# Stabilization of the Electroweak String by the Bosonic Zero Mode

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**Abstract**

In the *minimal* standard model we discuss stability of the $Z$-string configuration, which includes the zero mode connected with the broken gauge symmetry. The zero mode induces charge and current on the string and gives backreaction to the string profile changing the region of dynamical stability. For $\sin^2\theta_W = 0.23$ it is found that the string is stabilized for the Higgs mass $M_H < 132\ GeV$ in some finite range of the time-like currents.

The Nielsen-Olesen vortex solution of the abelian Higgs model [1] can be embedded into the $SU(2)_L \times U(1)_Y$ electroweak theory [2] and this solution is known as the electroweak or $Z$-string. It could trigger baryogenesis at the electroweak phase transition [3] and manifest itself in accelerator experiments. However, there are no topological arguments for stability of such a string. The dynamical stability is reached outside of the physical region of the parameters [4, 5, 6]. Different attempts have been undertaken to save the $Z$-strings. It was shown that bound states on the string may improve its stability [7]. Fermion zero modes do not provide the backreaction to the string forming fields [8] leaving the string unstable. Stabilization is possible in different extensions of the standard model where even topologically stable

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strings can be formed [9]. In the minimal model the string can be stabilized by strong external magnetic fields [10].

Any string solution inevitably breaks some internal and space-time symmetries generating massless excitations of the string ("zero modes") connected with the broken transformations. In some cases these excitations are very important for the string dynamics. A well known example is the string superconductivity in the Witten model [11] caused by the bosonic zero mode originated from the broken electromagnetic gauge symmetry inside the string core. Recently it has been shown that the Nielsen-Olesen string also has the zero mode of this kind [12] though the gauge symmetry is unbroken in the string center. It has been argued that the string fields interpolating between false and true vacua definitely break the gauge symmetry in the core inducing the zero mode with specific properties. This mode provides the backreaction to the string forming fields, induces charge and current on the string, and changes the excitation spectrum. As a result it will also change the region of dynamical stability for the $Z$-string.

In this paper the stability region for this current-carrying $Z$-string is analyzed. The string solution with a ground state zero mode is described first and its influence on the string profile is calculated numerically. Then the excitation spectrum is investigated and the range of stability is mapped on the Higgs mass - current plane for the physical value of the Weinberg angle.

The $Z$-string is the Nielsen-Olesen string embedded into the electroweak theory by such a way that the upper component of the Higgs doublet $\Phi_u = 0$ and the gauge bosons $W^{\pm}_{\mu} = A_{\mu} = 0$. The $Z$-boson and the down component of the doublet $\Phi_d$ are described by the lagrangian of the abelian Higgs model

$$\mathcal{L} = -\frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + (D_\mu \Phi_d)^* (D_\mu \Phi_d) - \lambda \left( |\Phi_d|^2 - \eta^2/2 \right)^2 , \tag{1}$$

where

$$Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu, \quad D_\mu \Phi_d = (\partial_\mu + iqZ_\mu)\, \Phi_d.$$

The charge $q$ is expressed via coupling constants $g$ and $g'$ of the groups $SU(2)_L$ and $U(1)_Y$, respectively, as $q = \sqrt{g^2 + g'^2}/2$ and masses of the $Z$ and Higgs bosons are

$$M_Z = q\eta, \quad M_H = \sqrt{2\lambda}\eta. \tag{2}$$

Let us consider the following ansatz for a unit winding string along the $z$-axis [12]

$$Z_i = \partial_i \varphi_0(x) f_0(r), \quad Z_\alpha = \frac{1}{qr^2}\epsilon_{\alpha\beta} y_\beta Z(r), \quad \Phi = \frac{\eta}{\sqrt{2}} f(r) e^{i\vartheta}, \tag{3}$$

where the notations $x^i = (t, z)$ for $i = 0, 1$ and $y^\alpha = (r\cos\vartheta, r\sin\vartheta)$ for $\alpha = 1, 2$ were introduced for longitudinal and transverse coordinates, respectively. If the function $\varphi_0(x) = 0$, one gets the standard vortex solution [1]. $Z_i = \partial_i\varphi_0(x) f_0(r)$ will describe a zero mode of this vortex connected with the broken gauge symmetry in the string core if the function $f_0(r)$ obeys the equation

$$f_0'' + \frac{1}{r} f_0' - M_Z^2 f^2 f_0 = 0. \tag{4}$$

One can check that in this case the equations of motion give $\partial_i\partial^i\varphi_0(x) = 0$ and, therefore, $\varphi_0(x)$ is a 2-dimensional massless field. The problem is that solutions of Eq.(4) are always singular at infinity or at the origin. If one supposes that $Z_i$ obeys usual boundary condition $Z_i \to 0$ at $r \to \infty$ and $Z_i$ is restricted at $r \to 0$, then one has to exclude the mode as having infinite energy. The boundary conditions for this zero mode of the Nielsen-Olesen string have been analyzed in Ref.[12], where it was found that $Z_i$ is restricted by the condition

$$\int d^2x d^2y \partial_\alpha \left( Z_i \partial_\alpha Z^i \right) =$$

$$2\pi \int d^2x \partial_i \left( \varphi_0 \partial^i \varphi_0 \right) \int dr \frac{d}{dr} \left( r f_0 f_0' \right) = 0. \tag{5}$$

Now we see that there is another way to meet (5): one can restrict the behavior of the field $\varphi_0(x)$ at the space-time $(t, z)$ boundaries in usual way instead of bound the $r$-behavior of $f_0(r)$. It was shown that $\varphi_0(x)$ will be normalized as a 2-dimensional field if $f_0(r)$ obeys the relaxed boundary conditions allowing logarithmic singularity at the origin:

$$f_0(r) f_0'(r) = -\frac{1}{2\pi r}, \; r = r_0 \to 0; \quad f_0(r) \to 0, \; r \to \infty. \tag{6}$$

Here the mathematical cutoff parameter $r_0 \to 0$ has been introduced.

The mode $\varphi_0(x)$ carries a finite energy

$$E_{zm} = \frac{1}{2} \int_0^L dx_1 \left[ (\partial_0 \varphi_0)^2 + (\partial_1 \varphi_0)^2 \right], \tag{7}$$

where $L$ is the string length, and propagating along the string with the speed of light. $E_{zm}$ does not increase with $L$ although the string solution has finite energy per unit length and, of course, excitations with finite energy per unit length should be permitted. This means that the functions $\varphi_0(x)$ with linear singularities at the space-time boundaries are also allowed. The mode $\varphi_0(x)$ which is linear in time and position obeys the equation of motion and carries the energy proportional to the string length $L$. It cannot be expanded in harmonic oscillators since it obeys different boundary conditions. For this reason we have to attribute such a mode to the background solution rather than the string perturbations. In this sense the mode is quite similar to the explicit collective coordinate [13]. For the background value of the zero mode one takes

$$\varphi_0(x) = b_i x^i = b_0 t - b_1 z, \tag{8}$$

where $b_i$ is a constant 2-vector. In this case the string is described by a time and position independent field configuration and its energy is

$$E = E_{core} + E_{zm}, \tag{9}$$

$$E_{core} = \pi \eta^2 L \int_0^\infty d\rho \rho \left\{ f'^2 + \frac{Z'^2}{\rho^2} + \frac{(1-Z)^2}{\rho^2} f^2 + \frac{1}{4} \beta \left( f^2 - 1 \right)^2 \right\},$$

$$E_{zm} = \pi \eta^2 L (a_0^2 + a_1^2) \int_{\rho_0 \to 0}^\infty d\rho \rho \left( f_0'^2 + f_0^2 f^2 \right) = \frac{\eta^2}{2} L (a_0^2 + a_1^2),$$

where the dimensionless variables were introduced

$$\rho = M_Z r, \quad \beta = \frac{M_H^2}{M_Z^2}, \quad a_i = \frac{b_i}{\eta}, \tag{10}$$

and primes denote differentiation with respect to $\rho$.

A string solution similar to (3) with $\varphi_0(x)$ as in (8) has been proposed [14] for a specially constructed non-abelian string with the only important difference that the solution was singular at infinity instead of the origin as in our case. There is also some resemblance of (3) and the dion solution of the $O(3)$ model [13] which, of course, comes from the fact that both solutions are connected with the zero mode of the same origin.

Solving Eq.(4) with the conditions (6) one finds that for small $\rho$ the $Z_i$ components of the vector potential are

$$Z_i = \eta a_i \frac{\ln(\overline{\rho}/\rho)}{\sqrt{2\pi \ln(\overline{\rho}/\rho_0)}}, \quad \rho \ll 1, \tag{11}$$

where $\overline{\rho} \sim 1$ is a constant. The first impression is that there are sources in the string center generating this potential. However, it is false because the current density induced by the mode

$$j_i = -M_Z^2 \eta a_i f_0(\rho) f^2(\rho) \tag{12}$$

disappears as $\rho^2 \ln \rho$ for small $\rho$. A real density has a maximum at $\rho \sim 1$ and goes to zero at both boundaries. Integrating (12) over the string cross section with the help of (4) and (6) one finds for the charge per unit length $J_0$ and the current $J_1$ along the string

$$J_i = -\frac{a_i \eta}{f_0(\rho_0)} = -a_i \eta \sqrt{\frac{2\pi}{\ln(\overline{\rho}/\rho_0)}}. \tag{13}$$

In what follows we will often save the term "current" for both components of $J_i$.

In the mathematical limit $\rho_0 \to 0$ the current goes to zero and interaction with the others string fluctuations disappears. Nevertheless, the zero mode is physical in this limit since it interacts with gravity. In physical reality we cannot believe in our equations for arbitrary small distances. A natural candidate for the cutoff parameter $r_0$ is the Plank length $M_P^{-1}$. It cannot be any scale of symmetry breaking in Grand Unification Theory since the field equations are still valid. However, if the particles in the standard model are composite, the compositness scale will play the role of $r_0$. Further we will suppose that $\rho_0 = M_Z/M_P$. The zero mode is concentrated at small distances $\sim 1/M_P$ but survives at $r \sim 1/M_Z$ because it decreases logarithmically.

Let us investigate now the backreaction of the zero mode to the string profile. Equations of motion following from (1) give for the functions in our string ansatz

$$f'' + \frac{1}{\rho} f' - \frac{(1-Z)^2}{\rho^2} f + \gamma f_0^2 f - \frac{1}{2} \beta \left( f^2 - 1 \right) f = 0,$$

$$Z'' - \frac{1}{\rho} Z' + (1-Z) f^2 = 0,$$

$$f_0'' + \frac{1}{\rho} f_0' - f^2 f_0 = 0, \tag{14}$$

where $\gamma = a_0^2 - a_1^2$. These equations have to be solved with the boundary conditions

$$\rho = \rho_0 : \qquad f = 0, \quad Z = 0, \quad f_0 f_0' = -\frac{1}{2\pi\rho_0};$$

$$\rho \to \infty : \qquad f \to 1, \quad Z \to 1, \quad f_0 \to 0, \tag{15}$$

There is no way to analyze the effect analytically, so one needs numerical investigation. The problem is that $\rho_0 \sim 10^{-17}$ is too small for numerical study. The way out is to shift the boundary condition to a point $\rho = \rho_1$, where $\rho_1$ is not too small. This can be done since for small $\rho$ one can solve Eq.(14) analytically.

The shift of the boundary allows to take the value of $\rho_1$ as large as 0.1. For the calculations it was chosen $\rho_1 = 0.05$. Even after this procedure a linear discretization of $\rho$ is not appropriate, so the variable $\xi = \ln \rho$ was used. Equations (14) were discretized on a lattice with 32, 64 or 128 points and the resulting system of nonlinear equations was solved by the Newton method. The results practically insensitive to the number of points in the range. Comparison of the linear and logarithmic discretization for the zero current showed a good agreement. After the solution has been found the independent check has been done with the Runge-Kutta algorithm. The width of the scalar core is increased with $|\gamma|$ for the space-like current ($\gamma < 0$) and the symmetry is restored near the center. For the time-like current ($\gamma > 0$) the Higgs field width is decreased with $\gamma$ increase. In both cases $Z(\rho)$ and $f_0(\rho)$ are much less sensitive to the value of $\gamma$. There is no natural restriction on the current value in contrast with the case of the Witten string [15].

One considers small excitations around the classical solution (3). Our goal is to find the stability conditions and for this reason one can set $\delta Z_\mu$ and $\delta\Phi_d$ to be zero because the $U(1)$ string is topologically stable and the zero mode cannot spoil the conclusion. The string solution does not break the electromagnetic $U(1)_{em}$ symmetry, therefore, $\delta A_\mu$ is not important for the stability problem. For the rest of the excitations one chooses the notations

$$\delta W^+_\mu = V^+_\mu, \qquad \delta W^-_\mu = V^-_\mu, \qquad \delta\Phi_u = \Psi. \tag{16}$$

In the description of these fields let us follow to the method developed by Goodband and Hindmarsh [6] and choose the background gauge condition

$$\partial^\mu V^+_\mu - ig\cos\theta_W Z^\mu V^+_\mu - i\frac{g}{\sqrt{2}}\Phi^*_d\Psi = 0, \tag{17}$$

In this gauge the fields $V^+_\mu$ and $\Psi$ will obey the linearized equations of motion

$$\mathcal{D}_\mu\mathcal{D}^\mu V^+_\nu + 2ig\cos\theta_W Z_{\mu\nu}V^{+\mu} - i\sqrt{2}g\left(\bar{D}_\nu\Phi_d\right)^*\Psi + \frac{1}{2}g^2\left|\Phi_d\right|^2 V^+_\nu = 0, \tag{18}$$

$$D_\mu D^\mu\Psi - i\sqrt{2}gV^{+\mu}\bar{D}_\mu\Phi_d + \left[\frac{1}{2}g^2\left|\Phi_d\right|^2 + 2\lambda\left(\left|\Phi_d\right|^2 - \eta^2/2\right)\right]\Psi = 0, \tag{19}$$

where

$$\bar{D}_\mu = \partial_\mu + iqZ_\mu,\ \mathcal{D}_\mu = \partial_\mu - 2iq\cos^2\theta_W Z_\mu,\ D_\mu = \partial_\mu - iq\cos 2\theta_W Z_\mu. \tag{20}$$

$V^-_\mu$ obey the conjugated equations. The gauge condition allows the residual gauge freedom with a gauge functions $\chi^+$ which is a solution of the equation

$$\mathcal{D}_\mu\mathcal{D}^\mu\chi^+ + \frac{1}{2}g^2\left|\Phi_d\right|^2\chi^+ = 0. \tag{21}$$

This means that the gauge fixing term must be accompanied by Fadeev-Popov ghost terms. Two of 5 fields $V^+_\mu$ and $\Psi$ are canceled by the ghost and the other 3 correspond to the spin states of the massive $W$ boson.

Of the total of 5 equations (18),(19) only 4 are coupled. To see it, one introduces the combinations

$$V^+ = \frac{a^i V_i^+}{a}, \qquad U^+ = \frac{\epsilon^{ij} a_j V_i^+}{a}. \tag{22}$$

where $a = \sqrt{|a^i a_i|}$. Then the field $U^+$ decouples and is canceled by the ghost. The other unphysical field is some linear combination of the remaining 4 fields. Since the string configuration is time and position independent, the $x$-dependence of the fields can be sought in the form $e^{-i\omega t + ikz}$. To solve the equations, one expands the fields in total angular momentum states

$$\begin{aligned}
\Psi(x,y) &= e^{-i\omega t+ikz} \sum_m \psi^m(\rho) e^{i(m+1)\vartheta}, \\
V_u^+(x,y) &= e^{-i\omega t+ikz} \sum_m -i v_u^m(\rho) e^{i(m-1)\vartheta}, \\
V_d^+(x,y) &= e^{-i\omega t+ikz} \sum_m i v_d^m(\rho) e^{i(m+1)\vartheta}, \\
V^+(x,y) &= e^{-i\omega t+ikz} \sum_m v^m(\rho) e^{im\vartheta},
\end{aligned} \tag{23}$$

where $V_{u,d}^+ = \frac{1}{\sqrt{2}}\left(V_1^+ \mp i V_2^+\right)$ are spin up and spin down states. Substituting it into (18) and (19) one finds the eigenvalue problem for the vector $\mathbf{Y}_m = (\psi^m, v_u^m, v_d^m, v^m)$. The functions $\mathbf{Y}_m(\rho)$ are restricted at the origin and vanish at infinity.

The string with zero current is unstable for $m = -1$ mainly due to interaction of the string "magnetic" field with $v_d$. The zero mode gives rise to the transverse components of the "electric" and "magnetic" fields confined in the string core ($Z_{\alpha i}$ components of the field tensor). Interaction of the excitations with these fields has non-abelian nature and plays a crucial role in the stability problem because it behaves as $1/\rho$ for small $\rho$. For the space-like current ($\gamma < 0$) the transverse components of the field strength work against the stability. On the contrary, the time-like current (if the current is not too large. Therefore, we expect some finite stability region for time-like currents in the string. This simple picture is spoiled by the components of the zero mode potential $Z_i$ which are combined with $\omega$ and $k$ in eigenvalue equations and by the first component of $\mathbf{Y}$ which is not governed by the field strength at all. Nevertheless, the calculations support this qualitative description.

Our calculations for the string with zero current agree well with the results of the previous works [4, 6]. All the other calculations were made for the physical value of $\sin^2\theta_W = 0.23$. It was checked that the pure current ($J_0 = 0$) makes lower the minimal eigenvalue $\omega^2$ of the problem which is always negative in the $m = -1$ sector. For the pure charge on the string ($J_1 = 0$) a finite range of $\gamma$ for which the lowest level $\omega$ is real was found for a fixed $\beta$ (see Fig.1). The level corresponds to a bound state. In this range the string is stable. The two lowest levels merge at the end points of the stability interval and become complex outside of this interval. When $\beta$ increases the range of stability shrinks. For $\beta > 2.1$ the string cannot be stabilized at any current and, therefore, the stable electroweak string can exist only if the Higgs mass is smaller than 132 $GeV$. The separate "star" in Fig.1 shows the result for the left boundary at $\beta = 0.02$ which is the smallest value we were able to consider. The range of small $\beta$ is important for description of the phase transition. The equations were also investigated for $m = 0$ and $m = 1$ where instabilities are possible in principle but we not found any.

We have considered only the bosonic sector of the standard model. Naculich [16] has analyzed the effects of fermions on the stability of the $Z$-string and has found that the fermions destabilize the string for all values of the parameters. The arguments given by Liu and Vachaspati [17] convince, however, that fermionic vacuum instability should lead to a distortion of the bosonic string but not be responsible for decay. These arguments are quite common and have to be true for the string configuration considered in this paper. Therefore, our conclusion is that for $M_H < 132$ $GeV$ there is a finite range of time-like currents for which the electroweak string is stable.

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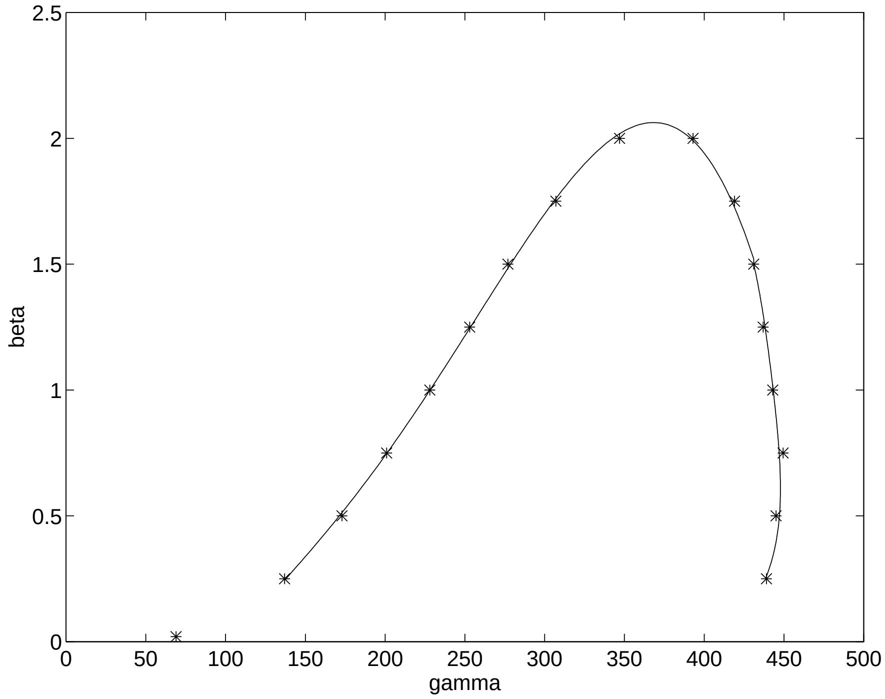


Figure 1.