Double-logarithmic (Sudakov) asymptotics in the theory of electroweak interactions *

B.I. Ermol ev[ab], M. Greco[c], S.M. Oliveir [b] nd S.I. Troy n[d]

[a]Ioffe Physico-Technical Institute, 194021 St.Petersburg, Russia

[b]CFTC, University of Lisbon Av. Prof. Gama Pinto 2, P-1649-003 Lisbon, Portugal

[c]University Rome III, Rome, Italy

[d]St.Petersburg Institute of Nuclear Physics, 188300 Gatchina, Russia

September 27, 2002

Abstract

Accounting for double-logarithmic contributions to high-energy ($\gg$ 100 GeV) e^+e^- annihilation into a quark or a lepton pair in the kinematics where the final particles are collinear to the e^+e^- beams leads to a sizable difference between the forward and backward scattering amplitudes, i.e. to a forward-backward asymmetry. When the annihilation is accompanied by emission of n electroweak bosons in the multi-Regge kinematics, it turns out that the cross sections for photon and Z production have the identical energy dependence and asymptotically their ratio depends on the Weinberg angle only (more explicitly it is equal to $\tan^{2n}\theta_W$) whereas the energy dependence of the cross section of the W production is suppressed by factor $s^{-0.4}$ compared to them.

1 Introduction

The double-logarithmic approximation (DLA) was introduced into the particle physics by V.V. Sudakov who first ound that the most important ra-

*talk given by B.I. Ermolaev

diative corrections to the orm actor $f(q^2)$ o electron at large q^2 are the double-logarithmic (DL) ones, i.e. $\sim (\alpha \ln^2(q^2/m^2))^n$ $(n = 1, 2, ..)$. with m being a mass scale,. A ter accounting them to all orders in α, it turns out[1] that asymptotically

$$f(q^2) \sim e^{-(\alpha/4\pi)\ln^2(q^2/m)} \tag{1}$$

when $q^2 \gg m^2$. The next important step towards studying DL asymptotics in QED was done in Re s. [2]. A ter that, calculations in DLA have become a technology rather than an art. The study o QCD scattering amplitudes has shown that there is no big technical difference between QED and QCD or calculating amplitudes o elastic processes (see e.g. Re . [3]) whereas inelastic (radiative) QCD -amplitudes are much harder to calculate (concerning the Sudakov logarithms in QCD see e.g. Re s. [4]). The methods o calculating the DL asymptotics can be applied also to electroweak (EW) processes provided the total energy is high enough to neglect the masses o the electroweak bosons. At such huge energies ($\gg$ 100 GeV), many important technical details learnt rom QED and QCD can be used or calculating EW amplitudes[5]. In the present talk I will discuss DL asymptotics or e^+e^- annihilation into a quark-anitquark or a lepton-antilepton pair [6] and [7].

2 DL contributions to elastic e^+e^- -annihilation into quarks and leptons

The conventional way or considering e^+e^- -annihilation into $\to q\bar{q}$ consists o two steps: the first one is the assumption that this process is mediated by a single virtual photon exchange: $e^+e^- \to \gamma^* \to q\bar{q}$; the second step is calculating QCD radiative corrections. However, electroweak radiative corrections can also be sizable when this process is considered in some particular kinematic regions. These are the orward and backward kinematics. The orward kinematics is defined when the scattering angle[1] between the momenta o the initial electron (positron) and o the final particles with the negative (positive) electric charges is $\ll 1$. The case when this angle is $\sim \pi$ gives the backward kinematics. Both kinematics are o the Regge type and the effect o accounting or the DL radiative corrections to all orders in the

[1]Through this paper when we refer to angles, we imply the angles in cmf.

electroweak couplings can be interpreted as exchanges o Reggeons propagating in the cross channel. This means that the expression or the orward and backward scattering amplitudes can be represented in the orm o the Sommer eld-Wotson (SW) integral:

$$M_j^{(\pm)}(s/\mu^2) = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} \left(\frac{s}{\mu^2}\right)^{\omega} \xi^{(\pm)}(\omega)\, F_j^{(\pm)}(\omega) \tag{2}$$

where the signs $\pm$ re er to the signatures o the amplitudes, the signature actors $\xi^{(+)} \approx 1$, $\xi^{(-)} \approx \imath\pi\omega/2$; $F_j^{(\pm)}(\omega)$ is called the SW amplitude (the partial wave); the Mandelstam variable s and the mass scale μ obey $\sqrt{s} \gg \mu \geq 100\mathrm{GeV}$. The integration contour in Eq. (2) runs to the right o all singularities o $F_j^{(\pm)}(\omega)$. Subscript j enumerates both the flavours o the produced quarks and the EW isospin state in the cross channel. For example, or the orward $e_R^+(p_2)e_L^-(p_1)$ annihilation into $u_L(p_2')\bar{u}_R(p_1')$ one gets

$$\begin{aligned} M^{(+)}(\rho,\eta) &= a \exp\left[-\frac{1}{8\pi^2}\left(\frac{3}{2}g^2 + \frac{Y_e^2+Y_q^2}{4}g'^2\right)\frac{\eta^2}{2}\right] \\ &\times \int_{-\imath\infty}^{\imath\infty} \frac{dl}{2\pi\imath} e^{\lambda l(\rho-\eta)} \frac{D_{p-1}(l+\lambda\eta)}{D_p(l+\lambda\eta)} \end{aligned} \tag{3}$$

where g and g' are the SU(2) and U(1) couplings, Y_e (Y_q) is the hypercharge o electron (quark), $a = (-3g^2 + g'^2 Y_e Y_q)/4$, $\lambda = -g'(Y_e + Y_q)/2$, $p = -a/\lambda^2$, $\rho = \ln(s/\mu^2)$, $\eta = \ln(-t/\mu^2)$ and D_p are the Parabolic cylinder unctions. Eq. (3) is obtained or the kinematic region $s = (p_1+p_2)^2 \gg -t = -(p_2 - p_1')^2$.

The exponent in Eq. (3) is the electroweak Sudakov orm actor or this process. It accumulates the so test radiative DL corrections, with virtualities o the virtual EW bosons $\leq -t$. The harder DL contributions are collected in the SW integral. The singularities o the integrand in Eq. (3) are the zeros o D_p. Forward scattering amplitudes or e^+e^- annihilation into quarks o other chiralities and flavours are represented by similar expressions. The only difference is in the values o   actors a, p, λ. The same is true or the backward scattering amplitudes.

I $F_j^{(\pm)}(\omega)$ are singular when $\omega = \Delta_{jr}^{(\pm)}$, $(r = 1, ...)$, then the asymptotic dependence o $M_j^{(\pm)}$ on s is

$$M_j^{(\pm)}(s/\mu^2) \sim \sum_r (s/\mu^2)^{\Delta_{jr}^{(\pm)}} \tag{4}$$

and $\Delta_{j_r}^{(\pm)}$ are the intercepts o the Reggeons. Re . [6] states that the value o the intercepts depends also on the flavours and chiralities o the final quarks. It turns out[6] that all intercepts or the backward amplitudes are negative whereas a part o intercepts or the orward amplitudes is positive. There ore, the backward amplitudes rapidly all when s increases whereas the orward amplitudes slowly grow with s. This result can be interpreted as a orward-backward charge asymmetry. In particular, the largest intercepts o the orward positive signature amplitudes M_j (we drop the superscript "+") are $\Delta_u = 0.11$ or $e_L^- e_R^+ \to u_L \bar{u}_R$ and $\Delta_d = 0.08$ or $e_L^- e_R^+ \to d_L \bar{d}_R$. The other intercepts are smaller. The asymmetry actor A is defined in terms o the orward and backward cross sections $d\sigma_{F,B}$ o detecting the quarks in the orward (backward) cones with very small opening angles $\theta < M_Z/\sqrt{s}$:

$$A = [d\sigma_F - d\sigma_B]/[d\sigma_F + d\sigma_B] \tag{5}$$

where $d\sigma_{F(B)}$ stands or orward (backward) differential cross section. Per- orming numerical calculations, we arrive at the result plotted in Fig. 1. The difference between the orward and backward scattering amplitudes leads also to the act that the average electric charge o the produced hadrons in the cone around o the e^- -beam (e^+ -beam) is negative (positive) and the value o the average charge grows with energy as shown in Fig. 2. It is possible to apply the plots o Figs. 1 and 2, to the situation when the produced quarks are in a wider angular region $1 \ll \theta < M_Z/\sqrt{s}$. To this end one should replace $\sqrt{s}$ in these Figs. by M_Z/θ.

3 Inelastic e^+e^- -annihilation into quarks

When e^+e^- annihilate into $q\bar{q}$ and electroweak bosons, with the final particles produced in the multi-Regge kinematics, there also appear DL electroweak corrections. The essence o the multi-Regge kinematics is that the longitudinal momenta o the produced particles are much greater than their transverse momenta. On the other hand, the transverse momenta $k_{i\perp}$ are assumed to be much greater than $M_{W,Z}$ so that all emission angles are $\ll 1$. With this assumption, the spontaneous broken $SU(2)U(1)$ symmetry in many respects can be regarded as restored. In particular, it becomes more convenient to consider emission o the isoscalar A_0 and isovector $A_{1,2,3}$ gauge fields and

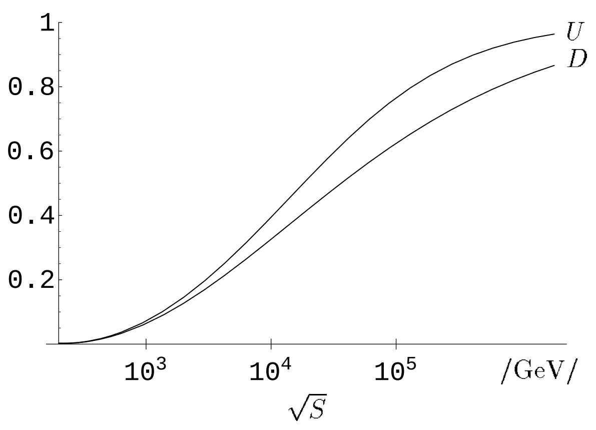

Figure 1: Asymmetry A or e^+e^- annihilation into quarks o different flavours in the "collinear angular region" in DLA.

then to proceed to the γ, W, Z emission, using the standard relations between these two sets. Also it makes possible to use arguments o Re s. [8] where the multi-Regge amplitudes or gluon production were calculated. It turns out[7] that amplitudes or the γ and Z production are governed by both the isoscalar and isovector Reggeons (with the intercepts 0.11 and 0.08) propagating in the cross channels, whereas the W production is controlled by the isovector Reggeons only, with the smaller (-0.08 and -0.27) intercepts. It means that the cross sections o the photon and the Z production have identical energy dependence. The only difference between them is due to the different EW couplings, so that asymptotically (at energies $\sqrt{s} \geq 10^6$ Gev)

$$\sigma^{nZ}(s)/\sigma^{n\gamma}(s) = \tan^{2n}\theta_W \tag{6}$$

whereas

$$\sigma^{nW}(s)/\sigma^{n\gamma}(s) \sim s^{-0.4}\ . \tag{7}$$

The results o the numerical calculations or these cross sections in the case o single boson production, covering the energy range rom 10^3 to 10^7 GeV, are finally shown in Figs. 3 and 4.

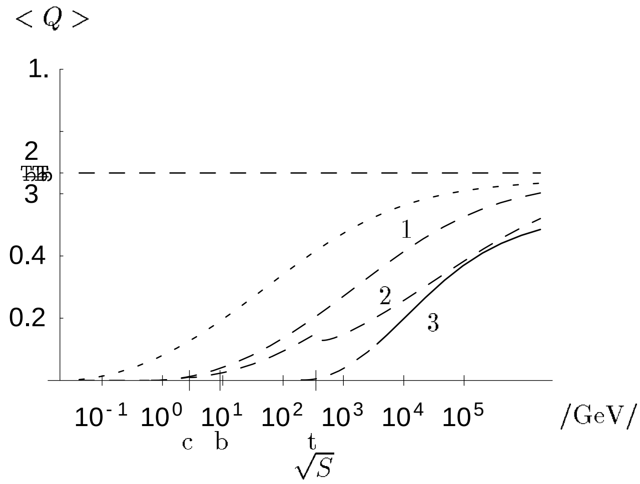

Figure 2: Average electric charge $< Q >$ o the hadron flow detected inside a narrow cone $\theta < M/\sqrt{s}$ in the direction o e^+ -beam. Short-dashed curve corresponds to the case o multiphoton annihilation to u, d -current quarks with $M \equiv \mu = 0.01$ Gev in QED. Dashed curve 1 corresponds to u, d -constituent quarks with $M \equiv \mu = 0.3$ GeV also in QED. Curves 2 and 3 account or the all quark flavours produced in e^+e^- -annihilation: the curve 2 is calculated in QED while the curve 3 corresponds to all EW -bosons exchanged in DLA with $M = M_Z$. Curve 2 shows how $< Q >$ would rise without account o EW interactions. The dashed part o the curve 3 corresponds to the region where subleading corrections to DLA could be important. The dashed horizontal line shows the asymptotic value o $< Q >$ as the u -quark contribution is dominating.

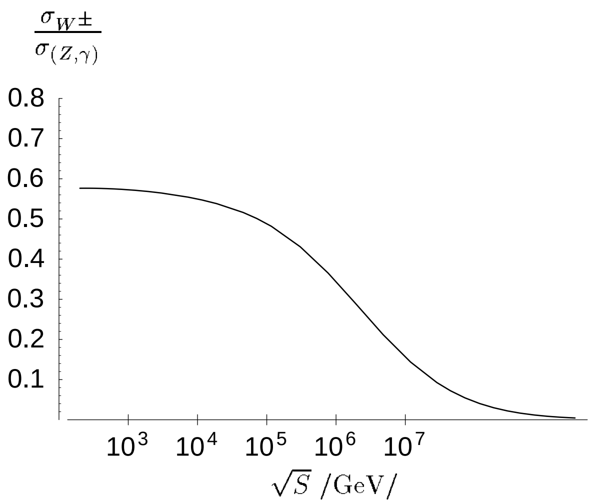

Figure 3: Total energy dependence o $W^{\pm}$ to (Z, γ) rate in e^+e^- annihilation.

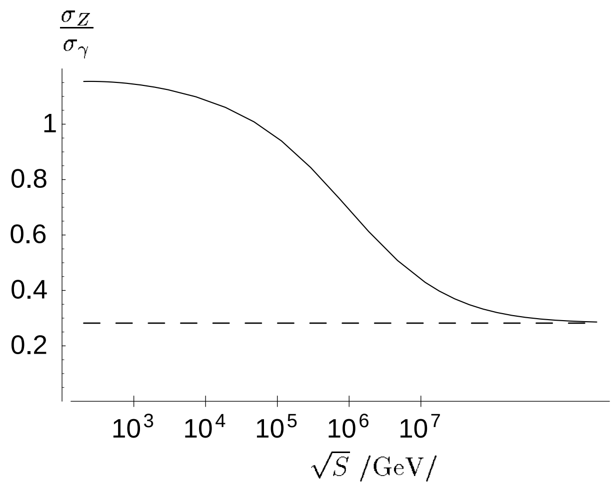

Figure 4: Total energy dependence o Z to γ rate in e^+e^- annihilation. The dashed line shows the asymptotical value o the ratio: $\tan^2\theta_W \approx 0.28$.

4 Acknowledgement

The work is supported by grants CERN/FIS/43652/2001, INTAS-97-30494, SFRH/BD/6455/2001 and RFBR 00-15-96610.

References

[1] V.V. Sudakov. Sov. Phys. JETP 3(1956)65.

[2] V.N. Gribov, V.G. Gorshkov, G.V. Frolov, L.N. Lipatov. Sov.J.Nucl.Phys. 6(1968)95; ibid 6(1968)262.

[3] J.J. Carazone, E.C. Poggio and H.R. Quinn. Phys. Rev. D 11(1975)2286; J.M. Cornwell and G. Tiktopolous. Phys. Rev. Lett. 35(1975)338; V.V. Belokurov and N.I. Usyukina. Phys. Lett. B94(1980)251; R. Kirschner, L.N. Lipatov. Nucl.Phys.B 213(1983)122.

[4] E.A. Kuraev and V.S. Fadin Yad. Fiz. 27(1978)1107; B.I. Ermolaev and V.S. Fadin JETP Letters 33(1981)269; Y. L. Dokshitzer, V. S. Fadin and V. A. Khoze, Phys. Lett. B **115** (1982) 242; B.I. Ermolaev, V.S. Fadin, L.N. Lipatov. Yad. Fiz. 45(1987)817.

[5] V.S. Fadin, L.N. Lipatov, A.D. Martin, and M. Melles, Phys.Rev. D**61** (2000) 094002.

[6] B.I. Ermolaev, M. Greco, S.I. Troyan. hep-ph/0205260.

[7] B.I. Ermolaev, S.M. Oliveira and S.I. Troyan. hep-ph/0201159.

[8] B.I. Ermolaev, L.N. Lipatov. Sov.J.Nucl.Phys. 48(1988)715; Int.J.Mod.Phys.A4(1989)3147.