Electrons as quasi-bosons in magnetic white dwarfs

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Abstract

We examine the possible role played by field angular momentum in highly magnetic white dwarf stars. White dwarfs achieve their equilibrium from the balancing of the gravitational compression by the Fermi degeneracy pressure of the electron gas. In field theory there are examples (*e.g.* the monopole-charge system) where a strong magnetic field can transform a boson into a fermion or a fermion into a boson. Based on these examples we argue that for magnetic white dwarfs the magnetic field may alter the statistics of some fraction of the white dwarf's electrons from fermionic to bosonic. This would effect the stars structure, giving it a smaller than expected radius, and a lower than expected temperature. In some extreme cases one could imagine that this effect could lead to the collapse of the white dwarf into a neutron star despite being below the Chandreshekar limit.

1 Introduction

In certain highly magnetic white dwarf stars the combination of the magnetic field with the Coulombic electric field of the electrons results in a field angular momentum. For certain values of the magnetic field the total angular momentum of the system – spin of the electron plus field angular momentum – can take on integer values, giving rise to an effective bosonic system. This proposed transformation of the electrons into effective bosons, as a result of the field angular momentum, has consequences for the structure of the white dwarf: the radius could be smaller than expected, and the temperature could be lower than expected.

2 Field angular momentum in magnetic white dwarfs

White dwarfs are "dead" stars which achieve their equilibrium by balancing the gravitational compression by a Fermi degeneracy pressure of the electrons [1, 2]. The stability argument for a white dwarf can be framed in terms of the Fermi energy of the electrons versus their gravitational binding energy. The Fermi energy for a relativistic electron is approximately

$$E_F \approx \frac{\hbar c N^{1/3}}{R} \tag{1}$$

where N is the number of electrons in the object, and R is the radius of the object. The gravitational energy per fermion is approximately

$$E_G \approx -\frac{GMm_n}{R} \tag{2}$$

where $m_n \approx 1.67 \times 10^{-24}$ g is the nucleon mass, and $M \approx 2Nm_n$ is roughly the total mass of the star The total energy is then $E_{tot} = E_F + E_G$. If the physical constants in E_F and E_G are such that $E_{tot} > 0$ then E_{tot} can be decreased by increasing R and a stable situation is eventually reached where the star is supported by its Fermi degeneracy pressure. If the physical constants in E_F and E_G are such that $E_{tot} < 0$ then E_{tot} decreases without bound by decreasing R and no equilibrium exists. The boundary between these two situations occurs when $\hbar c N^{1/3} = GNm_n^2$ which implies a maximum baryon number of $N_{max} \approx (\hbar c/2Gm_n^2)^{3/2} \approx 7.8 \times 10^{56}$ and a maximum total mass of $M_{max} \approx 2N_{max}m_n \approx 1.3M_{\odot}$. This simple argument gives an approximation of the Chandrasekhar mass limit for white dwarfs. Crucial to this argument (or more rigorous versions) is that the electrons should behave as fermions in order to give rise to the Fermi degeneracy pressure. In lower dimensional field theories there are examples, such as the sine-Gordon model in one space and one time dimension, where bosonic and fermionic degrees of freedom can be taken as dual or interchangeable [3, 4]. These lower dimensional examples can be extended to $3+1$ dimensions [5]. In Abelian and non-Abelian field theories [6, 7] there are configurations where, through the action of a magnetic field, the statistics of the system can be transformed (*i.e.* the system can be fermionic even though all the fields involved in its construction are bosonic). There are also certain condensed matter systems, where fermions can be converted into effective fermions or effective bosons. The fractional quantum Hall effect [8] offers one such example, where electrons in 2D systems in the presence of a large magnetic flux can act as effective fermions [9, 10] or effective bosons [11, 12] depending on which picture/approach one uses. Here we look at the possibility that in highly magnetized white dwarfs a similar transformation may occur for some fraction of the electrons of the star. Converting some fraction of the electrons of the star into effective quasi-bosons would mean that they would no longer be involved in giving rise to the Fermi degeneracy pressure. The N in eq. (1) would be reduced by the fraction of the electrons which are converted to effective bosons. This weakening of the Fermi degeneracy pressure would give the magnetic white dwarf a smaller than expected radius for its mass. In extreme cases, if a large enough fraction of the electrons were converted into bosons, the magnetic white dwarf could collapse into a neutron star despite being below the Chandrasekhar limit. Such a collapse might occur "gently" – without a supernova. This could offer an explanation for certain pulsar systems which have planets orbiting them [13]. If the neutron star formed via a supernova then the original planets of the progenitor star should have been blown out of the system. Also transforming electrons into quasi-bosons could result in these electrons Bose condensing, which could speed up the cooling rate of the white dwarf, so that such magnetic white dwarfs could be cooler than expected.

We now give a simple picture for how an electron inside a highly magnetized white dwarf can be transformed into an effective boson. Our arguments for this transformation are framed in terms of the field angular momentum of the system rather than the additional phase factor that arises in the exchange of the charge/magnetic flux composites. Various 2D [14] and 3D [15] examples have found that both approaches lead to an equivalent understanding of the change in statistics of these composite systems.

The angular momentum carried in the electric and magnetic fields can be written as [16]

$$\mathbf{L}_{em} = \frac{1}{4\pi c} \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) d^3\mathbf{r} \tag{3}$$

Now we consider an electron located at the origin in a uniform magnetic field whose direction is taken to define the z-axis (more details can be found in [17]). The electric field of the electron located inside the white dwarf will be screened due to the material of the white dwarf. The screening distance is denoted by R_{sc}. The electric potential associated with the screened field of the electron is of the Yukawa form – $\phi(\mathbf{r}) = \frac{e \exp(-r/R_{sc})}{r}$. The magnetic and electric field of this system are given as

$$\mathbf{B} = B_0 \hat{\mathbf{z}} \qquad \mathbf{E} = -\frac{e \exp(-r/R_{sc})}{r^2} \left[1 + \frac{r}{R_{sc}}\right] \hat{\mathbf{r}} \tag{4}$$

where B_0 is the magnitude of the magneic field. The field angular momentum which results from this combination of magnetic and electric fields is

$$\mathbf{L}^{em} = \frac{2eB_0 R_{sc}^2}{c} \hat{\mathbf{z}} \tag{5}$$

The direction of field angular momentum is in the $\hat{\mathbf{z}}$ direction, which is determined by the direction of the external magnetic field. The $B_0 R_{sc}^2$ part of this expression is proportional to the magnetic flux, Φ, "trapped" by the electron. For a typical white dwarf, R_{sc} , is generally small – on the order of 10^{-10} cm. There are two arguments for considering the electron and the trapped flux as a composite system with total angular momentum $L_{tot} = L_{spin} + L_{em}$. First, both the spin of the electron and the field angular momentum are localized in a small spatial region: a sphere of radius 10^{-10} cm. In contrast for the monopole/charge system the field angular momentum *density* is not necessarily well localized around the charge and yet is still taken as being part of the total angular momentum of the composite system. Second, L_{em}, by itself does not satisfy $[L_i, L_j] = i\epsilon_{ijk} L_k$. Thus L_{em} only makes sense as an angular momentum in combination with L_{spin}. This is analogous to the charge/monopole system where it is only the orbital plus field angular momentum that gives a proper quantum mechanical angular momentum [18].

In order for the composite system to behave as an effective boson we want L_{em} to be a half-integer multiple of $\hbar/2$ so that L_{tot} takes on an integer value. From eq. (5), and using the screening distance of $R_{sc} \approx 10^{-10}$ cm one can see that in order for L_{em} to equal $\hbar/2$ then $B_0 \approx 10^{11}$ G. (more detailed numbers can be found in Ref. [17]). Highly magnetic white dwarfs are known to have surface fields of $10^8 - 10^9$ G, thus it is not unreasonable to postulate interior fields of order 10^{11} G. In fact it is not uncommon to postulate interior fields as high as 10^{13} G [19].

There are two physical consequences of turning electrons into effective bosons in a highly magnetic white dwarfs. First, the radius would be smaller than expected for a given mass due to the reduction of the Fermi degeneracy pressure. The extent to which the radius of the white dwarf decreases depends on the number of electrons which become effective bosons, which in turn depends on the details of the structure of the magnetic field inside the star. In Ref. [17] a model was given where the magnitude of the magnetic field increased linearly from a surface value of 10^9 G to an interior value of 10^{13} G. This would give a series of concentric shells where the magnetic field would take on values that would give L_{tot} of $0, 1, 2, ...$ All the electrons within each of these shells would be transformed into effective bosons which would significantly decrease the radius of the white dwarf. In certain extreme cases where enough of the electrons are transformed one could envisions that the white dwarf would collapse into a neutron star despite being below the Chandrasekhar limit. Such an extreme collapse scenario might give an explanation of the planet-pulsar systems [13]. In these systems one has up to three planets orbiting a pulsar. If this pulsar formed in a supernova collapse then the initial planets should have been blown out of the system, which is taken to imply that these planets must have formed after the supernova. However, if the original star collapsed slowly due to the fermion $\rightarrow$ boson transformation, then the current planets might be the original planets of the star. The second possible consequence is that the cooling rate of these magnetic white dwarfs could be accelerated, since the quasi-bosonic electrons could Bose condense. In Ref. [20] a similar idea was advanced about the increased cooling rate of white dwarfs via Bose condensation of the nuclei of the star. Here we have an increased cooling rate via the Bose condensation of the transformed electrons.

It would appear that the above mechanism is not applicable to pure, ideal neutron stars. Even though the magnetic field strength of a neutron star can be several orders of magnitude larger than that of a white dwarf, a neutron carries no charge, and therefore no field angular momentum would be generated by placing the neutron in a magnetic field. This assumes that the neutron is a fundamental, chargeless object. As one goes to smaller distance/larger energy scales (*i.e.* as one considers neutron stars with increasing densities) there may be a transition where one needs to describe the matter, not in terms of neutrons, but in terms of a gas of charged quarks. At this point one might again consider applying the mechanism discussed in this paper, but now the magnetic field would be transforming the fermionic quarks into quasi-bosons. Such a neutron star shoud have a smaller than expected radius and be cooler than expected for the same reasons as in the white dwarf case – reduction of the Fermi degeneracy pressure and Bose condensation of the bosonic quarks. In this connection we would like to point to recent observational evidence [22] of neutron stars that exhibit both of these characteristics: smaller than expected radius and cooler temperature. The standard explanation of this result is that the neutron star has undergone a transition to a strange or quark star. The present transformation mechanism may also provide an explanation of these results. Recently [21] the mechanism presented in [17] has been used to explain the stability of magnetars.

3 Discussion and conclusion

We have presented arguments that some fraction of the electrons in highly magnetic white dwarf could undergo a transformation from fermions into effective bosons For this system the ambient magnetic field could combine with the electric field of the electrons, generating a field angular momentum. For certain values of the magnetic field the combined system of electron spin plus field angular momentum would take on integer values. Then as in the charge/monopole system [6, 7] or certain condensed matter systems, the composite system of electron plus trapped magnetic flux would behave as an effective boson. This transformation of electrons into effective bosons would lead to a reduction in the Fermi degeneracy pressure which supports the white dwarf, and it would open up the possibility of the transformed electrons Bose condensing. The reduction of the Fermi pressure could give the white dwarf a smaller than expected radius, while the Bose condensation would increase its cooling rate, giving it a colder than expected temperature. The above mechanism could also be applied to neutron stars at the level of the quarks. In this respect it can be pointed out that recently "neutron" stars have been observed which exhibit these two properties of smaller than expected radius and cooler temperature.

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