Could the process of neutrino photoproduction on nuclei, stimulated by a strong magnetic field, compete with URCA processes?

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Abstract

The recent studies [1] is reported of the neutrino photoproduction on nuclei, $\gamma + Ze \to Ze + \gamma + \nu + \bar{\nu}$, in a strong magnetic field. It is shown that the catalyzing influence of the field on the process decreases essentially because of the modification of the photon dispersion properties in a strong magnetic field. Therefore, at any field magnitude, neutrino photoproduction cannot compete with the URCA processes. This conclusion contradicts a recent statement in the literature [2].

1 Introduction

Strong magnetic fields which could be generated in the astrophysical cataclysms like a supernova explosion or a coalescence of neutron stars, make an active influence on quantum processes, thus allowing or enhancing the transitions which are forbidden or strongly suppressed in vacuum. However, the magnetic field influences significantly the quantum processes only in the case when it is strong enough. There exists a natural scale for the field strength which is the so-called critical value $B_e = m_e^2/e \simeq 4.41 \cdot 10^{13}$ G (we use natural units in which $c = \hbar = 1$, hereafter e is the elementary charge).

There exist arguments that field of such and essentially greater scale can appear in astrophysical objects. Thus, a class of stars exists, the so-called magnetars, which are neutron stars with magnetic fields $\sim 4 \cdot 10^{14}$ G [3, 4]. Models of astrophysical processes and objects are discussed, where magnetic fields of the order $10^{17} - 10^{18}$ G can be generated [5–8]. Thereby, physics of quantum processes in strong external fields presents itself as an interesting and important direction of studies, both from a conceptual standpoint, and in light of possible astrophysical implications.

Among others, the set of quantum processes is very interesting where only electrically neutral particles, such as neutrinos and photons, are presented in the initial and final states. The external field influence on these loop processes is provided, first, by the sensitivity of the charged virtual fermion to the field, and the electron plays the main role here as the particle with the maximal specific charge, e/m_e. Secondly, strong magnetic field essentially influences the dispersion properties of photons, and consequently it changes their kinematics.

In the recent paper [2] a contribution was studied, in particular, of the loop process of the neutrino pair photoproduction on nucleus

$$\gamma + Ze \to Ze + \gamma + \nu + \bar{\nu} \tag{1}$$

in strong external magnetic field, into the star cooling. An important conclusion was made there, that a contribution of this process could compete with the contribution of URCA - processes. Thereby, the process (1) should be taken into account in the description of a cooling of strongly magnetized neutron star, as one more channel of neutrino energy losses.

Here, we present the result of a new study of the process of neutrino pair photoproduction on nucleus, Fig. 1. We show that with the dispersion of a photon in strong magnetic field taken into account, a catalizing influence of the field on the process (1) vastly decreases. This effect was not considered in Ref. [2] with the result that the contribution of the loop process turned out to be overestimated in many orders of magnitude.

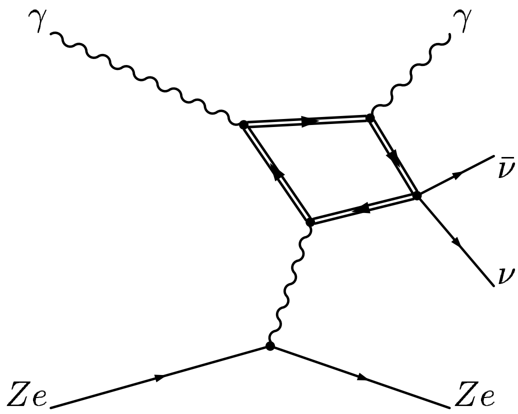

Figure 1: The Feynman diagram for the neutrino pair photoproduction on nucleus in a magnetic field.

2 The process $\gamma + \gamma + \gamma \to \nu + \bar{\nu}$ in a strong magnetic field

We start from this photon - neutrino process which is symmetric with respect to the photon interchange. The crossed process having a physical meaning is the photon - neutrino process $\gamma\gamma \to \nu\bar{\nu}\gamma$, and the history of its investigations is rather long. It was studied in vacuum by Van Hieu and Shabalin [9] and by Dicus and Repko [10]. The process was studied in a strong magnetic field ($B \gtrsim B_e$) in the paper [11].

The amplitude of the process in a strong field has the form

$$\begin{aligned} \mathcal{M} &= -\frac{8\,e^3\,G_F\,eB}{\sqrt{2}\pi^2\,m_e^6}\,(\varepsilon_1\tilde{\varphi}k_1)\,(\varepsilon_2\tilde{\varphi}k_2)\,(\varepsilon_3\tilde{\varphi}k_3)\;\times \\ &\times\;[C_V\,(j\tilde{\varphi}k_4) + C_A(j\tilde{\varphi}\tilde{\varphi}k_4)]I\left(\frac{k_1}{m_e},\frac{k_2}{m_e},\frac{k_3}{m_e}\right), \end{aligned} \tag{2}$$

where $C_V,\ C_A$ are the vector and axial-vector constants of the effective $\nu\nu ee$ Lagrangian,

$$C_V = \pm 1/2 + 2\sin^2\theta_W,\ C_A = \pm 1/2, \tag{3}$$

(here the upper signs correspond to ν_e, and the lower signs correspond to ν_μ and ν_τ); $\varepsilon_{1,2,3}$ and $k_{1,2,3}$ are the polarization 4-vectors and the momenta of photons, $j_\alpha = [\bar{\nu}(q_1)\gamma_\alpha(1+\gamma_5)\nu(-q_2)]$ is the Fourier transform of the neutrino current, $k_4 = q_1 + q_2$ is the neutrino pair momentum, $\tilde{\varphi}_{\alpha\beta} = \tilde{F}_{\alpha\beta}/B$ is the dimensionless dual tensor of external magnetic field, $\tilde{F}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\mu\nu}F_{\mu\nu}$. The tensor indices of four-vectors and tensors standing inside the parentheses are contracted consecutively, for example $(a\tilde{\varphi}b) = a_\alpha \tilde{\varphi}_{\alpha\beta} b_\beta$.

The formfactor $I\left(\frac{k_1}{m_e}, \frac{k_2}{m_e}, \frac{k_3}{m_e}\right)$ has a complicated form of a triple integral over the Feynman variables. In the case of low photon energies, $\omega_{1,2,3} \ll m_e$, the integral can be easily calculated:

$$I\left(\frac{k_1}{m_e}, \frac{k_2}{m_e}, \frac{k_3}{m_e}\right) \simeq \frac{1}{60}. \tag{4}$$

In this case, the above amplitude corresponds to the effective local Lagrangian of the $\gamma\gamma\gamma\nu\bar{\nu}$ interaction:

$$\begin{aligned} \mathcal{L}_{eff} &= -\frac{e^3\, G_F\, eB}{45\,\sqrt{2}\pi^2\, m_e^6} \left(\frac{\partial A^\alpha}{\partial x_\beta}\,\tilde{\varphi}_{\alpha\beta}\right)^3 \times \\ &\times \frac{\partial}{\partial x_\sigma}[\bar{\nu}\gamma^\rho(1+\gamma_5)\nu]\,[C_V\,\tilde{\varphi}_{\rho\sigma} + C_A\,(\tilde{\varphi}\tilde{\varphi})_{\rho\sigma}]. \end{aligned} \tag{5}$$

We note that $\gamma\gamma\gamma\nu\bar{\nu}$ interaction in the low-energy limit was studied earlier by Loskutov and Skobelev [11], however, their Lagrangian has an extra factor of 2.

The dimensional analysis of the amplitude with respect to a typical photon energy, $|k_1| \sim |k_2| \sim |k_3| \sim \omega$, shows an essential distinction of the cases of low energies ($\mathcal{M} \sim \omega^5$), and high energies, ($\mathcal{M} \sim \omega^{-3}$).

3 Photon dispersion and kinematics in a strong magnetic field

In analyses of the photon processes in a strong magnetic field, the field influence on the photon dispersion properties is a crucial factor, and it has to be taken into account. We remind that only photons of transversal polarization [13] participate in the processes in a strong magnetic field. For virtual photon, instead of the propagator $\sim q^{-2}$, one should use the propagator with the vacuum polarization in a magnetic field:

$$D^{(B)}(q_\parallel^2, q_\perp^2) \;=\; \frac{1}{q^2 - P(q_\parallel^2)}, \tag{6}$$

here $q_\parallel^2 = q_0^2 - q_z^2$, $q_\perp^2 = q_x^2 + q_y^2$, $q^2 = q_\parallel^2 - q_\perp^2$ (the magnetic field is directed along the z axis), $P(q_\parallel^2)$ is the photon polarization operator in the field, which has a rather simple form in a strong field, $B \gg B_e$, and in an approximation $|q_\parallel^2| \ll m_e^2$ [14]:

$$P(q_\parallel^2) \simeq -\frac{\alpha}{3\pi}\,\frac{B}{B_e}\,q_\parallel^2. \tag{7}$$

It is convenient to introduce a dimensionless parameter which defines the field influence in expressions below:

$$\beta = \frac{\alpha}{3\pi}\,\frac{B}{B_e}. \tag{8}$$

For the field values $10^3\,B_e$ and $10^4\,B_e$, the parameter β is 0.77 and 7.7 consequently, so, it is not the small one.

Finally, taking $q_0 = 0$ for a virtual photon coupled with a nucleus at rest, one obtains the propagator:

$$D^{(B)} \;\simeq\; -\,\frac{1}{q_\perp^2 + (1+\beta)q_z^2}. \tag{9}$$

On the other hand, real photons participating in the process are also under the influence of a strong magnetic field, which leads to the renormalization of the photon wave-functions:

$$\varepsilon_\alpha \longrightarrow \sqrt{\mathcal{Z}}\,\varepsilon_\alpha, \tag{10}$$

where the renormalization factor $\mathcal{Z}$ takes the form

$$\mathcal{Z} = \left(1 - \frac{dP(q_\parallel^2)}{dq_\parallel^2}\right)^{-1} = \frac{1}{1+\beta}. \tag{11}$$

The kinematics of photons is also modified by the field. The photon dispersion equation $k^2 - P(k_\parallel^2) = 0$ can be rewritten to the form $\omega^2 = \mathbf{k}^2(1 + \beta\cos^2\theta)/(1+\beta)$, and the element of the momentum space becomes

$$d^3k = (1+\beta)\,\omega^2 d\omega\,dy\,d\varphi, \quad y = \cos\theta\sqrt{1+\beta}/\sqrt{1+\beta\cos^2\theta},$$

where θ, φ are the polar and the azimuthal angles.

4 The process of neutrino photoproduction on nuclei

Using the effective local Lagrangian of the $\gamma\gamma\gamma\nu\bar{\nu}$ interaction, with the field influence on the photon properties, and with the substitution of the photon $\perp$ polarization vectors

$$\varepsilon_\alpha^{(\perp)} = \sqrt{\mathcal{Z}}\, \frac{(\tilde{\varphi}k)_\alpha}{\sqrt{k_\parallel^2}}, \tag{12}$$

the amplitude for the process

$$\gamma + Ze \to Ze + \gamma + \nu + \bar{\nu}$$

can be presented in the form

$$\mathcal{M} = \frac{32\pi\alpha Z\, G_F}{5\,\sqrt{2}\, m_e^4}\, \frac{\beta}{1+\beta}\, \frac{2m_N\, q_z \sqrt{k_{1\parallel}^2 k_{2\parallel}^2}}{q_\perp^2 + (1+\beta) q_z^2}\, [C_V\,(j\tilde{\varphi}k_4) + C_A(j\tilde{\varphi}\tilde{\varphi}k_4)], \tag{13}$$

where m_N is the nucleus mass, q is the momentum transferred, $q^\alpha = (0, \mathbf{q})$.

Our amplitude differs essentially from the amplitude obtained in the paper [2], where the strong magnetic field influence on the photon dispersion properties was not taken into account.

5 The neutrino emissivity

The neutrino emissivity is the energy carried out by neutrinos from unit volume per unit time. It is defined in terms of the process amplitude (13) as follows

$$\begin{aligned} Q_\nu \;&=\; \frac{(2\pi)^4 n_N}{2m_N} \int |\mathcal{M}|^2\, (\varepsilon_1 + \varepsilon_2)\, \delta^4(k_1 - k_2 - q_1 - q_2 - q) \times \\ &\times\; \frac{d^3k_1}{(2\pi)^3 2\omega_1} f(\omega_1) \frac{d^3k_2}{(2\pi)^3 2\omega_2} [1 + f(\omega_2)] \;\times \\ &\times\; \frac{d^3q_1}{(2\pi)^3 2\varepsilon_1}\, \frac{d^3q_2}{(2\pi)^3 2\varepsilon_2}\, \frac{d^3q}{(2\pi)^3 2m_N}, \end{aligned} \tag{14}$$

where n_N is the nuclei density, ε_1 and ε_2 are the neutrino and antineutrino energies, $f(\omega) = [\exp(\omega/T) - 1]^{-1}$ is the density of the photon gas in equilibrium at the temperature T. One obtains

$$Q_\nu = \frac{8\,(2\pi)^9}{225}\; Z^2\,\alpha^2\,G_F^2\,m_e^6\,n_N \left(\frac{T}{m_e}\right)^{14} J\,(\beta). \tag{15}$$

The dependence of the value Q_ν on the field parameter β (8) is defined by the function $J\,(\beta)$ as follows

$$\begin{aligned} J\,(\beta) \;&=\; \beta^2 \int\limits_{-1}^{1} du\,(1-u^2) \int\limits_{-1}^{1} dv\,(1-v^2) \int\limits_{0}^{1} ds\, s^3\,(1-s)^8 \int\limits_{0}^{1} dr\, r^2 \,\times \\ &\times \int\limits_{-1}^{1} dx[u - sv - (1-s)rx]^2 (1 - r^2x^2)\,\times \\ &\times \left[\overline{C_V^2}(1-r^2) + \overline{C_A^2} r^2(1-x^2)\right] \int\limits_{0}^{2\pi} \frac{d\varphi_1}{2\pi} \int\limits_{0}^{2\pi} \frac{d\varphi_2}{2\pi}\,\frac{1}{[F(\beta)]^2}, \end{aligned} \tag{16}$$

$$\begin{aligned} F(\beta) \;&=\; (1+\beta)\Big\{1 - u^2 + s^2(1-v^2) - 2s\sqrt{1-u^2}\sqrt{1-v^2}\cos\varphi_1 + \\ &+\; [u - sv - (1-s)rx]^2\Big\} - 2\sqrt{1+\beta}(1-s)r\sqrt{1-x^2}\,\times \\ &\times\; \left[\sqrt{1-u^2}\cos\varphi_2 - s\sqrt{1-v^2}\cos(\varphi_2 - \varphi_1)\right] + \\ &+\; (1-s^2)r^2(1-x^2). \end{aligned} \tag{17}$$

The constants $\overline{C_V^2} = 0.93$ and $\overline{C_A^2} = 0.75$ under the integral are summarized over all channels of the neutrino production, ν_e, ν_μ, ν_τ.

The dependence of the function $J(\beta)$ on the field parameter β is shown in Fig. 2.

The upper bound for the value Q_ν in the asymptotically strong field is

$$Q_\nu \lesssim 2.3 \cdot 10^{27} \left(\frac{T}{m_e}\right)^{14} \left\langle \frac{Z^2}{A} \right\rangle \left(\frac{\rho}{\rho_0}\right) \frac{\text{erg}}{\text{cm}^3\,\text{s}}, \tag{18}$$

where Z is the charge number and A is the mass number of a nucleus, $< Z^2/A >$ means the averaging over all nuclei, $\rho_0 = 2.8 \cdot 10^{14}\,\text{g/cm}^3$ is the typical nuclear density, and ρ is the averaged density of a star.

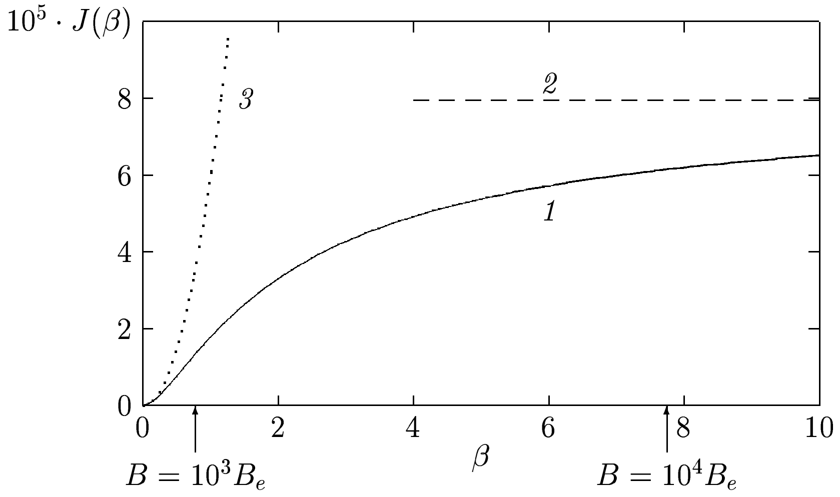

Figure 2: The dependence of the function $J(\beta)$ on the field parameter β (the curve *1*). The line *2* shows the asymptotics of the function at large values of β, $J(\beta) \to 8 \cdot 10^{-5}$. The curve *3* shows the dependence $\sim \beta^2$, which would take place without taking account of the magnetic field influence on the photon dispersion.

The Eq. (18) should be compared with the neutrino emissivity via the standard channel of the modified URCA process

$$Q_\nu(\mathrm{URCA}) \sim 10^{27} \left(\frac{T}{m_e}\right)^8 \left(\frac{\rho}{\rho_0}\right)^{2/3} \frac{\mathrm{erg}}{\mathrm{cm}^3\ \mathrm{s}}. \tag{19}$$

At first glance, these values for the emissivities could compete. However, a big numerical factor in the neutrino photoproduction emissivity arises from the integral over the initial photon energy ω_1 $(x = \omega_1/T)$

$$\int\limits_0^\infty \frac{x^{13}\, dx}{e^x - 1} \;=\; 13!\, \zeta(14) \;=\; \frac{(2\pi)^{14}}{24} \;\simeq\; 6.2 \cdot 10^9. \tag{20}$$

It is obvious, that the integral (20) acquires its big value in the region of the argument

$$x \sim 10 \div 20, \quad \omega_1 \sim (10 \div 20)\, T.$$

Thus, as the amplitude of the neutrino photoproduction is obtained within the approximation

$$\omega \lesssim m_e,$$

the above expression for the neutrino emissivity is true at the photon gas temperatures

$$T \lesssim (1/10)\, m_e,$$

however, it is obviously incorrect at $T \sim m_e$.

Consequently, the assumption that the factor $(T/m_e)^{14}$ could be taken of the order unity [2], is wrong. Within the area of applicability one obtains

$$(T/m_e)^{14} \lesssim 10^{-14}.$$

In summary, the neutrino photoproduction on nuclei cannot compete with URCA processes for any values of the magnetic field strength.

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