T

~~to explain t~~he matter/anti-matter asymmetry of the Universe in terms of the

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physics of the electroweak phase transition. Such scenarios typically depon (among other things) t

process in high-temperature non-Abelian plasmas—has long been a source of
theoretical confusion. In fact, it is only recently becoming clear how the scales with the Γne structure c

For non-Abelian plasmas at high temperatures, non-perturbative =uctuations of the gauge Γeld are associated with magnetic =uctuations over

distance scales $L \sim 1/g$

processes was also of order $1/g$

therefore of order T. Two years ago, we $1/L \sim t \sim \alpha$ argued [4] that damping

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eΓFects in the plasma slow the time scale down to $t \sim 1/g$$T$, giving a rate of
$\Gamma \sim 1 \alpha a t \alpha_3$ $t \sim$. More recently, BБdeker [5] has claimed that there is an
additional logarithmic suppression of the time scale, so ΓFt $1g$ 5 4 4

$T \ln(1/g,$ and $\Gamma \sim \alpha T$ $\ln(1/\alpha\alpha)$. 5BБdeker has also prop

numerical coe¿cient c of the logarithm:

$\Gamma \simeq c$ 5 4 $\ln(b/\alpha\alpha)$

stant b under the log, and one should expect there to be sub-leading corrections suppressed only by powers of $1/$ ln $\alpha\alpha$.) We with to show that BБdeker's

^{2}0e

results can be reproduced and interpreted through a simple argument based

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on the fact that plasmas are conductivBefore presenting the essential argument, let us review the

gin of the length scale $1/g$

Non-perturbative means that, for example, gA is not a perturbation in the covariant derivative $\mathbf{D} = \boldsymbol{\nabla} - ig\mathbf{A}$. So non-perturbative means A $\alpha\alpha O(1/gR$and hence the energy E

$O(1/g$

RT) Boltzmann factor $\exp(-<E) \sim \exp[-1/(2$ $O(1>2\, T)$. Because of entropy effects, non-perturbative processes will be dominated by the smallest size scale for which the probability is unsuppressed (since there are more small-wavelength degrees of freedom than large-wavelength ones), and so the characteristic length scale of non-perturbative physics is $R \sim 1/g\ T$. Static electric fields are screened by the Debye effect on smaller distance scales, of order $1/gT$. For this reason, non-perturbative physics in the hot plasma is essentially magnetic. More technically, it is only the transverse degrees of freedom of the gauge field which are important.

dynamics of magnetic fluctuations in plasmas is slowed down by Lenz's law: conducting media resist changes in magnetic field. (In the context of high temperature baryon number violation, this qualitative explanation of the slow time scale for non-perturbative processes has been emphasized by Guy Moore [6].) Let's make it quantitativImagine splitting the gauge field into soft degrees of freedom—th

$_g T$, and hard degrees of associated with much higher momenta

this split is made will not be relevant at the order we shall consider. The[3] amplitude of fluctuations is non-perturbative for the soft modes but perturbative for the hard ones. As is well known [8], the soft modes are effectively classical—the

R). The probabilit),of such a fluctuation in e

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[3] 00a

cause of Bose statistics. Now, treating the soft modes classically, start with

the Maxwell $\mathbf{D} \times \mathbf{B} = D$

$\mathbf{E} + \mathbf{J}$ for the soft degrees of freedom, where $\mathbf{B} = \mathbf{D} \times \mathbf{A}$

derivatives are to be understood as only involving the soft gauge field degrees of freedom. $\mathbf{J}_{\mathrm{hard}}$ is the color current^{e0}

been implicitly disregarded, and we will need to put it in if we wish to de-

$(t)\xi$

d

d

scribe equilibrium =uctuations. Fortunately, this is simple to do after fact because noise and d

from thermal noise and their dissipative decay.

To be more speciΓc, note that (7) has the general form

$$\sigma \qquad t\alpha\alpha\mathbf{q} = -V(\mathbf{q}),$$

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$\mathbf{x}$

case i). tSuch sy-Abelisarmagnetic energy $\frac{1}{2}$ B^2

in physics, and a simple way to incorporate thermal noise is to include a random force $\boldsymbol{\xi}\sigma$

$$\overline{\mathbf{q}} = -\boldsymbol{\nabla}_{\mathbf{q}} V(\mathbf{q}) + \boldsymbol{\xi}.$$

This is a typical exampl

choice of thermal noise, Gaussian white noise, reproduces the correct equi-

librium distribution $\exp(-<V)$ if the noise variance is suitably scaled with the amount of dissipation,

$${}_j(t')\rangle\rangle = 2\sigma T\,{}_{ij} > (t-t') \quad .$$

(9)This well-known resul

(9) into a Fokker-Planck equation for the probability distribution. (See, for example, chapter 4 of ref. [9].)

Let us introduce noise as in (9). Translating back to our particular system (7), we obtain the following eΓFective theory for the soft moσ

$$\frac{d}{dt}\mathbf{A} = -\mathbf{D}\times\mathbf{B} + \boldsymbol{\xi},$$

$\langle\xi(t,\ \mathbf{x})\ {}_j\xi(t$

~~The effective equation (2) turns out to have the wo~~nderful property that

it is insensitive to how the soft modes are cut off at large momentum [1]. means that one can ignore t

write (3) but which was never specified in detail. It means that (2) will binsensitive to short-dis

Finally, it also means that such simulations will not be plagued by lattice artifacts, such as loss of rotational invariance, that were thought to arise other approaches ¿From

g of non-perturbative dynamics is given bDσ t

$$-A \sim R\ {}_{-2}$$

$$^2\ \sigma \sim \quad \sigma T \quad {}_{\mathrm{g}}(14)$$

Thus, one need only know the color conductivity σ. There has been some confusion in the literature (described later) about this quantity, but the correct value was first presented by Selikhov and Gyulassy [10]. The color conductivity is of order

$\sigma - T\underline{\ln(1/g)}$:We will review below how to understand this physica

into Eq. (14) then gives the time scale

$$t\ 1 \quad T\ln(1/g) \quad ,$$

$^5\xi\ln(1/\alpha\alpha)$, which has the logarithmic enhancement claimed by BБdeker. Earlier estimates [4] of the time scale as $t \sim 1/(g^4 T)$ correspond to ignoring the effects of collisions on the conductivity. Note that ignoring the time derivative term in (5) was justified since the characteristic time (16) is much greater than

–determined by On a more quantitative level,

$$4\sigma \approx$$

pl/

$$\frac{m^2}{}$$

,

$A,$ [6] 00e rea0er 00 re00 0100

where m

f

BБdeker g**3** **Color conductivit**Begin by considering the current respo

is the plasma frequency and

$/_{\rm g}$ $_{\rm A}T\ln(1/g)$

Casimir, conventionally normalized as $p\alpha_{\rm A}= N$ for the gauge group SU(N), and "≈" means equality up to relative corrections suppressed by powers of ln($1/g$). That is, no claim is made about discriminating ln($1/g$) from

ln($2/g$7 The only place where the matter content of the theory enters is in the value of the plasma frequency. For hot electroweak theory, it is given by m

n$[1 + O(g)]$, (19)

$_0 g\mathbf{E}\ \Delta t$ ^{2}

where n is the number of fermion families. The Langevin equation (2) (21)

, with the value (17) of σ precisely reproduces the effective theory derived by

collisionless ultra-relativistic plasma. For simplicity of notation, consider a QED plasma for the moment rather than a non-Abelian one. If the external field were static and homogeneous, particles in the plasma with charge g would respond to the field by a change in momen$\Delta\mathbf{p} = g\mathbf{E}\ \ \Delta t$

particle whose energy is order T would then by $\Delta\mathbf{v}\,\Delta\mathbf{p}$

and the resulting current would be

$T\ \Delta t)\ \ \mathbf{E}$, $\mathbf{J} \sim$(22) $g\ \ \Delta\mathbf{v} \sim (_2{}^2$

$^7{}_0$

where $n \sim$ is the density of hard particles. The current is dominated by

$\underline{g\quad}$

the most prevalent particles in the plasma: those with momentum of order TThe current (22) grows i

is applied. There are two things which can cut off this growth of the curren(a) collisions, and (b

Stick with the collisionless plasma for a moment and consider oscillations of
E. As we've discussed, the time scale for non-perturbative processes Tout to be slow. So suppose, fo

carriers, which have an rms z velocity of $1/\sqrt{3}$, will move from regions of
positive E to regions of negative E

$-t\alpha\alpha$(23)

This change in direction of the electric field felt by the charge carriers then
limits the average current response to a $J \sim$

^{2} E(24) ^{2} k

If we identify the (k-dependent) conductivity $\sigma(k) \sim$

(collisionless) (25) $\underline{^{2}\quad}$

T and take k to be the inverse spatial scale, for non-perturbative the time scale t for non-perturbative physics would be

$$t \sim \frac{}{g^2 k} \sim {}_{g}T \quad ,$$

provided we could indeed ignore the effects of collisions on the conductivity. This is the qualitative physics behind the more formal and quantitative
discussions of refs. [4, 12, 13]. [8]

The divergence of the conductivity (25) as $k \to 0$ is cut off in real physical systems by the effects of collisions, as pointed out by Drude in 4900. Let's continue to focus on a QED plasma for the moment. A charge accelerated by the electric field eventually experiences a collision with other particles in the

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z

[8] $\sigma\gamma k\gamma\ \gamma$

~~plasma which changes the charged particle's direction by a large angle. Such~~

collisions randomize the direction of the particle and so randomize its conbution to the current. So

(21) becomes the mean collision time for large angle tri-$_{-1}$

$$_{\text{large}} \sim T \left(\frac{T}{m} \right) .$$

The g

above just comes from the square of the scattering matrix elemenThe logarithm arises because

either through a single large-angle scattering or through the cumulative eΓFect of many (individually more probable) small-angle scatterings. ^{9} $_{\text{large}}$ wplthe relevant mean free

collisions on the conductivity $\sigma(k)$ could safely be ignored when investigating non-perturbative=uctuations. That's because $_{\text{large}} \gg 1/k \sim 1/g^2T$, and so it would be the collisionless time scale $1/k$ instead of τ_{large} that determines Δt and so $\sigma(k)$.

However, Selikhov and Gyulassy [10] have pointed out that is not the relevant mean free time in the non-Abelian case. In the non-Abelian case, even arbitrarily small angle scatterings can randomize the current, not by randomizing the velocity of the current carriers, but by randomizing their color charge. The crucial di;erence with QED is that an exchanged Abelian gauge boson, no matter h

color charge of the scatterers, whereas an exchanged photon is neutral. relevant time scale for the

for *any*-angle scattering, which is much shorter than the mean free time for large-angle scattering. SpeciΓcally, t-channel gauge boson exchange gives a cross-section $\hat{\sigma}$ such τ

$$^{-1} \sim n\hat{\sigma} \sim ng^{dt} \ {}^{\text{M}}_{\text{M}} ,$$

where $n \sim T$

is the density of particles and t

If τ