Double-logarithmic (Sudakov) asymptotics in the theory of electroweak interactions *

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September 27, 2002

Abstract

Accounting for double-logarithmic contributions to high-energy ($\gg$ 100 GeV) e^+e^- annihilation into a quark or a lepton pair in the kinematics where the final particles are collinear to the e^+e^- beams leads to a sizable difference between the forward and backward scattering amplitudes, i.e. to a forward-backward asymmetry. When the annihilation is accompanied by emission of n electroweak bosons in the multi-Regge kinematics, it turns out that the cross sections for photon and Z production have the identical energy dependence and asymptotically their ratio depends on the Weinberg angle only (more explicitly it is equal to $\tan^{2n}\theta_W$) whereas the energy dependence of the cross section of the W production is suppressed by factor $s^{-0.4}$ compared to them.

1 Introduction

The double-logarithmic approximation (DLA) was introduced into the particle physics by V.V. Sudakov who first found that the most important ra-

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diative corrections to the form factor $f(q^2)$ of electron at large q^2 are the double-logarithmic (DL) ones, i.e. $\sim (\alpha \ln^2(q^2/m^2))^n \quad (n = 1, 2, ..)$. with m being a mass scale,. After accounting them to all orders in α, it turns out[1] that asymptotically

$$f(q^2) \sim e^{-(\alpha/4\pi)\ln^2(q^2/m)} \tag{1}$$

when $q^2 \gg m^2$. The next important step towards studying DL asymptotics in QED was done in Refs. [2]. After that, calculations in DLA have become a technology rather than an art. The study of QCD scattering amplitudes has shown that there is no big technical difference between QED and QCD for calculating amplitudes of elastic processes (see e.g. Ref. [3]) whereas inelastic (radiative) QCD -amplitudes are much harder to calculate (concerning the Sudakov logarithms in QCD see e.g. Refs. [4]). The methods of calculating the DL asymptotics can be applied also to electroweak (EW) processes provided the total energy is high enough to neglect the masses of the electroweak bosons. At such huge energies ($\gg$ 100 GeV), many important technical details learnt from QED and QCD can be used for calculating EW amplitudes[5]. In the present talk I will discuss DL asymptotics for e^+e^- annihilation into a quark-anitquark or a lepton-antilepton pair [6] and [7].

2 DL contributions to elastic e^+e^- -annihilation into quarks and leptons

The conventional way for considering e^+e^- -annihilation into $\to q\bar{q}$ consists of two steps: the first one is the assumption that this process is mediated by a single virtual photon exchange: $e^+e^- \to \gamma^* \to q\bar{q}$; the second step is calculating QCD radiative corrections. However, electroweak radiative corrections can also be sizable when this process is considered in some particular kinematic regions. These are the forward and backward kinematics. The forward kinematics is defined when the scattering angle[1] between the momenta of the initial electron (positron) and of the final particles with the negative (positive) electric charges is $\ll 1$. The case when this angle is $\sim \pi$ gives the backward kinematics. Both kinematics are of the Regge type and the effect of accounting for the DL radiative corrections to all orders in the

[1]Through this paper when we refer to angles, we imply the angles in cmf.

electroweak couplings can be interpreted as exchanges of Reggeons propagating in the cross channel. This means that the expression for the forward and backward scattering amplitudes can be represented in the form of the Sommerfeld-Wotson (SW) integral:

$$M_j^{(\pm)}(s/\mu^2) = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} \left(\frac{s}{\mu^2}\right)^{\omega} \xi^{(\pm)}(\omega)\, F_j^{(\pm)}(\omega) \tag{2}$$

where the signs $\pm$ refer to the signatures of the amplitudes, the signature factors $\xi^{(+)} \approx 1$, $\xi^{(-)} \approx \imath\pi\omega/2$; $F_j^{(\pm)}(\omega)$ is called the SW amplitude (the partial wave); the Mandelstam variable s and the mass scale μ obey $\sqrt{s} \gg \mu \geq 100$GeV. The integration contour in Eq. (2) runs to the right of all singularities of $F_j^{(\pm)}(\omega)$. Subscript j enumerates both the flavours of the produced quarks and the EW isospin state in the cross channel. For example, for the forward $e_R^+(p_2)e_L^-(p_1)$ annihilation into $u_L(p_2')\bar{u}_R(p_1')$ one gets

$$\begin{aligned} M^{(+)}(\rho,\eta) &= a \exp\left[-\frac{1}{8\pi^2}\left(\frac{3}{2}g^2 + \frac{Y_e^2+Y_q^2}{4}g'^2\right)\frac{\eta^2}{2}\right] \\ &\quad \times \int_{-\imath\infty}^{\imath\infty} \frac{dl}{2\pi\imath}\, e^{\lambda l(\rho-\eta)} \frac{D_{p-1}(l+\lambda\eta)}{D_p(l+\lambda\eta)} \end{aligned} \tag{3}$$

where g and g' are the SU(2) and U(1) couplings, Y_e (Y_q) is the hypercharge of electron (quark), $a = (-3g^2 + g'^2 Y_e Y_q)/4$, $\lambda = -g'(Y_e + Y_q)/2$, $p = -a/\lambda^2$, $\rho = \ln(s/\mu^2)$, $\eta = \ln(-t/\mu^2)$ and D_p are the Parabolic cylinder functions. Eq. (3) is obtained for the kinematic region $s = (p_1+p_2)^2 \gg -t = -(p_2 - p_1')^2$.

The exponent in Eq. (3) is the electroweak Sudakov form factor for this process. It accumulates the softest radiative DL corrections, with virtualities of the virtual EW bosons $\leq -t$. The harder DL contributions are collected in the SW integral. The singularities of the integrand in Eq. (3) are the zeros of D_p. Forward scattering amplitudes for e^+e^- annihilation into quarks of other chiralities and flavours are represented by similar expressions. The only difference is in the values of factors a, p, λ. The same is true for the backward scattering amplitudes.

If $F_j^{(\pm)}(\omega)$ are singular when $\omega = \Delta_{j_r}^{(\pm)}$, $(r = 1, ...)$, then the asymptotic dependence of $M_j^{(\pm)}$ on s is

$$M_j^{(\pm)}(s/\mu^2) \sim \sum_r (s/\mu^2)^{\Delta_{jr}^{(\pm)}} \tag{4}$$

and $\Delta_{j_r}^{(\pm)}$ are the intercepts of the Reggeons. Ref. [6] states that the value of the intercepts depends also on the flavours and chiralities of the final quarks. It turns out[6] that all intercepts for the backward amplitudes are negative whereas a part of intercepts for the forward amplitudes is positive. Therefore, the backward amplitudes rapidly fall when s increases whereas the forward amplitudes slowly grow with s. This result can be interpreted as a forward-backward charge asymmetry. In particular, the largest intercepts of the forward positive signature amplitudes M_j (we drop the superscript "+") are $\Delta_u = 0.11$ for $e_L^- e_R^+ \to u_L \bar{u}_R$ and $\Delta_d = 0.08$ for $e_L^- e_R^+ \to d_L \bar{d}_R$. The other intercepts are smaller. The asymmetry factor A is defined in terms of the forward and backward cross sections $d\sigma_{F,B}$ of detecting the quarks in the forward (backward) cones with very small opening angles $\theta < M_Z/\sqrt{s}$:

$$A = [d\sigma_F - d\sigma_B]/[d\sigma_F + d\sigma_B] \tag{5}$$

where $d\sigma_{F(B)}$ stands for forward (backward) differential cross section. Performing numerical calculations, we arrive at the result plotted in Fig. 1. The difference between the forward and backward scattering amplitudes leads also to the fact that the average electric charge of the produced hadrons in the cone around of the e^- -beam (e^+ -beam) is negative (positive) and the value of the average charge grows with energy as shown in Fig. 2. It is possible to apply the plots of Figs. 1 and 2, to the situation when the produced quarks are in a wider angular region $1 \ll \theta < M_Z/\sqrt{s}$. To this end one should replace $\sqrt{s}$ in these Figs. by M_Z/θ.

3 Inelastic e^+e^- -annihilation into quarks

When e^+e^- annihilate into $q\bar{q}$ and electroweak bosons, with the final particles produced in the multi-Regge kinematics, there also appear DL electroweak corrections. The essence of the multi-Regge kinematics is that the longitudinal momenta of the produced particles are much greater than their transverse momenta. On the other hand, the transverse momenta $k_{i\perp}$ are assumed to be much greater than $M_{W,Z}$ so that all emission angles are $\ll 1$. With this assumption, the spontaneous broken $SU(2)U(1)$ symmetry in many respects can be regarded as restored. In particular, it becomes more convenient to consider emission of the isoscalar A_0 and isovector $A_{1,2,3}$ gauge fields and

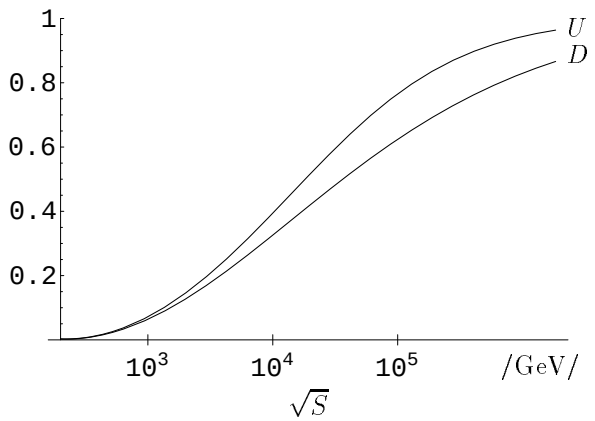

Figure 1: Asymmetry A for e^+e^- annihilation into quarks of different flavours in the "collinear angular region" in DLA.

then to proceed to the γ, W, Z emission, using the standard relations between these two sets. Also it makes possible to use arguments of Refs. [8] where the multi-Regge amplitudes for gluon production were calculated. It turns out[7] that amplitudes for the γ and Z production are governed by both the isoscalar and isovector Reggeons (with the intercepts 0.11 and 0.08) propagating in the cross channels, whereas the W production is controlled by the isovector Reggeons only, with the smaller (-0.08 and -0.27) intercepts. It means that the cross sections of the photon and the Z production have identical energy dependence. The only difference between them is due to the different EW couplings, so that asymptotically (at energies $\sqrt{s} \geq 10^6$ Gev)

$$\sigma^{nZ}(s)/\sigma^{n\gamma}(s) = \tan^{2n}\theta_W \tag{6}$$

whereas

$$\sigma^{nW}(s)/\sigma^{n\gamma}(s) \sim s^{-0.4} \ . \tag{7}$$

The results of the numerical calculations for these cross sections in the case of single boson production, covering the energy range from 10^3 to 10^7 GeV, are finally shown in Figs. 3 and 4.

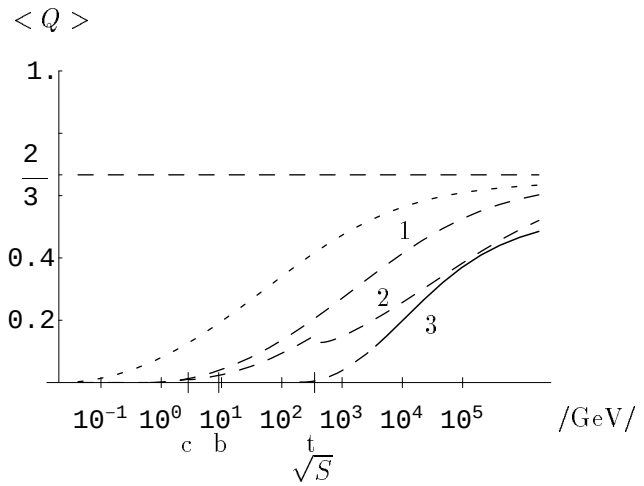

Figure 2: Average electric charge $< Q >$ of the hadron flow detected inside a narrow cone $\theta < M/\sqrt{s}$ in the direction of e^+ -beam. Short-dashed curve corresponds to the case of multiphoton annihilation to u, d -current quarks with $M \equiv \mu = 0.01$ Gev in QED. Dashed curve 1 corresponds to u, d -constituent quarks with $M \equiv \mu = 0.3$ GeV also in QED. Curves 2 and 3 account for the all quark flavours produced in e^+e^- -annihilation: the curve 2 is calculated in QED while the curve 3 corresponds to all EW -bosons exchanged in DLA with $M = M_Z$. Curve 2 shows how $< Q >$ would rise without account of EW interactions. The dashed part of the curve 3 corresponds to the region where subleading corrections to DLA could be important. The dashed horizontal line shows the asymptotic value of $< Q >$ as the u -quark contribution is dominating.

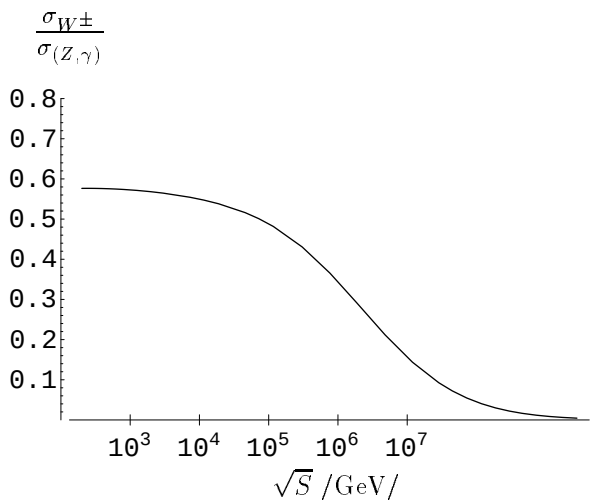

Figure 3: Total energy dependence of $W^\pm$ to (Z, γ) rate in e^+e^- annihilation.

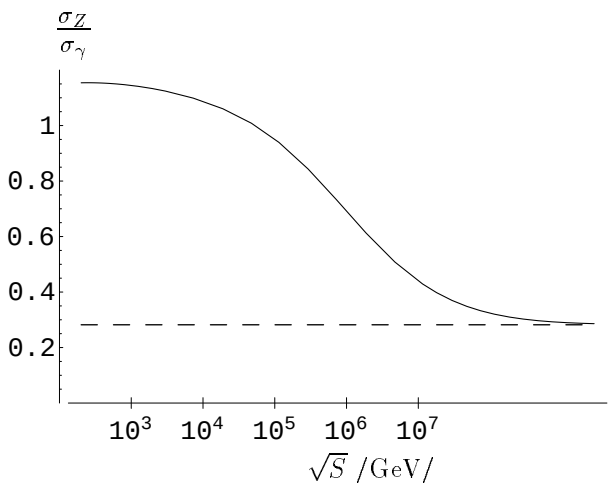

Figure 4: Total energy dependence of Z to γ rate in e^+e^- annihilation. The dashed line shows the asymptotical value of the ratio: $\tan^2\theta_W \approx 0.28$.

4 Acknowledgement

The work is supported by grants CERN/FIS/43652/2001, INTAS-97-30494, SFRH/BD/6455/2001 and RFBR 00-15-96610.

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