Fading of chiral strings loops to vortons

E. Babichev, V. Dokuchaev

September 27, 2002

Abstract

The damping of oscillations of chiral string loops due to electromagnetic and gravitational radiation is studied. We derive expressions for energy losses into the gravitational and electromagnetic waves from the closed superconducting chiral cosmic strings of arbitrary form. For the case of large current on the string we describe the asymptotic behaviour of chiral loops and their fading to the stationary state (vortons). General limits on the gravitational and electromagnetic energy losses by a near stationary chiral loops are found. For these loops we estimate the oscillation damping time. The analytical dependence of string energy with time is found in the case of the chiral ring with small amplitude radial oscillations.

1 Introduction

We study the gravitational and electromagnetic radiation of energy from superconducting closed cosmic strings with chiral current and their fading into the stationary state (vortons). Cosmic strings are linear topological defects, that may have been created during phase transitions in the early Universe (see e. g. reviews in [1, 2]). Witten [3] has shown that strings could be superconducting in certain particle physics models. The presence of current on a string leads to the principal specific feature: the superconducting string loop may form a stable stationary configuration [4, 5, 6]. Cosmic strings lose their energy on gravitational and electromagnetic radiation (if string is superconducting). As a result, the "ordinary" not extremely long cosmic strings without the current evaporate completely during the cosmological time. On the contrary the superconducting string loops could survive due to the presence of conserved "charge" and tend to the stable configuration which is named the chiral vorton [4].

While moving cosmic string sweeps out a two-dimensional world-sheet in the Minkowskian space-time. The four-dimensional coordinates of string are functions of two world-sheet parameters $x^\mu = x^\mu(\sigma^a)$, where indexes a take values 0, 1 and σ^a are correspondingly the coordinates on a two-dimensional world-sheet. The convenient gauge choice is such that σ^0 is the Minkowskian time t and σ^1 parameterizes the string total energy:

$$E = \mu \int d\sigma. \tag{1}$$

In this gauge the general solution of the equations of motion of the chiral string is [7, 8, 9]:

$$x^0 = t, \quad \mathbf{x}(t,\sigma) = \frac{\mathcal{L}}{4\pi}\left[\mathbf{a}(\xi) + \mathbf{b}(\eta)\right], \tag{2}$$

where $\mathcal{L}$ is the invariant length of the string, $\mathbf{a}(\xi)$ and $\mathbf{b}(\eta)$ are arbitrary vector functions of $\xi = (2\pi/\mathcal{L})(\sigma - t)$ and $\eta = (2\pi/\mathcal{L})(\sigma + t)$ obeying the following conditions:

$$\mathbf{a}'^2 = 1, \quad \mathbf{b}'^2 = k^2(\eta) \le 1. \tag{3}$$

In the case of closed chiral strings (loops) the vector functions $\mathbf{a}(\xi)$ and $\mathbf{b}(\eta)$ form closed loops, called a- and b- loops. The function $k(\eta)$ in (3) may be expressed as follows [9]:

$$k^2(\eta) = 1 - \frac{4F'^2(\eta)}{\mu}, \tag{4}$$

where function $F(\eta)$ defines in turn the auxiliary scalar field

$$\phi(\sigma,t) = \frac{\mathcal{L}}{2\pi}F(\eta). \tag{5}$$

According to (5) the scalar field $\phi(\sigma,t)$ is an arbitrary function of the only parameter η. The four-dimensional current on the string is expressed through this scalar field $\phi(\sigma,t)$ in the following way [10]:

$$j^\mu(\mathbf{x},t) = q \int d\sigma \phi'(\sigma,t)(x'^\mu - \dot{x}^\mu)\delta^{(3)}\left(\mathbf{x} - \mathbf{x}(\sigma,t)\right), \tag{6}$$

where x' denotes $\partial x/\partial\sigma$ and $\dot{x}$ denotes $\partial x/\partial t$. The energy-momentum tensor of the string in this gauge is

$$T^{\mu\nu} = \mu \int d\sigma \left(\dot{x}^\mu \dot{x}^\nu - x'^\mu x'^\nu\right)\delta^{(3)}\left(\mathbf{x} - \mathbf{x}(\sigma,t)\right). \tag{7}$$

2 Radiation from chiral loops

For any periodic system the gravitational and electromagnetic energy radiation rates (averaged over the period T) per solid angle $d\Omega$ is given by series

$$\frac{d\dot{E}^{\rm gr}}{d\Omega} = \sum_{l=1}^{\infty} \frac{d\dot{E}^{\rm gr}(\omega_l)}{d\Omega}, \quad \frac{d\dot{E}^{\rm em}}{d\Omega} = \sum_{n=1}^{\infty} \frac{d\dot{E}^{\rm em}(\omega_l)}{d\Omega}, \tag{8}$$

where $\omega_l = 2\pi l/T$, $\mathbf{n}$ is an arbitrary unit vector and [11, 12]

$$\frac{d\dot{E}^{\rm gr}(\omega_l)}{d\Omega} = \frac{G\omega_l^2}{\pi}[\tau_{pq}^* \tau_{pq} - \frac{1}{2}\tau_{qq}^* \tau_{pp}], \quad \frac{d\dot{E}^{\rm em}(\omega_l)}{d\Omega} = \frac{\omega_l^2}{2\pi}\tilde{\iota}_p^* \tilde{\iota}_p, \tag{9}$$

where τ_{pq} and ι_p are correspondingly Fourier-transforms of an energy-momentum tensor and current in the corotating basis $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3) \equiv (\mathbf{n}, \mathbf{v}, \mathbf{w})$. Note that only indexes p, q with values 2 and 3 appear in the equation (9).

Usually the total rates per unit time (averaged over the period) are calculated by summing of losses in different modes. In practical numerical calculations the values of $\dot{E}$, $\dot{P}$ and $\dot{L}$ are determined with the accuracy up to the l of a few hundred. Such calculations may be not correct because of the slow convergence of the corresponding sums over l as was pointed out by Allen et al. [13]. The summation over radiation modes for energy, momentum and angular momentum losses were done analytically in Ref. [14]. The corresponding gravitational and electromagnetic energy radiation rates per unit solid angle is given

$$\begin{aligned} \frac{d\dot{E}^{\rm gr}}{d\Omega} &= \frac{G\mu^2}{16\pi^3}\int d^4\xi \mathcal{P}^{\rm gr}(\Delta x \bmod 2\pi - \pi)^2, \\ \frac{d\dot{E}^{\rm em}}{d\Omega} &= \frac{q^2\mu}{32\pi^3}\int d^4\xi \mathcal{P}^{\rm em}(\Delta x \bmod 2\pi - \pi)^2, \end{aligned} \tag{10}$$

where:

$$\begin{aligned} \Delta x &= \xi - \xi' - (\eta - \eta') + \mathbf{n}[\mathbf{a}(\xi) - \mathbf{a}(\xi') + \mathbf{b}(\eta) - \mathbf{b}(\eta')], \\ \mathcal{P}^{\rm gr} &= \mathcal{T}'_{pq}\mathcal{T}_{pq} - \frac{1}{2}\mathcal{T}'_{qq}\mathcal{T}_{pp}, \quad \mathcal{P}^{\rm em} = \mathcal{J}'_p \mathcal{J}_p, \end{aligned} \tag{11}$$

where, in turn,

$$\mathcal{T}_{ij} = \mathcal{I}_i\mathcal{Y}_j + \mathcal{I}_j\mathcal{Y}_i, \quad \mathcal{J}_i = \mathcal{I}_i\mathcal{X}, \tag{12}$$

here the functions $\mathcal{I}_i$, $\mathcal{Y}_j$ and $\mathcal{X}$ are given by

$$\mathcal{I}_i = \left[\frac{\mathbf{a}'\mathbf{e_i}}{1 + \mathbf{n}\mathbf{a}'}\right]', \quad \mathcal{Y}_j = \left[\frac{\mathbf{b}'\mathbf{e_j}}{1 - \mathbf{n}\mathbf{b}'}\right]', \quad \mathcal{X} = \left[\frac{\sqrt{1 - |\mathbf{b}'|^2}}{1 - \mathbf{n}\mathbf{b}'}\right]'. \tag{13}$$

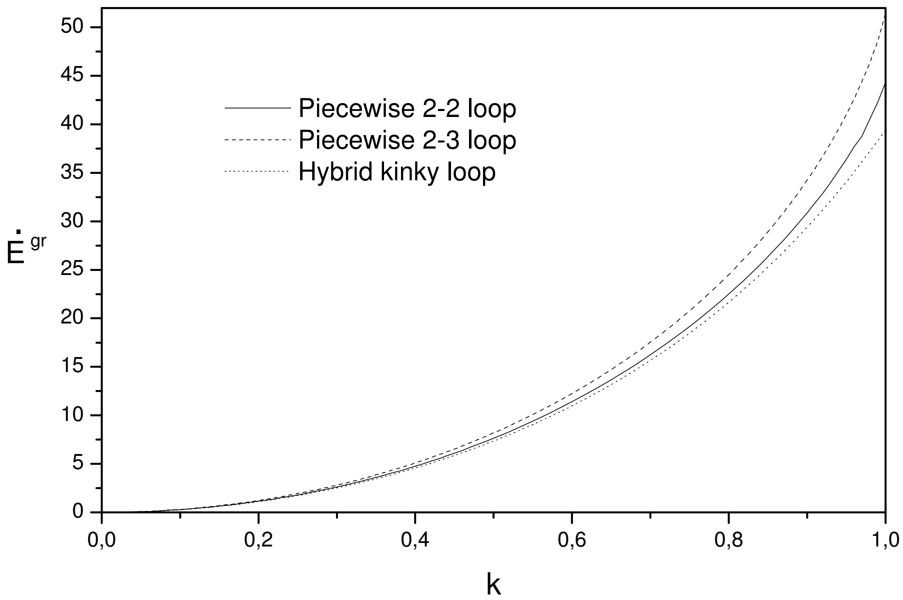

Figure 1: Total radiated gravitational energy $\dot{E}^{gr}$ in units $G\mu^2$ for the 2-2, 2-3 piecewise and hybrid kinky loops is shown as a function of parameter k. For 2-2 loop $\alpha = \pi/2$, for 2-3 loop $\beta = \pi/4$, for hybrid loop $\gamma = 0$.

One can observe from (10) that rates (averaged per oscillation period) of energy losses into electromagnetic and gravitational waves can be expressed generally in the following form:

$$\dot{E}^{\rm gr} = \Gamma_E^{\rm gr} G\mu^2, \quad \dot{E}^{\rm em} = \Gamma_E^{\rm em} \mu q^2, \tag{14}$$

where the coefficients $\Gamma_E^{\rm gr}$ and $\Gamma_E^{\rm em}$ depend only on the loop configuration.

Let us now consider some examples of chiral loops and calculate the gravitational and electromagnetic radiation from them. First example is piece-wise linear loop:

$$\mathbf{a} = \mathbf{A} \begin{cases} (\xi - \pi/2)\,, & \xi \in [0, \pi), \\ (3\pi/2 - \xi)\,, & \xi \in [\pi, 2\pi), \end{cases} \quad \mathbf{b} = k\mathbf{B} \begin{cases} (\eta - \pi/2)\,, & \eta \in [0, \pi), \\ (3\pi/2 - \eta)\,, & \eta \in [\pi, 2\pi), \end{cases} \tag{15}$$

Using (10) we calculate numerically the gravitational and electromagnetic radiation (see Fig.1 and 2).

As the second example we consider the more complicated configuration of piece-wise linear loops. Let a-loop consist of 2 segments and lie along the z-axis. One kink of a-loop is positioned at the origin ($\xi = 0$) and the another kink ($\xi = \pi/2$) has coordinates $(0, 0, \pi/2)$. The positions of kinks of b-loop are given by the following coordinates: the first kink at $\eta = 0$ is positioned at the

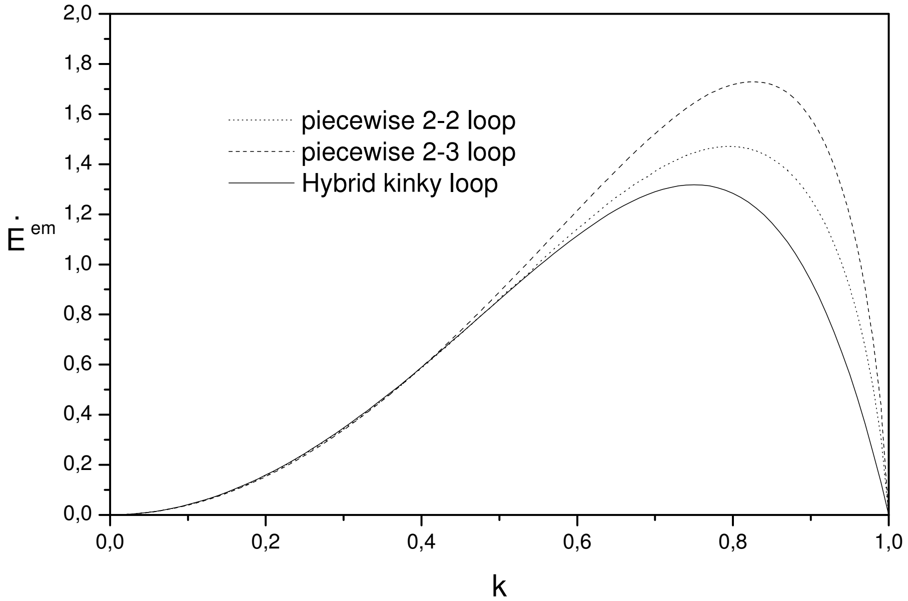

Figure 2: Total radiated electromagnetic energy $\dot{E}^{em}$ in units $q^2\mu$ for the 2-2, 2-3 piece-wise and hybrid kinky loop is shown as a function of parameter k. For 2-2 loop $\alpha = \pi/2$, for 2-3 loop $\alpha = \pi/4$, for hybrid loop $\alpha = 0$.

origin; the second kink at $\eta = \pi/3$ has coordinates $-(k\pi/3)(\cos\beta, \sqrt{3}, \sin\beta)$ and the third kink at $\eta = 2\pi/3$ has coordinates $(k\pi/3)(\cos\beta, -\sqrt{3}, \sin\beta)$. The results for gravitational and electromagnetic radiation are presented on Figs. 1 and 2.

The third example is the hybrid loop of the following kind:

$$\mathbf{a} = (\sin\xi,\ -\cos\xi,\ 0), \quad \mathbf{b} = k\mathbf{B} \left\{ \begin{array}{ll} (\eta - \pi/2)\,, & 0 \le \eta \le \pi, \\ (-\eta + 3\pi/2)\,, & \pi \le \eta \le 2\pi. \end{array} \right. \tag{16}$$

In this example the a-loop lies in the (x, y) plane and $\mathbf{B} = (\cos\gamma; 0; \sin\gamma)$. For $\gamma = \pi/2$ the gravitational and electromagnetic radiated energy and angular momentum are shown on the Figs. 1 and 2.

Let us now consider the case of large currents (almost at the maximum value). It means that the string loop is close to the stationary vorton state. First we estimate the upper limits for the gravitational and electromagnetic radiation. Using the smallness of k from (10) we find [16]:

$$\left|\dot{E}^{\rm gr}\right| \le 32\pi^4 b_3^2 G\mu^2, \quad \left|\dot{E}^{\rm em}\right| \le \frac{4}{3}\pi^4 b_3^2 q^2 \mu. \tag{17}$$

where b_3 is a maximum value $|b'''(\eta)|$ on the segment $\eta \in (0, 2\pi)$.

Let us further assume for simplicity that the current is constant along the string, $k = const$. Then from (10) we see that the radiated power can be rewritten in the following way:

$$\dot{E}^{\rm gr} = K^{\rm gr} G\mu^2 k^2, \quad \dot{E}^{\rm em} = K^{\rm em} q^2 \mu k^2, \tag{18}$$

where $K^{\rm em}$ and $K^{\rm gr}$ are numerical factors, depending only on the loop geometry. We see that radiation power of the near stationary chiral loops is proportional to k^2. The geometrical numerical factors $K^{\rm gr}$ and $K^{\rm em}$ in turn is connected with the corresponding parameters $\Gamma_E^{\rm gr}$ and $\Gamma_E^{\rm em}$ in equation (14) by relation

$$\Gamma^{\rm gr} = K^{\rm gr} k^2, \quad \Gamma^{\rm em} = K^{\rm em} k^2. \tag{19}$$

For the following three examples we calculate the coefficients $K^{\rm gr}$ and $K^{\rm em}$: (i) the radially oscillating ring, which configuration is given by

$$\mathbf{a} = (\cos\xi,\ -\sin\xi,\ 0), \quad \mathbf{b} = k(\cos\eta,\ -\sin\eta,\ 0), \tag{20}$$

(ii) the piece-wise linear loop (15) and (iii) the hybrid loop of the following configuration:

$$\mathbf{a} = \mathbf{A} \begin{cases} (\xi - \pi/2)\,, & 0 \le \xi \le \pi, \\ (-\xi + 3\pi/2)\,, & \pi \le \xi \le 2\pi, \end{cases} \quad \mathbf{b} = k(\sin\eta,\ -\cos\eta,\ 0). \tag{21}$$

We obtain respectively for the first example $K^{\rm gr} = 4.73$ and $K^{\rm em} = 2.28$; for the second example $K^{\rm gr} = 7.63$ and $K^{\rm em} = 3$; for the third loop $K^{\rm gr} = 7.63$ and $K^{\rm em} = 3$.

3 Damping of loop oscillations

Let us evaluate now the damping time of small amplitude oscillations of a near stationary chiral string loops corresponding to the limit $k \ll 1$ in (3). For simplicity we assume that k does not depend on η in the considered limit (this assumption is held true in the considered above solvable examples). Then a total loop charge conservation in (6) gives

$$\frac{q\sqrt{\mu}}{2} L\sqrt{1-k^2} = const. \tag{22}$$

From this equation we find the relation between energy E and parameter k of the chiral string with small amplitude oscillations:

$$E \simeq E_v \left(1 + \frac{k^2}{2}\right), \tag{23}$$

where $E_v = L\mu$ is an energy of the stationary (vorton) chiral loop configuration at $k = 0$. Comparing (23) with (18) we estimate the damping time of string oscillations

$$\tau \sim \frac{E_v}{2(K^{\rm gr}G\mu^2 + K^{\rm em}q^2\mu)}. \tag{24}$$

It is convenient to express the damping time (24) through the vorton physical length:

$$\tau \sim \frac{L_{\rm ph}}{K^{\rm gr}G\mu + K^{\rm em}q^2}, \tag{25}$$

where the physical string length $L_{\rm ph}$ is connected with an invariant length of the string by the relation $L_{\rm ph} = L/2$ [15]. In the case of the chiral ring (20) it is naturally to assume that evolving ring saves its form (due to the symmetry) during the damping of small amplitude oscillations. In this case therefore we can find precisely the evolution of the radiating string with time. Only k varies in time if the shape of the string is invariable. Solving together (18) and (23), we find the law of oscillation damping in the near stationary chiral ring [16]

$$k^2 \simeq k_0^2 e^{-t(1/\tau_c^{\rm gr} + 1/\tau_c^{\rm em})}, \tag{26}$$

where $k_0 = k(t = 0)$, and the damping times due to gravitational and electromagnetic radiation correspondingly:

$$\tau_c^{\rm gr} = \frac{L_{\rm ph}}{K^{\rm gr}G\mu}, \quad \tau_c^{\rm em} = \frac{L_{\rm ph}}{K^{\rm em}q^2}. \tag{27}$$

Substituting (26) in (23) we obtain [16]

$$E \simeq E_v \left[1 + \frac{k_0^2}{2} e^{-t(1/\tau_c^{\rm gr} + 1/\tau_c^{\rm em})}\right]. \tag{28}$$

An effective number of oscillations during the damping time (oscillator quality) is

$$Q = \frac{\tau}{T} = \frac{2}{L}\frac{\tau^{\rm gr}\tau^{\rm em}}{\tau^{\rm gr} + \tau^{\rm em}}. \tag{29}$$

The ratio of damping times due to gravitational and electromagnetic radiation is

$$\tau^{\rm gr}/\tau^{\rm em} \sim 1.4 \times 10^{-4}\frac{q_e^2}{\mu_6}\frac{K^{\rm em}}{K^{\rm gr}}, \tag{30}$$

where $G\mu/c^2 = 10^{-6}\mu_6$ and $q_e = q/e$. If $q_e^2/\mu_6 \gtrsim 1.4 \times 10^{-3}$, then electromagnetic radiation prevails in the chiral loop evolution (it is valid for the standard values of $\mu_6 \sim 1$ and $q_e \sim 1$). If on the contrary $q_e^2/\mu_6 \lesssim 1.4 \times 10^{-3}$ (for example, if a current is neutral and there is no electromagnetic radiation at all), then pure gravitational radiation determines the evolution.

4 Astrophysical consequences

Let us estimate a characteristic size of the string $L_{\rm v}$ with oscillation damping time (25) equals to the universe lifetime $t_0 \simeq 10^{18}$ s. In the case of the gravitational radiation predominance we find

$$L_{\rm v}^{\rm gr} \simeq \frac{G\mu K^{\rm gr} t_0}{c} \simeq 10^2 \mu_6 \text{ kpc} \tag{31}$$

for $K^{\rm gr} \sim 1$. So the chiral strings with length $L < L_{\rm v}^{\rm gr}$ (i. e. with a size of typical galactic halo or less) have had enough time to fade into vortons. On the other hand if the electromagnetic radiation prevails, we have

$$L_{\rm v}^{\rm em} \simeq \frac{q^2 K^{\rm em} t_0}{\hbar} \simeq 70 q_e^2 \text{ Mpc} \tag{32}$$

for $K^{\rm em} \sim 1$ and so the electromagnetically radiated chiral loops with the length shorter than the size of galactic clusters have had transformed now to vortons.

From the damping time estimation it follows that only sufficiently long superconducting cosmic strings oscillate up to the present time. On the contrary the small scale chiral loops transformed into the stationary vortons due to the oscillation damping. Namely, the minimal length of presently oscillating chiral loop varies from $L_{\rm v}^{\rm gr} \sim 10^2 \mu_6$ kpc for gravitational radiation domination to $L_{\rm v}^{\rm em} \sim 70 q_e^2$ Mpc for electromagnetic radiation domination depending on the relations between μ and q. It appears that the oscillator quality of chiral loops (29) is independent on the loop length and is determined only by the loop shape through geometric parameters $K^{\rm gr}$ and $K^{\rm em}$. For characteristic values of $K^{\rm gr} \sim 10$ and $K^{\rm em} \sim 1$ the corresponding oscillator qualities for the gravitational and electromagnetic radiation are $Q^{\rm gr} \sim 10^5/\mu_6$ and $Q^{\rm em} \sim 137/q_e^2$.

References

[1] E. P. S. Shellard and A. Vilenkin, Cosmic Strings and other Topological Defects (Cambridge University Press, Cambridge, England, 1994).

[2] M. B. Hindmarsh and T. W. B. Kibble, Rep. Prog. Phys. **58**, 477 (1995).

[3] E. Witten, Nucl. Phys. **B249**, 557 (1985).

[4] R. L. Davis and E. P. S. Shellard, Phys. Lett. **B209**, 485 (1988).

[5] D. Haws, M. Hindmarsh and N. Turok, Phys. Lett. **B209**, 255 (1988).

[6] E. Copeland, D. Haws, M. Hindmarsh and N. Turok, Nucl. Phys. **B306**, 908 (1988).

[7] B. Carter and P. Peter, Phys. Lett. **B466**, 41 (1999).

[8] A. C. Davis, T. W. B. Kibble, M. Pickles and D.A. Steer, Phys. Rev. **D62** (2000) 083516.

[9] J.J. Blanco-Pillado, Ken D. Olum and A. Vilenkin, Phys. Rev. **D63**, 103513.

[10] A. Vilenkin and T. Vachaspati, Phys.Rev.Lett **58**, 1041 (1987).

[11] S. Weinberg, Gravitation and Cosmology (Wiley, New York, 1974).

[12] R. Durrer, Nucl. Phys. **B328**, 238 (1989).

[13] B.Allen and P.Casper, Phys.Rev. **D50**, 2496 (1994).

[14] E.Babichev and V.Dokuchaev, to be published in Nucl. Phys. B. (2002).

[15] D.A. Steer, Phys.Rev. **D63**, (2001) 083517.

[16] E. Babichev and V. Dokuchaev, Phys. Rev. **D66**, (2002) 025007.