Lambda-inflation versus observations

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Abstract. Lambda-inflation is tested against CMB and LSS observational data. The constraints for inflationary parameters are considered.

1 Introduction

An increasing precision of measurements in observational cosmology requires the adequate accuracy of predictions in the early Universe theory. It is particularly true for numerious recent and on-going experiments to measure the CMBR anisotropy (BOOMERanG [1], MAXIMA [2], DASI [3], VSA [4], CBI [5], ARCHEOPS, MAP) and to reconstruct the power spectrum of density perturbations by means of spatial distribution of galaxies [6] and galaxy clusters [7].

The inflationary paradigm is still the only viable theory of the early Universe. There is no doubt that inflation explains the observable features of the Universe much better than all available alternative theories of the early Universe. Moreover, all these theories have significant difficulties. For example, a string theory fails to reproduce both observable CMBR anisotropy and the spatial distribution of matter. Another theory, the ekpirotic one [9], needs a fine-tuning to create cosmological perturbations with appropriate spectrum [10].

Proposed two decades ago, inflation successfully solved in elegant way a majority of existed cosmological problems (see [11]). For example, one of these problems was a spatial flatness of the Universe predicted by inflation and well established now [1, 2, 3, 4, 8]. In addition, inflation creates

favorable conditions for the parametric amplification of cosmological perturbations [12]; scalar perturbations (S) appear as density fluctuations, tensor perturbations (T) are cosmological gravitational waves [1]. Assuming some inflationary potential one can calculate spectra of both kinds of perturbations.

From the other hand, a majority of papers dedicated to the numerical analysis of cosmological models uses a simplified phenomenological approach, where the spectra of cosmological perturbations are scale-free and independent of each of them. Both above-mentioned features contradict present inflationary models since almost all of them predict more complicated spectra (than just power-law) and significant amount of gravitational waves.

So, the cosmological model reconstruction problem may be solved in two ways. The first one is testing given inflationary models. This method requires accurate calculations of scalar and tensor perturbation spectra and allows to test inflationary models straightforwardly. The accuracy of such a method is related with the validity of slow-rolling approximation. The second way of cosmological model reconstruction is phenomenological one. It is based on assumption that the density perturbation spectrum can be approximated by power law, k^n, where n may be constant or slowly depends on scale. In the latter case a new parameter, the running $\partial n/\partial \ln k$, appears (see, e.g., [13] and references wherein).

My present analysis is related with the first way.

2 Lambda-inflation

One calls by Lambda-inflation [14, 15] a model with a single scalar field φ and the following inflaton potential:

$$V(\varphi) = V_0 + \frac{1}{\kappa}\lambda_\kappa \varphi^\kappa, \tag{1}$$

where V_0, λ_κ, κ are constants. Here I will concentrate on the case with $\kappa = 4$.

Lambda-inflation is often referred in literature as "hybrid inflation". I prefer to reserve the latter name for the case of Lambda-inflation, where some particular way of decay of the effective Lambda-term (V_0) is assumed [16].

[1] Strickly speaking, vector perturbations can be amplified too. However, to create them in the first order of perturbations, some vector source in Lagrangian or inflaton potential is necessary. Such a source is absent in inflationary models based on the existence of scalar field(s) in the early Universe. So, a vector mode can appear in the second order of perturnations only.

It is convinient to introduce two new parameters. The first of them, c, is the following combination of inflaton potential constants:

$$c \equiv \frac{1}{2}\sqrt{\frac{V_0}{\lambda}} \tag{2}$$

(here and below $\lambda \equiv \lambda_\kappa$). c is closely related to the critical field value where both terms of the inflaton potential are equal ($\varphi_{cr} \equiv (4V_0/\lambda)^{1/4} = 2\sqrt{c}$). The second parameter, k_{cr}, is the scale where φ reaches φ_{cr} which allows to relate a size of horizon during inflation with its value now. If $c \gg 1$ then slow-rolling approximation is valid. In this case the spectra of perturbations are as follows:

$$\mathrm{S}: \qquad q_k = \frac{H}{2\pi\sqrt{2\gamma}} = \frac{\sqrt{2\lambda/3}}{\pi}\left(c^2 + x^2\right)^{3/4}, \tag{3}$$

$$\mathrm{T}: \qquad h_k = \frac{H}{\pi\sqrt{2}} = \frac{2c\sqrt{\lambda/3}}{\pi}\left(1 + \frac{x}{\sqrt{c^2 + x^2}}\right)^{-1/2}, \tag{4}$$

where H is the Hubble constant at inflation, $\gamma \equiv -\dot{H}/H^2$, $x \simeq \ln(k/k_{cr})$ (see details in [14, 15]). Figure 1 illustrates these spectra.

3 Cosmological model

Cosmological parameters can be separated into few groups. The first one contains inflationary parameters describing spectra of the cosmological perturbations. Another group of cosmological parameters is related to matter and energy in the Universe. These parameters are the matter density parameter, Ω_m, dark energy parameter, Ω_Λ, the baryon abundance, Ω_b, the massive neutrino abundance, Ω_ν, the number of massive neutrino species N_ν (the cold dark matter abundance $\Omega_c = \Omega_m - \Omega_b - \Omega_\nu$). This short list must be added by a value of the Hubble constant, $H_0 = 100h$ km s^{-1}Mpc^{-1} and may be by some others (optical depth of reionization, τ, the running parameter, $\partial n/\partial \ln k$, etc).

The complete analysis of complex cosmological model is a hard task: one has to use both a large number of free parameters and all available experimental data related to the determination of cosmological model ($\Delta T/T$, galaxy and galaxy cluster abundance data, Ly_α data, and others).

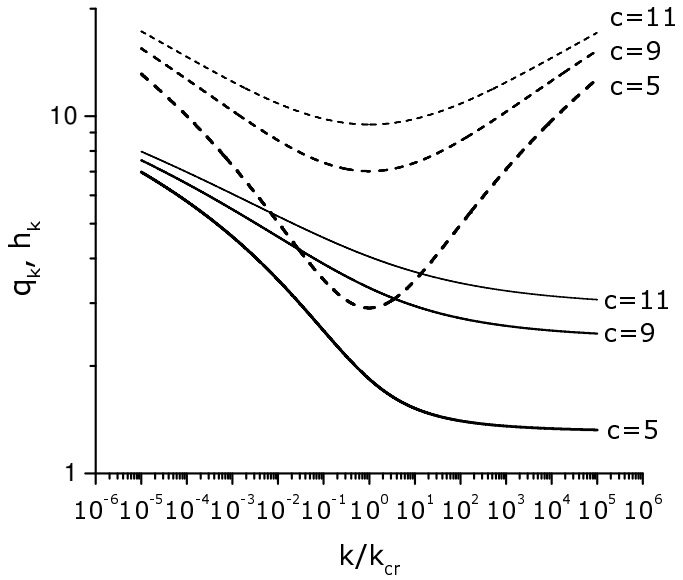

Figure 1: Scalar (q_k, dash lines) and tensor (h_k, solid lines) perturbation spectra in Lambda-inflation (arbitrary normalizations).

To simplify this task let us assume that our present knowledge about the matter and energy in the Universe is reasonable. It means that we may consider the cosmological model with the following parameters: $\Omega_\Lambda = 0.65$, $\Omega_m = 0.35$, $\Omega_c = 0.29$, $\Omega_b = 0.05$, $\Omega_\nu = 0.01$, $N_\nu = 3$, $h = 0.65$[2]. Below, we will take into account only two observational tests: COBE measurement of $\Delta T/T$ at 10th spherical harmonic (C_{10}), and σ_8 which normalizes the spectrum of density perturbations. Both density perturbations and cosmic gravitational waves contribute to the value of C_{10}. The measure of such contribution is usually described by parameter T/S as follows:

$$\mathrm{T/S} \equiv \frac{C_{10}^{\mathrm{T}}}{C_{10}^{\mathrm{S}}} \simeq -6n_T \equiv -3\frac{d\ln h_k}{d\ln k}. \tag{5}$$

The characterictic scale of COBE measurement is $k_{COBE} \equiv k_1 \simeq 1.5 \times 10^{-3}h$ Mpc^{-1}. As for σ_8, the characteristic scale is $k_{\sigma_8} \equiv k_2 \simeq 1.5 \times 10^{-1}h$ Mpc$^{-1} \simeq 100k_1$.

[2]This cosmological model with the scale-invariant spectrum of density perturbations and negligible abundance of cosmic gravitational waves reproduces both observational large scale CMBR anisotropy and galaxy cluster abundance (σ_8).

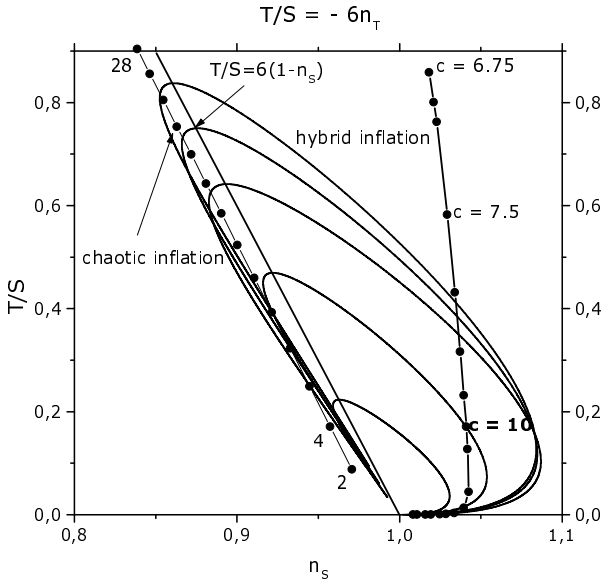

Figure 2: T/S versus $n_{\rm S}$ and c in Lambda-inflation.

After simple calculations we derive the following equation:

$$(1 + \mathrm{T/S}) \left(\frac{q_{k_1}}{q_{k_2}} \right)^2 = 1, \tag{6}$$

The numerical solution of this equation can be approximated by the linear fit:

$$\ln \frac{k_1}{k_{cr}} = 0.32\, c - 1.92, \tag{7}$$

the statistical error is in the last digits of both numbers. An important consequence of this solution is that the critical scale should be higher than the COBE scale, thus the former is either close to or out of the present horizon.

Figure 2 presents the corresponding T/S evaluated by means of the famous consistency relation T/S$\simeq -6n_{\rm T}$. The inflationary models, located along the line with solid circles (marked by "c=...") , satisfy both observational tests mentioned above. Assuming that T/S is not large (T/S$<$ 0.5) we may constrain $n_{\rm S}$ ($1 < n_{\rm S} < 1.05$) and inflationary parameter c ($c > 8$).

Recall that $n_{\rm S}$ is estimated at COBE scale and can vary at different scales. Figure 3 demonstrates the running parameter $\partial n_{\rm S}/\partial \ln k$. On this

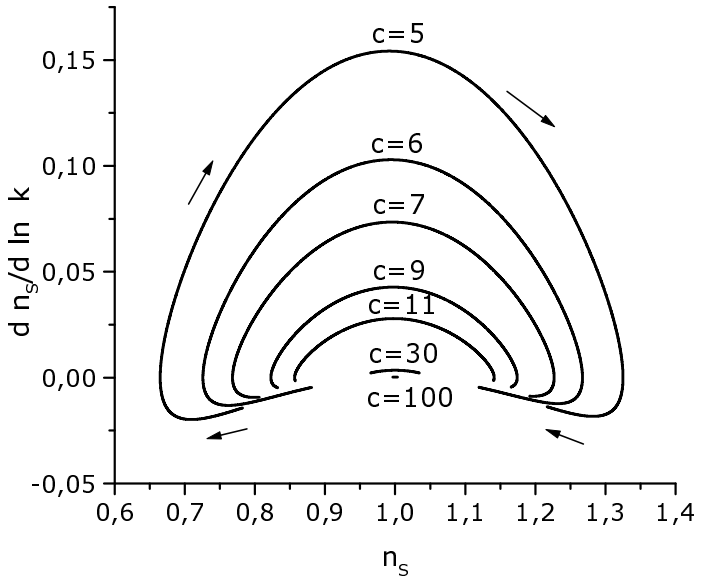

Figure 3: $\partial n_{\rm S}/\partial \ln k$ versus $n_{\rm S}$ in Lambda-inflation.

figure k/k_{cr} is the affine parameter which varies as shown by arrows. We see that the density perturbation slope at LSS scale (k_2) depends on c and can vary from 1 till 1.2.

4 Discussion

The demonstrated example of Λ-inflation favors slightly blue spectra of density perturbations ($n_{\rm S} \gtrsim 1$). However, phenomenological constraints for $n_{\rm S}$ based on LSS observational data, while consistent with $n_S \simeq 1$, are still uncertain about the sign of $(n_S - 1)$. E.g., galaxy cluster data prefer $n_S > 1$ [17], whereas galactic surveys indicate slightly red spectra ($n_{\rm S} < 1$) (e.g. [13]). Certainly, better data are required to delimit the slope of the fundamental spectrum. If future analysis reveals a blue spectrum, the Λ-inflation model will gain strong support as the theory of the very early Universe.

Acknowledgments

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