudes is that counter terms for the CKM matrix elements K_{ij} are by themselves gauge independent. This condition is fulfilled by the CKM counter term proposed in [10] as it is in minimal subtraction [6], [11].

Other proposals to handle charged vertex renormalization exist in the literature [11]. In all these works either the external wfr. proposed originally in [8, 9] are used, or the issue is sidestepped altogether. In either case the absorptive part of the self-energies are not taken into account. As we shall see doing so leads to S -matrix elements which are gauge dependent, and this irrespective of the method one uses to renormalize K_{ij} provided the redefinition of K_{ij} is gauge independent and preserves unitarity.

A more detailed account of this work is presented in [12]

2 $\mathbf{W}^+$ and top decay

We shall consider $W^+(q) \to f_i(p_1)\,\bar{f}_j(p_2)$ and $f_i(p_1) \to W^+(q)\,f_j(p_2)$. Latin indices correspond to families. For the first process there are at the one-loop level two different type of Lorentz structures that appear

$$\begin{aligned} M_L^{(1)} &= \bar{u}_i(p_1)\,\not{\varepsilon}(q)\,Lv_j(p_2)\ , \qquad (L \leftrightarrow R)\ , \\ M_L^{(2)} &= \bar{u}_i(p_1)\,Lv_j(p_2)\,p_1\cdot\varepsilon(q)\ , \qquad (L \leftrightarrow R)\ . \end{aligned} \tag{2.1}$$

and for the second one

$$\begin{aligned} M_L^{(1)} &= \bar{u}_j(p_2)\ \not{\varepsilon}^*(q)\, L u_i(p_1)\ , \qquad (L \leftrightarrow R)\ , \\ M_L^{(2)} &= \bar{u}_j(p_2)\, L u_i(p_1)\, p_1 \cdot \varepsilon^*(q)\ , \qquad (L \leftrightarrow R)\ . \end{aligned} \tag{2.2}$$

At tree level

$$\mathcal{M}_0 = -\frac{eK_{ij}}{2s_W} M_L^{(1)}\,, \tag{2.3}$$

where Eq. (2.1) is used for $M_L^{(1)}$ in W^+ decay and Eq. (2.2) instead for $M_L^{(1)}$ in t decay. The one-loop corrected transition amplitude can be written as

$$\mathcal{M}_1 = -\frac{e}{2s_W} M_L^{(1)} \left[K_{ij} \left(1 + \frac{\delta e}{e} - \frac{\delta s_W}{s_W} + \frac{1}{2}\delta Z_W \right) + \frac{1}{2}\left(\delta \bar{Z}_{ir}^{Lu} K_{rj} + K_{ir} \delta Z_{rj}^{Ld} \right) \right.$$
$$\left. + \delta K_{ij} \right] - \frac{e}{2s_W} \left(\delta F_L^{(1)} M_L^{(1)} + M_L^{(2)} \delta F_L^{(2)} + M_R^{(1)} \delta F_R^{(1)} + M_R^{(2)} \delta F_R^{(2)} \right). \tag{2.4}$$

In this expression $\delta F_{L,R}^{(1,2)}$ are the electroweak form factors from one-loop vertex diagrams. The renormalization constants for e, s_W and the wfr. of the gauge boson can be found in [5]. δK_{ij} and the fermion wfr. will be discussed next.

3 The Role of Ward Identities

There is a SU(2) Ward identity [10] that relates the CKM counterterms and wfr. constants. Let us see how the argument goes. In the weak basis, doublets renormalize with a common wfr. constant

$$\begin{pmatrix} u_0 \\ d_0 \end{pmatrix}_L = Z^{L\frac{1}{2}} \begin{pmatrix} u \\ d \end{pmatrix}_L . \tag{3.1}$$

On the other hand, in the mass diagonal basis there is no reason for up-type and down-type quarks to renormalize in the same way

$$\begin{pmatrix} \tilde{u}_0 \\ \tilde{d}_0 \end{pmatrix}_L = \begin{pmatrix} Z^{uL\frac{1}{2}} u \\ Z^{dL\frac{1}{2}} d \end{pmatrix}_L . \tag{3.2}$$

The passage from one basis to the other is accomplished with the help of unitary matrices V^0 and V for the up-type and down-type quarks, namely

$$\tilde{u} = V_u^\dagger u \qquad \tilde{u_0} = V_u^{0\dagger} u_0, \qquad \tilde{d} = V_d^\dagger u \qquad \tilde{d_0} = V_d^{0\dagger} d_0. \tag{3.3}$$

Elementary manipulations allow us to arrive at the following identity involving wfr. constants in the mass diagonal basis and the CKM matrix

$$Z^{uL\frac{1}{2}}K = K^0 Z^{dL\frac{1}{2}}, \tag{3.4}$$

and, writing $K^0 = K + \delta K$, we arrive at

$$\delta K_{jk} = \frac{1}{4}\left[\left(\delta\hat{Z}^{uL} - \delta\hat{Z}^{uL\dagger}\right)K - K\left(\delta\hat{Z}^{dL} - \delta\hat{Z}^{dL\dagger}\right)\right]_{jk}, \tag{3.5}$$

where we have changed notation and used $\hat{Z}$ for the wfr. constants appearing in the above expression. Indeed, they are not necessarily the same ones that have to be used to renormalize and guarantee the proper on-shell residue for the external legs and we anticipate that they will not. The reason is clear: the above wfr. constants $\hat{Z}$ are introduced so as to preserve the diagonal character of the mass matrix and they do not necessarily keep the kinetic terms diagonal on-shell.

From the above expressions it is also straightforward to derive the following Ward identity

$$(\hat{Z}^{uL}\hat{Z}^{uL\dagger})^{\frac{1}{2}}K = K(\hat{Z}^{dL}\hat{Z}^{dL\dagger})^{\frac{1}{2}} \tag{3.6}$$

Notice that in order to arrive to (3.5) we have used the fact that both K and K^0 are unitary matrices. It is perfectly possible, though perhaps a bit strange, to use a renormalized CKM matrix that is not unitary. If this is the case, the appropriate expression for the counterterm would simply be

$$\delta K_{jk} = \frac{1}{2}\left[\delta\hat{Z}^{uL}K - K\delta\hat{Z}^{dL}\right]_{jk}. \tag{3.7}$$

The previous Ward identity is certainly a necessary condition for the gauge invariance of the results, but it is not sufficient.

Any renormalization scheme that is manifestly gauge invariant and in addition mass independent, will obviously fulfill the above Ward identity automatically. This can be seen explicitly from the calculations presented in [6, 10] that use minimal subtraction and a mass independent scheme, respectively. However, the on-shell conditions to be imposed on the external legs are manifestly different for different quarks, since they have different masses. It is therefore impossible that the external leg wfr. obey the previous Ward identity and they cannot be used to define the CKM counterterms.

4 Renormalization of External Legs

We want to define an on-shell renormalization scheme that guarantees the correct properties of the fermionic propagator in the $p^2 \to m_i^2$. The conditions necessary for that purpose were first given by Aoki et. al. in [7]. We renormalize the bare fermion fields as $\Psi_0 = Z^{\frac{1}{2}}\Psi$ and $\bar{\Psi}_0 = \bar{\Psi}\bar{Z}^{\frac{1}{2}}$ For reasons that will become clear along the discussion, we shall allow Z and $\bar{Z}$ to be independent renormalization constants. Due to radiative corrections the propagator mixes fermion of different family indices. Namely

$$iS^{-1}(p) = \bar{Z}^{\frac{1}{2}}\Big(\not{p} - m - \delta m - \Sigma(p)\Big) Z^{\frac{1}{2}}\,, \tag{4.1}$$

where the bare self-energy Σ is non-diagonal and is given by $-i\Sigma = \sum 1\text{PI}$. Within one-loop accuracy we can write $Z^{\frac{1}{2}} = 1 + \frac{1}{2}\delta Z$ etc. Introducing the family indices explicitly we have

$$iS_{ij}^{-1}(p) = (\not{p} - m_i)\,\delta_{ij} - \hat{\Sigma}_{ij}(p)\ , \tag{4.2}$$

where the one-loop renormalized self-energy is given by

$$\hat{\Sigma}_{ij}(p) = \Sigma_{ij}(p) - \frac{1}{2}\delta\bar{Z}_{ij}(\not{p} - m_j) - \frac{1}{2}(\not{p} - m_i)\,\delta Z_{ij} + \delta m_i \delta_{ij}\,. \tag{4.3}$$

h The conditions [7] necessary to avoid mixing will be

$$\begin{aligned}
\hat{\Sigma}_{ij}(p)\,u_j^{(s)}(p) &= 0\,, \qquad (p^2 \to m_j^2)\,, \quad \text{(incoming particle)} && (4.4)\\
\bar{v}_i^{(s)}(-p)\,\hat{\Sigma}_{ij}(p) &= 0\,, \qquad (p^2 \to m_i^2)\,, \quad \text{(incoming anti}-\text{particle)} && (4.5)\\
\bar{u}_i^{(s)}(p)\,\hat{\Sigma}_{ij}(p) &= 0\,, \qquad (p^2 \to m_i^2)\,, \quad \text{(outgoing particle)} && (4.6)\\
\hat{\Sigma}_{ij}(p)\,v_j^{(s)}(-p) &= 0\,, \qquad (p^2 \to m_j^2)\,, \quad \text{(outgoing anti}-\text{particle)} && (4.7)
\end{aligned}$$

where no summation over repeated indices is assumed and $i \neq j$. These relations determine the non-diagonal parts of Z and $\bar{Z}$.

Here it is worth to make one important comment regarding the above conditions. First of all we note that in the literature the relation

$$\bar{Z}^{\frac{1}{2}} = \gamma^0 Z^{\frac{1}{2}\dagger}\gamma^0\,, \tag{4.8}$$

is taken for granted. This relation is tacitly assumed in [7] and explicitly required in [9]. Imposing Eq. (4.8) would guarantee hermiticity of the Lagrangian written in terms of the renormalized physical fields. However, we

are at this point concerned with external leg renormalization, for which it is perfectly possible to use a different set of renormalization constants. In fact, using two sets of renormalization constants is a standard practice in the on-shell scheme [5]. In case one is worried about the consistency of using a set of wfr. constants not satisfying (4.8) for the external legs while keeping a Hermitian Lagrangian, it should be pointed out that there is a complete equivalence between the set of renormalization constants we shall find out below and a treatment of the external legs where diagrams with self-energies (including mass counter terms) are inserted instead of the wfr. constants; provided that the mass counter term satisfy the on-shell condition. This gives results identical to ours and different from those obtained using the wfr. proposed in [9], which do fulfil (4.8).

In any case, self-energies develop absorptive terms and this makes Eq. (4.8) incompatible with the diagonalizing conditions (4.4)-(4.7). Therefore in order to circumvent this problem one can give up diagonalization conditions (4.4)-(4.7) or alternatively the hermiticity condition (4.8). The approach taken originally in [9] and works thereafter was the former alternative, while in this work we shall advocate the second one.

The approach of [9] consists in dropping out absorptive terms from conditions (4.4)-(4.7). Two severe problems arise if one follows this approach: (a) Since only the dispersive part of the self-energies enters into the diagonalizing conditions the on-shell propagator remains non-diagonal. (b) The on-shell scheme based in this prescription leads to gauge parameter dependent physical amplitudes. The reason for this unwanted dependence is the dropping of absorptive gauge parameter dependent terms in the self-energies that are necessary to cancel absorptive terms appearing in the vertices.

We shall now present the renormalization constants derived solely from the on-shell conditions (4.4)-(4.7) and allowing for $\bar{Z}^{\frac{1}{2}} \neq \gamma^0 Z^{\frac{1}{2}\dagger}\gamma^0$. In a rather obvious notation

$$\delta Z^L_{ij} = \frac{2}{m_j^2 - m_i^2}\left[\Sigma^{\gamma R}_{ij}\left(m_j^2\right) m_i m_j + \Sigma^{\gamma L}_{ij}\left(m_j^2\right) m_j^2 + m_i \Sigma^L_{ij}\left(m_j^2\right) + \Sigma^R_{ij}\left(m_j^2\right) m_j\right]$$

$$\delta Z^R_{ij} = \frac{2}{m_j^2 - m_i^2}\left[\Sigma^{\gamma L}_{ij}\left(m_j^2\right) m_i m_j + \Sigma^{\gamma R}_{ij}\left(m_j^2\right) m_j^2 + m_i \Sigma^R_{ij}\left(m_j^2\right) + \Sigma^L_{ij}\left(m_j^2\right) m_j\right]$$

$$\delta \bar{Z}^L_{ij} = \frac{2}{m_i^2 - m_j^2}\left[\Sigma^{\gamma R}_{ij}\left(m_i^2\right) m_i m_j + \Sigma^{\gamma L}_{ij}\left(m_i^2\right) m_i^2 + m_i \Sigma^L_{ij}\left(m_i^2\right) + \Sigma^R_{ij}\left(m_i^2\right) m_j\right]$$

$$\delta\bar{Z}_{ij}^{R}=\frac{2}{m_i^2-m_j^2}\left[\Sigma_{ij}^{\gamma L}\left(m_i^2\right)m_im_j+\Sigma_{ij}^{\gamma R}\left(m_i^2\right)m_i^2+m_i\Sigma_{ij}^{R}\left(m_i^2\right)+\Sigma_{ij}^{L}\left(m_i^2\right)m_j\right] \tag{4.9}$$

It is immediate to check that $\delta\bar{Z}_{ij}^{L}-\delta Z_{ij}^{L\dagger}\neq 0$. This non-vanishing difference is due to the presence of branch cuts in the self-energies that invalidate the pseudo-hermiticity relation $\Sigma_{ij}(p)\neq\gamma^0\Sigma_{ij}^{\dagger}(p)\gamma^0$. If we, temporally, ignore those branch cut contributions our results reduces to the ones depicted in [9] or [8]. In the SM these branch cuts are generically gauge dependent as a cursory look to the appropriate integrals shows at once. The proper consideration of the branch cuts is absolutely essential.

Once the off-diagonal wfr. are obtained we focus our attention in the diagonal sector. Here, even after using the on-shell conditions some freedom remains. We obtain

$$\delta m_i=-\frac{1}{2}Re\left\{m_i\Sigma_{ii}^{\gamma L}\left(m_i^2\right)+m_i\Sigma_{ii}^{\gamma R}+\Sigma_{ii}^{L}\left(m_i^2\right)+\Sigma_{ii}^{R}\left(m_i^2\right)\right\}. \tag{4.10}$$

This condition defines a mass and a width that agrees at the one-loop level with the ones given in [13]. Finally,

$$\begin{aligned}
\delta\bar{Z}_{ii}^{L} &= \Sigma_{ii}^{\gamma L}\left(m_i^2\right)-X-\frac{\alpha_i}{2}+D\,,\\
\delta\bar{Z}_{ii}^{R} &= \Sigma_{ii}^{\gamma R}\left(m_i^2\right)+X-\frac{\alpha_i}{2}+D\,,\\
\delta Z_{ii}^{L} &= \Sigma_{ii}^{\gamma L}\left(m_i^2\right)+X+\frac{\alpha_i}{2}+D\,,\\
\delta Z_{ii}^{R} &= \Sigma_{ii}^{\gamma R}\left(m_i^2\right)-X+\frac{\alpha_i}{2}+D\,,
\end{aligned} \tag{4.11}$$

$$X = \frac{1}{2}\frac{\Sigma_{ii}^{R}\left(m_i^2\right)-\Sigma_{ii}^{L}\left(m_i^2\right)}{m_i}\,, \tag{4.12}$$

$$D = m_i^2\left(\Sigma_{ii}^{\gamma L\prime}\left(m_i^2\right)+\Sigma_{ii}^{\gamma R\prime}\left(m_i^2\right)\right)+m_i\left(\Sigma_{ii}^{L\prime}\left(m_i^2\right)+\Sigma_{ii}^{R\prime}\left(m_i^2\right)\right) \tag{4.13}$$

Note that since $X=0$ at the one-loop level and choosing $\alpha_i=0$ we obtain $\delta\bar{Z}_{ii}^{L}=\delta Z_{ii}^{L}$ and $\delta\bar{Z}_{ii}^{R}=\delta Z_{ii}^{R}$. However we have the freedom to choose $\alpha_i\neq 0$. This does not affect the mass terms or neutral current couplings, but changes the charged coupling currents by multiplying the CKM matrix K by diagonal matrices. These redefinitions do not change the physical observables provided the α_i are pure imaginary numbers. This ambiguity corresponds in perturbation theory to the well know freedom in phase redefinitions of the CKM matrix. Except for this last freedom, the on-shell conditions determine one unique solution, the one presented here, with $\bar{Z}^{\frac{1}{2}}\neq\gamma^0 Z^{\frac{1}{2}\dagger}\gamma^0$.

5 Gauge Invariance

Let us briefly recapitulate where we stand at this point. In section 3 the relation (3.5) relating the CKM counterterm to a set of wfr. renormalization constants that we denoted by $\hat{Z}$ was discussed. This set of renormalization constants has to fulfill the Ward identity (3.6). The diagonal character of the mass matrix is preserved along the renormalization process for any value of the momenta when these wfr. constants are used.

On the other hand, in the previous section another set of wfr. constants has been introduced by requiring the diagonalization of the whole kinetic term at specific values of the momenta (on-shell) as well as the corresponding unit residue condition.

It is important to realize that both set of wfr. constants *do not coincide*. However, the poles are identical in both set of wfr. constants, as they correspond, in fact, to different choices of schemes that have to render the same set of Green functions finite. In the case of the external leg wfr. we claim that we have no choice if we are to implement the proper LSZ conditions. In fact, the prescription in [9] does not achieve the diagonalization of the absortive parts of the self-energies. On the other hand, for the CKM counterterms one does have a choice; one can use for example minimal subtraction (or variations thereof), subtraction at a given q^2 or whatever other method yields mass independent renormalization conditions. They will simply give different values of the renormalized entries of the CKM mixing matrix.

How can be sure that our procedure for the renormalization of the external legs is indeed correct? As we have seen in section 3, gauge invariance is an issue when dealing with CKM renormalization. Let us therefore use gauge invariance as an additional check. We will then discover that the presciption proposed here is the only one that provides gauge invariant amplitudes

Let us go back to eq. (2.4). We know [14] that the combination $\frac{\delta e}{e} - \frac{\delta s_W}{s_W}$ is gauge parameter independent. All the other vertex functions and renormalization constants are gauge dependent. We want the amplitude (2.4) to be exactly gauge independent —not just its modulus— so the gauge dependence must cancel between all the remaining terms.

To determine the gauge dependence of the different self-energies appearing in the external leg counterterms and the vertex 1PI Green function we shall appeal to the so-called Nielsen identitie[15, 16]. We urge the interested reader to check [12] for details. The outcome of the analysis is that three of the form factors appearing in the vertex (2.4) are by themselves gauge

independent, namely $\partial_\xi \delta F_L^{(2)} = \partial_\xi \delta F_R^{(1)} = \partial_\xi \delta F_R^{(2)} = 0$. ξ is the gauge-fixing parameter. We shall also see that the gauge dependence in the remaining form factor $\delta F_L^{(1)}$ cancels exactly with the one contained in δZ_W and in δZ and $\delta \bar{Z}$. Indeed the Nielsen identities lead to

$$\partial_\xi \left(M_L^{(1)} \delta F_L^{(1)} \right) = -M_L^{(1)} \partial_\xi \left(\delta \bar{Z}_{ir}^{uL} K_{rj} + K_{ir} \delta Z_{rj}^{dL} + \delta Z_W K_{ij} \right) \tag{5.1}$$

where Eq. (2.4) and the gauge independence of the electric charge and Weinberg angle has been used in the last equality. Note that Eq. (5.1) states that the gauge dependence of the on-shell bare one-loop vertex function cancels out the renormalization counter terms appearing in Eq. (2.4). This is one of the crucial results and special care should be taken not to ignore any of the absorptive parts —including those in the wfr. constants. As a consequence

$$\partial_\xi \mathcal{M}_1 = -\frac{e}{2 s_W} M_L^{(1)} \partial_\xi \delta K_{ij} \,,$$

so the counterterm for K_{ij} must be separately gauge independent, as originally derived in [10]. If one uses, for instance, the on-shell prescription wfr. constants to determine the CKM counterterms in eq. (3.5), the latter will be gauge dependent and so will be the amplitude, which is unacceptable. On the other hand, minimal subtraction for instance is fine.

The difficulties related to a proper definition of δK were first pointed out in [10, 15], where it was realized that using the on-shell Z's of [8] in Eq. (3.5) led to a gauge dependent K and amplitude. They suggested a modification of the on-shell scheme based on a subtraction at $p^2 = 0$ for all flavours that ensured gauge independence. We want to stress that the choice for δK is not unique and different choices may differ by gauge independent finite parts [17].

However, this is only half the story. Assuming that a gauge independent scheme has been used to define the $\hat{Z}$ and, accordingly, the CKM counterterms δK_{ij}, the Nielsen identities ensure the gauge independence of the amplitude if and only if the set of wfr. constants derived in section 4 are used for the external legs. If instead of using our prescription for δZ and $\delta \bar{Z}$ one makes use of the wfr. constants of [9] to renormalize the external fermion legs, it turns out that the gauge cancellation dictated by the Nielsen identities does not actually take place in the amplitude. The culprit is of course the neglect of the absorptive parts. Notice, that the vertex contribution has gauge dependent absorptive parts and they remain in the final result.

One might think of absorbing these additional terms in the counter term for δK. This does not quite work. Indeed one can see from explicit calculations that the 'left-over' absorptive parts would lead to a non-unitary CKM matrix[12].

It turns out that in the SM these gauge dependent absorptive parts, leading to a gauge dependent amplitude if they are dropped, do actually cancel, at least at the one-loop level, in the modulus of the S-matrix. However, in [12] it is shown that gauge independent absorptive parts do survive even in the modulus of the amplitude for top or anti-top decay (and only in these cases).

Acknowledgements

This work was triggered by many fruitful discussions with B. A. Kniehl. The work of D.E. and P.T. is supported in part by TMR, EC–Contract No. ERBFMRX–CT980169 (EURODAPHNE) and contracts MCyT FPA2001-3598 and CIRIT 2001SGR-00065. It is a true pleasure to thank V.Rubakov and all the organizing committe of the Quark 2002 Workshop for having arranged such a superb meeting.

References

[1] See the contributions by J.Chauveau and B.Schwartz in these proceedings.

[2] S. Amato *et al.* [LHCb Collaboration], CERN-LHCC-98-4.

[3] D. Espriu and J. Manzano, Phys. Rev. D **63**, 073008 (2001) [arXiv:hep-ph/0011036].

[4] A. Pich, arXiv:hep-ph/9711279.

[5] W.F.L. Hollik, Fortsch. Phys. 38 (1990) 165-260, Fortsch. Phys. **34** (1986) 687-751.

[6] C. Balzereit, T. Mannel and B. Plumper, Eur. Phys. J. C **9**, 197 (1999) [arXiv:hep-ph/9810350].

[7] K.I. Aoki, Z. Hioki, M. Konuma, R. Kawabe, T. Muta, Prog. Theor. Phys. Suppl. **73** (1982) 1-225.

[8] A. Denner and T. Sack, Nucl. Phys. B **347** (1990) 203.

[9] A. Denner, Fortsch. Phys. **41** (1993) 4, 307-420.

[10] P. Gambino, P.A. Grassi, F. Madricardo, Phys. Lett. B **454** (1999) 98-104.

[11] K. P. Diener and B. A. Kniehl, Nucl. Phys. B **617**, 291 (2001) [arXiv:hep-ph/0109110]; A. Barroso, L. Brucher and R. Santos, Phys. Rev. D **62** (2000) 096003 [arXiv:hep-ph/0004136]; Y. Yamada, Phys. Rev. D **64** (2001) 036008].

[12] D.Espriu, J.Manzano and P.Talavera, to appear in Phys. Rev. D [arXiv:hep-ph/0204085]

[13] A. Sirlin, Phys. Rev. Lett. **67** (1991) 2127; A. Sirlin, arXiv:hep-ph/9811467; P. A. Grassi, B. A. Kniehl and A. Sirlin, Phys. Rev. Lett. **86** (2001) 389 [arXiv:hep-th/0005149]; P. A. Grassi, B. A. Kniehl and A. Sirlin, arXiv:hep-ph/0109228.

[14] W. J. Marciano and A. Sirlin, Phys. Rev. D **22** (1980) 2695 [Erratum-ibid. D **31** (1980) 213].

[15] P. Gambino, P. A. Grassi, Phys. Rev. D **62** (2000) 076002.

[16] N. K. Nielsen, Nucl. Phys. B **101**, (1975) 173; O. Piguet, K. Sibold, Nucl. Phys. B **253** (1985) 517-540; E. Kraus, K. Sibold, hep-th/9406115.

[17] B. A. Kniehl, F. Madricardo, M. Steinhauser, Phys. Rev. D **62** (2000) 073010.