Solitonic Baryons in a Chiral Quark Meson Model

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Abstract

We consider a simple SU(2) quark soliton model based on chiral invariant quark-meson couplings. We find axially symmetric, classically stable soliton solutions with B=2,3 and 4 (where B is the baryon number). The energy distributions corresponding to these solutions are toroidal in nature. The symmetries of the B=2 solution indicate that the ground state of this solution has the quantum numbers of the deuteron.

1 Introduction

Two simple models based on the chiral symmetry of hadronic interactions have been studied in recent times to explain the properties of the individual baryons in the low energy region. In the Skyrme model [1] one considers purely mesonic degrees of fredom, and baryons are obtained as the solitonic configurations of the meson fields. There are no explicit quark fields in this model. On the other hand, chiral quark meson models [2], [3] are hybrid in nature and both quark and meson mesonic degrees of freedom are present in the theory. Phenomenologicaly these models are somewhat better. General arguments based on QCD indicate that the low energy effective action for hadrons should include both quark and meson fields. Moreover it has been shown recently that parton distributions inside a hadron at a low normalisation point can be explicitly calculated in a chiral quark-soliton model. All these indicate the importance of the chiral quark meson models.

In the simplest version of the chiral quark meson model based on SU(2) x SU(2) symmetry, the Lagrangian is

$$L = i\overline{\Psi}\partial\Psi - gF_\pi(\overline{\Psi}_L U \Psi_R + \overline{\Psi}_R U^\dagger \Psi_L)$$
$$+\frac{F_\pi^2}{4} Tr(\partial_\mu U \partial^\mu U^\dagger) + \frac{F_\pi^2 m_\pi^2}{4} Tr(U + U^\dagger - 2). \quad (1)$$

Here Ψ denotes the iso-doublet of quark fields u,d and $U = exp(-i\vec{\tau}.\vec{\phi})$ where τ_i are the usual iso-spin matrices and ϕ_i are the pionic degrees of freedom. F_π is the pion decay constant and g is the coupling constant (with our normalisation $F_\pi = 93Mev$). For describing nucleons,one considers a hedgehog ansatz $\vec{\phi} = \hat{r}\Theta(r)$(as in the Skyrme model), that is, $U = \cos\Theta - i\vec{\tau}.\hat{r}\sin\Theta$ and

$$\Psi = \begin{pmatrix} G(r)\chi \\ i\vec{\sigma}.\hat{r}F(r)\chi \end{pmatrix} exp(-i\epsilon t) \quad (2)$$

where the hedgehog spinor

$$\chi = \frac{1}{\sqrt{2}}(u \downarrow - d \uparrow) \quad (3)$$

satisfies

$$(\vec{\tau} + \vec{\sigma}) = 0. \quad (4)$$

Here ϵ is the quark eigen-energy. Also $\Theta(0) = \pi$ and $\Theta(\infty) = 0$. One considers a soliton solution of the Lagrangian with three quarks which which will be identified with the nucleons after projecting to spin-isospin eigenstates. It is found that a good phenomenological fit is obtained for $gF_\pi = 500Mev$ or $g = 5.367$. In Ref.[3] actually a linear σ model has been used. However we have checked that there is very little change when we work with the nonlinear σ model corresponding to L in Eq.(1).

Kopeliovich and Shtern [4] were the first to show that the lowest energy soliton solutions for B=2,3,4... in the Skyrme model correspond to toroidal configurations. Later Braaten and Carson [5]showed that after collective quantization, the lowest energy state for B=2 has the quantum numbers of the deuteron. In the Skyrme model,for $B = n \geq 2$, U has the form [4]

$$U = \cos\Theta - i\vec{\tau}.\hat{n}.\sin\Theta, \quad (5)$$

where,

$$\hat{n} = (\sin\alpha \ \cos n\phi, \sin\alpha \ \sin n\phi, \cos\alpha). \quad (6)$$

When Θ and α are considered as field variables which are functions of ρ and z (the cylindrical coordinates), one gets classically stable(against decay into lower B states) solutions with toridal distributions of energy.

The purpose of this paper is to investigate toroidal solitons in the chiral quark meson model [6].

2 B=2 toroidal soliton in the chiral quark meson model

We study the B=2 soliton solution with six quarks, with cylindrical symmetry for a U_2 corressponding to n=2 in U given by Eq.(6). For the B=2 toroidal Skyrmion,

$$(L_3 + 2T_3) \mid Physical \;\; state >= 0, \tag{7}$$

where L_3 and T_3 are the z-components of the orbital angular momentum and the isosspin generator respectively. This is simply the statement of cylindrical symmetry in operator form,namely that a spatial rotation of α about the z-axis can be compensated by an isorotation of -2α about the z-axis. For the qquark fields in our model, the relevant operator is $K_3 = L_3 + S_3 + 2T_3$ where S_3 is the z-component of the spin angular momentum. As L_3 is integral and S_3 and T_3 are $\pm\frac{1}{2}$ for the quark fields, $e^{-i\phi}u\uparrow, u\downarrow, e^{i\phi}d\uparrow$, and $e^{2i\phi}d\downarrow$ have $K_3 = +\frac{1}{2}$ and $e^{-2i\phi}u\uparrow, e^{-i\phi}u\downarrow, d\uparrow$, and $e^{i\phi}d\downarrow$ have $K_3 = -\frac{1}{2}$. For a U_2 of the form given in Eq.(6), the Dirac spinor with $K_3 = +\frac{1}{2}$ or$-\frac{1}{2}$ would have a complicated structure and it is very difficult to solve the exact equations of motion. We have found that a spinor Ψ of the following form minimizes the energy:

$$\Psi = \left(\begin{array}{c} G(\rho, z)\chi(\theta,\phi) \\ i\vec{\sigma}.\hat{r}F(\rho, z)\chi(\theta,\phi) \end{array} \right) exp(-i\epsilon t) \tag{8}$$

where χ is a column vector in spin-isospin space which satisfies

$$(\vec{\tau}.\hat{n} + \vec{\sigma}.\hat{r})\chi = 0. \tag{9}$$

The following two solutions χ_1 and χ_2 with $K_3 = +\frac{1}{2}$ and $-\frac{1}{2}$ respectively satisfy the above equation:

$$\chi_1 = \frac{1}{\sqrt{2(1-\cos\theta\cos\alpha)}}[\sin\alpha u\downarrow - \sin\theta e^{i\phi}d\uparrow - (\cos\alpha - \cos\theta)e^{2i\phi}d\downarrow], \tag{10}$$

$$\chi_2 = \frac{1}{\sqrt{2(1-\cos\theta\cos\alpha)}}[\sin\alpha d\uparrow - \sin\theta e^{-i\phi}u\downarrow + (\cos\alpha - \cos\theta)e^{-2i\phi}u\uparrow]. \tag{11}$$

It can be easily checked that the two solutions are degenerate in energy.

U_2 and $\chi_{1,2}$ satisfy the following symmetry properties.

(1)Rotation by an arbitrary angle around the z-axis:

$$U_2(\rho,\phi+\beta,z) = e^{-i\beta\tau_3}U_2(\rho,\phi,z)e^{i\beta\tau_3}, \tag{12}$$
$$\chi_{1,2}(\theta,\phi+\beta) = e^{i\beta/2}e^{-i\beta(\tau_3+S_3)}\chi_{1,2}(\theta,\phi). \tag{13}$$

(2)Rotation by π around the x-axis:

$$U_2(x,-y,-z) = \tau_1 U_2(x,y,z)\tau_1, \tag{14}$$
$$\chi_{1,2}(x,-y,-z) = \tau_1 S_1 \chi_{2,1}(x,y,z). \tag{15}$$

(3)Rotation by π around the y-axis:

$$U_2(-x,y,-z) = \tau_1 U_2(x,y,z)\tau_1, \tag{16}$$
$$\chi_{1,2}(-x,y,-z) = i\tau_1 S_2 \chi_{2,1}(x,y,z). \tag{17}$$

Under these transformations, the energy is invariant as χ_1 and χ_2 are degenerate in energy. The symmetry properties of U_2 are the same as in the Skyrme model and are important for the existence of the soliton solution [5], [7]. All these symmetry requirements are met by our ansatz and emphasise its correctness.

Each quark state can accomodate three colours. Hence we can construct a six quark state, with three quarks each in χ_1 and χ_2 states. Hence,

$$(L_3 + S_3 + 2T_3) \mid six \;\; quark \;\; state >= 0. \tag{18}$$

The energy corresponding to the ansatz in Eqs.(6),(9) and (10) (in units of gF_π) for a B=2 state with six quarks is

$$E = 6\epsilon + \frac{\pi}{g^2}\int \rho d\rho dz[(\Theta^{'})^2 + (\Theta)^2 + \sin^2\Theta((\alpha^{'})^2 + (\alpha)^2 + \frac{n^2\sin^2\alpha}{\rho^2})$$
$$+2\beta^2(1-\cos\Theta)], \tag{19}$$

where ϵ, the quark eigenenergy is a functional of G and F given by

$$\epsilon = 2\pi \int \rho d\rho dz [G(\frac{\rho}{r}F' + \frac{z}{r}\dot{F} + \frac{2F}{r} + \eta F) - F(\frac{\rho}{r}G' + \frac{z}{r}\dot{G} - \eta G) + (G^2 - F^2)\cos\Theta - 2GF\sin\Theta]. \quad (20)$$

Here prime and dot denote differentiation with respect to ρ and z, $\beta = m_\pi/gF_\pi$ and η is given by

$$\eta = -\frac{1}{r} + \frac{n\sin^2\alpha}{r} + \frac{\alpha' \sin\alpha\rho z}{r^2 c} - \frac{\dot{\alpha}\sin^2\alpha\rho}{r^2 c}, \quad (21)$$

with $c = 2(1 - \cos\alpha\cos\theta)$. In Eqns.(15) and (16), the spatial variables ρ, z and r have been made dimensionless by scaling with gF_π. We have used a relaxation procedure to minimise the energy subject to the boundary conditions. A lattice of 40x40 points in the $\rho - z$ plane was adequate. Trial functions for Θ, α, G and F which satisfy the boundary conditions are iterated using Newton-Raphson method, so that the energy functional is minimised. The energy functional converges rapidly after a few iterations.

We find the energy of the configuration to be 2019 Mev. As the classical energy of the B=1 state is 1149 Mev, the classical binding energy is 279 Mev. In the Skyrme model, the binding energy is around 170 Mev if we use the physical value of F_π, but has the value 74 Mev if we choose $F_\pi =$ 54 Mev. In Fig.1 we give the contour plot of the energy density function $\mathcal{E}(\rho, z)$ which clearly indicates a toroidal structure of the soliton solution. We have drawn the figures with variables which are scaled in units of gF_π. A naive value for the mean square radius of the charge distribution coresponding to our soliton would be given by $< r^2 >= 2\pi \int \rho d\rho dz r^2 (G^2 + F^2)$. We find $< r^2 >^{1/2}$ to be 0.934 fm to be compared with the value of o.92 fm in the Skyrme model and the experimental value of 2.09 fm. We can define the mean radius of the torus as:

$$< \rho >= \int d^3x \rho (G^2 + F^2) = 2\pi \int \rho^2 d\rho dz (G^2 + F^2). \quad (22)$$

We find $< \rho > =$ 0.70 fm compared with the value of 0.78 fm in the Skyrme model.

In the Skyrme model, collective quantization of the rotational and isospin degrees of freedom of the soliton has been performed and it is shown that the ground state is the deuteron. As all the symmetries of the B=2 soliton have equivalents in the chiral quark meson model, the ground state of the toroidal soliton in this model would also have the quantum numbers of the deuteron. The toroidal distribution of energy in the deuteron in the chiral soliton models is perhaps understandable, as even in the traditional potential approach to the deuteron, the baryon dendity of p and n orbiting classically in the nuclear potential is toroidal due to short range repulsion between the nucleons.

3 Toroidal solitons with B=3 and 4

For $B > 2$, we use the simpler ansatz:

$$\Psi = \begin{pmatrix} G(r)\chi(\theta,\phi) \\ i\vec{\sigma}.\hat{r}F(r)\chi(\theta,\phi) \end{pmatrix} exp(-i\epsilon t) \tag{23}$$

where χ again satisfies
$(\vec{\tau}.\hat{n} + \vec{\sigma}.\hat{r})\chi = 0.$

We dont make any simplifying assumptions about $\Theta(\rho, z)$ and $\alpha(\rho, z)$. The generalisation of Eqn.(7) would be

$$(L_3 + S_3 + nT_3) \mid 3n - quark\ \ state >= 0. \tag{24}$$

where n=B.

Boundary conditions are the same as for B=2. We find the minima of energy using the relaxation procedure. We again find soliton solutions with toroidal distribution of energy. In fig.2 we give the contour plot of the energy density $\mathcal{E}(\rho, z)$ for the B=3 state. We obtain a similar contour plot for B=4 also. The B=3 state has an energy 3221 Mev corresponding to a binding energy of 226 Mev and the B=4 state has an energy 4404 Mev with a binding energy of 192 Mev. it is gratifying to note that the toroidal states are independent of the particular chiral invariant model.

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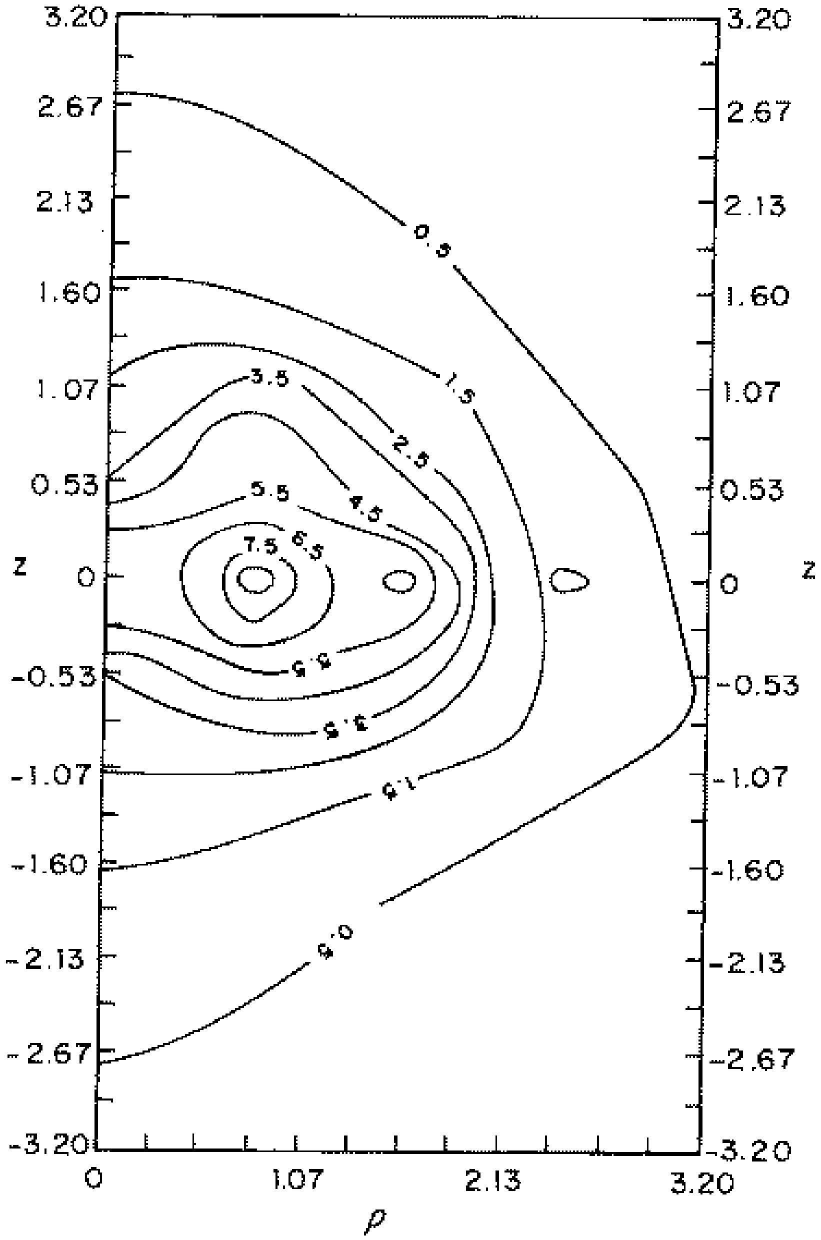

Fig.1 The contour plot for the energy density $\epsilon(\rho, z)$ for the B=2 toroidal soliton.

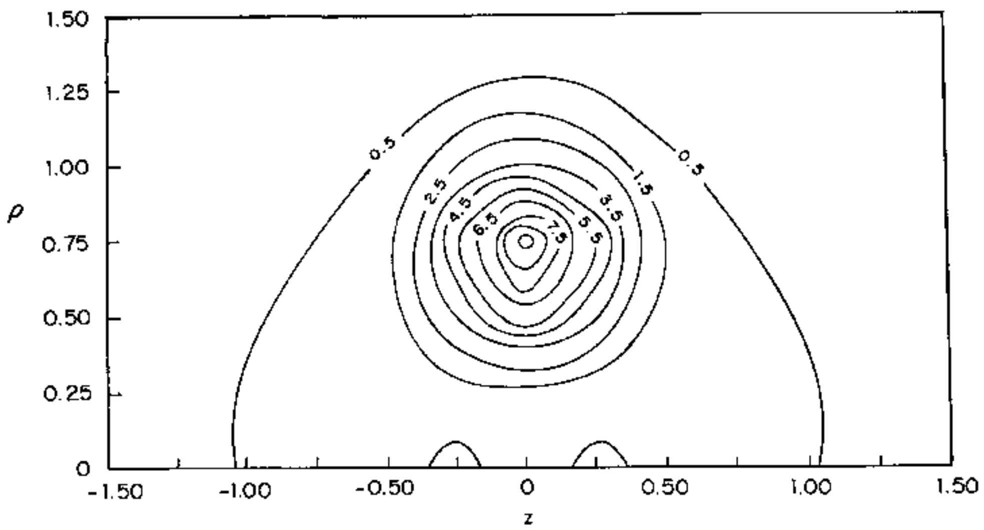

Fig. 2 The contour plot for the energy density $\epsilon(\rho, z)$ for the B=3 toroidal soliton.