Active neutrino oscillations in the early Universe.

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Abstract

In this talk I present the results of the study of three-flavor neutrino oscillations in the early universe in the presence of neutrino chemical potentials which was performed in [1]. Our calculations show that the effective flavor equilibrium between all active neutrino species is established well before the big-bang nucleosynthesis (BBN) epoch for large mixing angle (LMA) solution of the solar neutrino problem. The BBN limit on the ν_e degeneracy parameter, $|\xi_\nu| \lesssim 0.07$, now applies to all flavors. Therefore, a putative extra cosmic radiation contribution from degenerate neutrinos is limited to such low values that it is neither observable in the large-scale structure of the universe nor in the anisotropies of the cosmic microwave background radiation. Existing limits and possible future measurements, for example in KATRIN, of the absolute neutrino mass scale will provide unambiguous information on the cosmic neutrino mass density, essentially free of the uncertainty of the neutrino chemical potentials.

1 Introduction

The cosmic matter and radiation inventory is known with ever increasing precision, but many important questions remain open. The cosmic neutrino background is a generic feature of the standard hot big bang model, and its presence is indirectly established by the accurate agreement between the calculated and observed primordial light-element abundances. However, the exact neutrino number density is not known as it depends on the unknown chemical potentials for the three flavors. (In addition there could be a population of sterile neutrinos, a hypothesis that we will not discuss here.) The standard assumption is that the asymmetry between neutrinos and anti-neutrinos is of order the baryon asymmetry $\eta \equiv (n_B - n_{\bar{B}})/n_\gamma \simeq 6 \times 10^{-10}$. This would be the case, for example, if $B - L = 0$ where B and L are the cosmic baryon and lepton asymmetries, respectively. While $B - L = 0$ is motivated by scenarios where the baryon asymmetry is obtained via leptogenesis [2], there are models for producing large L and small B [3].

In order to quantify a putative neutrino asymmetry we assume that well before thermal neutrino decoupling a given flavor is characterized by a Fermi-Dirac distribution $f_\nu(p,T) = [\exp(E_p/T - \xi_\nu) + 1]^{-1}$, where $\xi_\nu \equiv \mu_\nu/T$ is the degeneracy parameter, μ_ν is a chemical potential and $E_p \simeq p$ since we may neglect small neutrino mass effects on the distribution function. For anti-neutrinos the distribution function is given by $\xi_{\bar{\nu}} = -\xi_\nu$.

A neutrino chemical potential modifies the outcome of primordial nucleosynthesis in two different ways [4]. The first effect appears only in the electron sector because electron neutrinos participate in the beta processes which determine the primordial neutron-to-proton ratio so that $n/p \propto \exp(-\xi_e)$. Therefore, a positive ξ_e decreases Y_p, the primordial ^{4}He mass fraction, while a negative ξ_e increases it, leading to an allowed range

$$-0.01 < \xi_e < 0.07 \;, \tag{1}$$

compatible with $\xi_e = 0$ (see Refs. [5] and Sec. 3). A second effect is an increase of the neutrino energy density for any non-zero ξ which in turn increases the expansion rate of the universe, thus enhancing Y_p. This applies to all flavors so that the effect of chemical potentials in the ν_μ or ν_τ sector can be compensated by a positive ξ_e. Altogether the big-bang nucleosynthesis (BBN) limits on the neutrino chemical potentials are thus not very restrictive.

Another consequence of the extra radiation density in degenerate neutrinos is that it postpones the epoch of matter-radiation equality. In the cosmic microwave background radiation (CMBR) it boosts the amplitude of the first acoustic peak of the angular power spectrum and shifts all peaks to smaller scales. Moreover, the power spectrum of density fluctuations on small scales is suppressed [9], leading to observable effects in the cosmic large-scale structure (LSS).

A recent analysis of the combined effect of a non-zero neutrino asymmetry on BBN and CMBR/LSS yields the allowed regions [10]

$$-0.01 < \xi_e < 0.22, \qquad |\xi_{\mu,\tau}| < 2.6, \tag{2}$$

in agreement with similar bounds in [11]. These limits allow for a very significant radiation contribution of degenerate neutrinos, leading many authors to discuss the implications of a large neutrino asymmetry in different physical situations. These include the explanation of the former discrepancy between the BBN and CMBR results on the baryon asymmetry [12] or the origin of the cosmic rays with energies in excess of the Greisen-Zatsepin-Kuzmin cutoff [13]. In addition, if the present relic neutrino background is strongly degenerate, it would enhance the contribution of massive neutrinos to the total energy density [14] and affect the flavor oscillations of the high-energy neutrinos [15] which are thought to be produced in the astrophysical accelerators of high-energy cosmic rays.

The limits in Eq. (2) ignore neutrino flavor oscillations, an assumption which is no longer justified in view of the experimental signatures for neutrino oscillations by solar and atmospheric neutrinos. For zero initial neutrino chemical potentials, the flavor neutrinos have the same spectra so that oscillations produce no effect. This is true up to a small spectral distortion caused by the heating of neutrinos from e^+e^- annihilations, an effect which is different for electron and muon/tau neutrinos and which causes a small relative change in the final production of ^{4}He of order 10^{-3} [16]. This relative change is slightly enhanced by neutrino flavor oscillations [17]. In the presence of neutrino asymmetries, flavor oscillations equalize the neutrino chemical potentials if there is enough time for this relaxation process to be effective [18]. If flavor equilibrium is reached before BBN, then the restrictive limits on ξ_e in Eq. (1) will apply to all flavors, in turn implying that the cosmic neutrino radiation density is close to its standard value. As a consequence, it is no longer necessary to use the neutrino radiation density as a fit parameter for CMBR/LSS analyses, unless one considers exotic models with decaying massive particles.

The effects of flavor oscillations on possible neutrino degeneracies have been considered in [15], where it was concluded that flavor equilibrium was achieved before the BBN epoch if the solar neutrino problem was explained by the large-mixing angle (LMA) solution. The LMA solution is favored by the current solar neutrino data. Thus, it was concluded that in the LMA case a large cosmic neutrino degeneracy was no longer allowed.

We revisit this problem because the flavor evolution of the neutrino ensemble is more subtle than previously envisaged if medium effects are systematically included. Contrary to the treatment of Ref. [15], the refractive effect of charged leptons can not be ignored, and actually is one of the dominant effects. While the background neutrinos produce an even larger refractive term, its effect is to synchronize the neutrino oscillations [19] which remain sensitive to the charged-lepton contribution. Still, equilibrium is essentially, but not completely, achieved in the LMA case so that our final conclusion is qualitatively similar to that of Ref. [15]. Recently the authors of refs. [20] have also analyzed the equilibration of neutrino asymmetry from flavor oscillations, providing further analytical insight and confirming our conclusions make in ref.[1]. Method developed in [1] was also recently applied to the supernova case in ref. [21].

2 Three-flavor oscillations

The neutrino flavor eigenstates ν_e, ν_μ, and ν_τ are related to the mass eigenstates via the mixing matrix

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix} . \qquad (3)$$

Here $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ for ij = 12, 23, or 13, and we have assumed CP conservation. The set of oscillation parameters is now five-dimensional (see for instance [27]),

$$\begin{array}{ll} \Delta m^2_{\rm sun} & \equiv \Delta m^2_{21} = m_2^2 - m_1^2 \\ \Delta m^2_{\rm atm} & \equiv \Delta m^2_{32} = m_3^2 - m_2^2 \\ \theta_{\rm sun} & \equiv \theta_{12} \\ \theta_{\rm atm} & \equiv \theta_{23} \\ \theta_{13} & \end{array} \qquad (4)$$

We do not perform a global analysis of all possible values of these parameters, but fix them to be in the regions that solve the atmospheric and solar neutrino problems [27, 28].

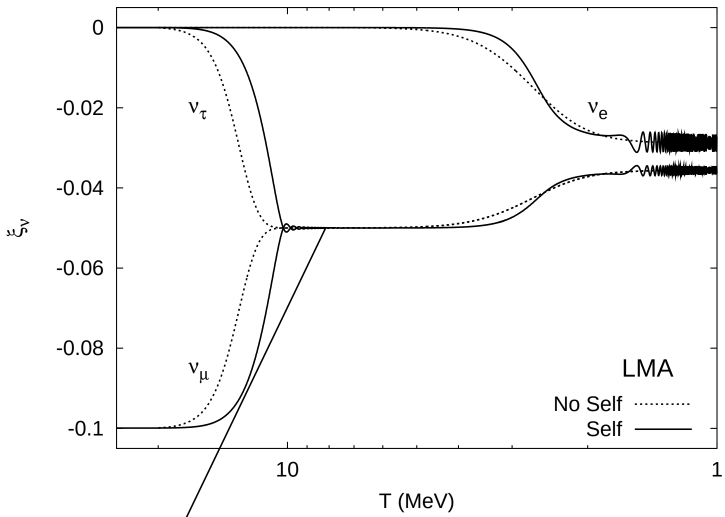
In particular we take $\Delta m^2_{\rm atm} = 3 \times 10^{-3}$ eV^2 and maximal mixing for $\theta_{\rm atm}$ from the former, while from the solar analyses we consider the following values for $\Delta m^2_{\rm sun}$ in eV^2: 4.5×10^{-5}, 7×10^{-6}, 1×10^{-7}, 8×10^{-11} for the Large Mixing Angle (LMA), Small Mixing Angle (SMA), LOW and Vacuum regions, respectively. For the angle $\theta_{\rm sun}$ we take the approximation of maximal mixing for all cases except SMA where we use $\theta_{\rm sun} = 1.5°$.

The equations for neutrino distribution functions, taking into account all essential effects of the medium, i.e. collisional damping, the refractive effects from charged leptons, and in particular neutrino self-interactions that synchronize the neutrino oscillations was presented in [1]. The relative impotance of each term in those equations was discussed in details in [1]. Here we just present the results for the evolution of the neutrino asymmetries for the case $\xi_\mu = -0.1$, $\theta_{13} = 0$ is shown in Figs. 1 and 2 for the LMA and LOW cases, respectively, both with and without the neutrino self-interactions. For this choice of oscillation parameters the three-flavor oscillations effectively separate as two two-flavor problems for the atmospheric and solar parameters, respectively. The oscillations caused by the largest Δm^2 are effective at $T \simeq 20$ MeV, as soon as the $\mu^\pm$ background disappears completely. The presence of the self-term causes only a slight delay in the equilibration of ξ_μ and ξ_τ.

The oscillations due to $\Delta m^2_{\rm sun}$ and $\theta_{\rm sun}$ are effective only when the vacuum term overcomes the $e^\pm$ potential. In the LMA case, the conversions takes place above $T \simeq 1$ MeV, leading to nearly complete flavor equilibrium before the onset of BBN. For the LOW parameters the synchronized oscillations just start at that epoch. The presence of the neutrino self-potential does not significantly change the picture in the LMA case while for the LOW case one clearly observes the phenomenon of synchronized oscillations. For the SMA and Vacuum regions primordial oscillations involving ν_e are not effective before BBN if $\theta_{13} = 0$. For the other solutions of the solar neutrino problem, partial flavor equilibrium may be achieved if the angle θ_{13} is close to the experimental limit $\tan^2\theta_{13} \lesssim 0.065$.

3 New limits on neutrino degeneracy

We conclude that in the LMA region the neutrino flavors essentially equilibrate long before n/p freeze out, even when θ_{13} is vanishingly small. For the other cases the outcome depends on the magnitude of θ_{13}. In the LMA case


0
-0.02
-0.04
-0.06
-0.08
-0.1
ξ_ν
10
1
T (MeV)
ν_τ
ν_μ
ν_e
LMA
No Self
Self

it is thus justified to derive new limits on the cosmic neutrino degeneracy parameters under the assumption that all three neutrino flavors are characterized by a single degeneracy parameter, independently of the primordial initial conditions. We do not derive the corresponding limits for the other solar neutrino solutions, since they would strongly depend on the value of a non-zero θ_{13}. However, if that angle is close to the experimental limit, the bounds that we describe would approximately apply.

We first note that the energy density in one species of neutrinos and anti-neutrinos with degeneracy parameter ξ is

$$\rho_{\nu\bar{\nu}} = T_\nu^4 \frac{7\pi^2}{120}\left[1 + \frac{30}{7}\left(\frac{\xi}{\pi}\right)^2 + \frac{15}{7}\left(\frac{\xi}{\pi}\right)^4\right]. \tag{5}$$

It is clear that the BBN limit will imply $\xi \ll 1$ for all flavors so that the modified energy density and the resulting change of the primordial helium abundance $Y_{\rm p}$ will be negligibly small. If there are additional relativistic species, such as sterile neutrinos or majorons, then Eq. (2) will simply apply to all the active neutrinos $|\xi| < 0.22$.

Therefore, the only remaining BBN effect is the shift of the beta equilibrium by ξ_e. We recall that $Y_{\rm p}$ is essentially given by n/p at the weak-interaction freeze-out, and that $n/p \propto \exp(-\xi_e) \simeq 1-\xi_e$ where the latter expansion applies for $|\xi_e| \ll 1$. Therefore, $\Delta Y_{\rm p} \simeq -Y_{\rm p}(1 - Y_{\rm p}/2)\xi_e \simeq -0.21\,\xi_e$. Modifications of $Y_{\rm p}$ by new physics are frequently expressed in terms of the equivalent number of neutrino flavors ΔN_ν which would cause the same modification due to the changed expansion rate at BBN. If the radiation density at BBN is expressed in terms of N_ν, the helium yield can be expressed by the empirical formula $\Delta Y_{\rm p} = 0.012\,\Delta N_\nu$ [30]. Therefore, the effect of a small ξ_e on the helium abundance is equivalent to $\Delta N_\nu \simeq -18\xi_e$. A conservative standard limits holds that BBN implies $|\Delta N_\nu| < 1$ which thus translates into $|\xi_e| \lesssim 0.057$.

A more detailed recent analysis reveals that the measured primordial helium abundance implies a 95% CL range $N_\nu = 2.5 \pm 0.8$ or $\Delta N_\nu = -0.5 \pm 0.8$ [6, 10]. We conclude that the BBN-favored range for the electron neutrino degeneracy parameter is at 95% CL

$$\xi_e = 0.03 \pm 0.04\ . \tag{6}$$

If all degeneracy parameters are the same, then this range applies to all flavors.

It should be noted that the actual limit we obtain on the neutrino degeneracy depends on the adopted BBN analysis. For instance ΔN_ν could be as high as 1.2 when the primordial abundance of lithium is used instead of that of deuterium [31]. At any rate, a limit of $|\xi_e| \lesssim 0.1$ seems rather conservative and does not modify our conclusions.

Using $|\xi| < 0.07$ as a limit on the one degeneracy parameter for all flavors, the extra radiation density is limited by $(\Delta\rho_{\nu\bar{\nu}})/\rho_{\nu\bar{\nu}} < 3 \times 0.0021 = 0.0064$, i.e. $\Delta N_\nu < 0.0064$. If the same radiation density were to be produced by the asymmetry of one single species, this would correspond to $|\xi| < 0.12$.

For comparison with the future satellite experiments MAP and PLANCK that will measure the CMBR anisotropies, it was calculated that they optimistically will be sensitive to a single-species ξ above 0.5 and 0.25, respectively [32]. However, with proper consideration of the degeneracy with the matter density, ω_M, and the spectral index, n, a more realistic sensitivity is $\xi \approx 2.4$ and 0.73, respectively [33]. Turning this around we conclude that our new limits are so restrictive that the CMBR is certain to remain unaffected by neutrino degeneracy effects so that $|\xi|$ can be safely neglected as a fit parameter in future analyses.

If our new limits apply the number density of relic neutrinos is very close to its standard value. Therefore, existing limits and possible future measurements of the absolute neutrino mass scale, for example in the forthcoming tritium decay experiment KATRIN [34], will provide unambiguous information on the cosmic mass density in neutrinos, free of the uncertainty of neutrino chemical potentials.

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