Renormalization of the Semiclassical Hamiltonian Field Theory

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Abstract

The Hamiltonian approach to the quantum field theory is considered. Since there are additional difficulties such as the Haag theorem and Stueckelberg divergences, renormalization of the time-dependent dynamical quantum field theory is much more complicated than renormalization of the S-matrix. It is necessary to consider the regularized theory with ultraviolet and infrared cutoffs and impose the conditions not only on the dependence of the Hamiltonian on the cutoffs (as usual) but also on the dependence of the initial states. It happens that one should consider the initial states to be singulary dependent on the cutoffs in order to avoid the Stueckelberg divergences. The problem of imposing conditions on the semiclassical initial states is discussed. Different prescriptions for choice of initial conditions are analysed.

1. Introduction

There are two different languages to construct quantum field theory. One of them is based on the notion of the S-matrix, another uses the formalism of equations of motion.

These approaches are somehow equivalent. One can start from the S-matrix and obtain the Green functions [1] which determine in the unique fashion all the properties of the quantum field theory according to the well-known

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Whightman reconstruction theorem [2]. On the other hand, the scattering theory also can be constructed as a corollary of the equations of motion.

However, there are several advantages and disadvantages of each of formulations of quantum field theory. The S-matrix theory is manifestly Poincare-invariant and can be renormalized by the procedure of the Bogoliubov-Parasiuk R-operation [1]. On the other hand, to describe time evolution, one needs a dynamical formalism based on the equations of motion. It is also much more convenient to apply non-perturbative methods such as semi-classical technique [3] to the equations rather than to the S-matrix.

The main disadvantage of the formalism of equations of motion is that the structure of divergences is very complicated. For example, there is a Haag theorem (see, for example, [2]) that tells us that the interaction representation cannot be defined in quantum field theory if the interaction is non-zero.

The Haag theorem leads, for example, to the problem of the Stueckelberg divergences [4] (see also [1]). Let us consider the virtual process of emission of the photon by the electron. When one considers the S-matrix, i.e. the infinite time interval, such a process is forbidden by the conservation laws. However, at the finite time intervals this process takes place. Consider the initial one-electron state. The first-order perturbative evolution operator contains the term that transforms this state to the state containing two particles, electron and photon. The amplitude $\Phi_{\mathbf{pk}}$ that the momentum of the photon is $\mathbf{k}$ and the momentum of the electron is $\mathbf{q}$ has the following asymptotic behaviour at $\mathbf{k}, \mathbf{p} \to \infty$, $\mathbf{k} + \mathbf{p} = const$: $\Phi_{\mathbf{pk}} \sim |\mathbf{k}|^{-3/2}$. Thus, the full probability to emit the virtual photon diverges as $\int \frac{d\mathbf{k}}{|\mathbf{k}|^3}$ at larges $\mathbf{k}$.

Certainly, one can consider the more physical processes as decay of the Z-boson into a lepton-antilepton pair. The same difficulty will arise.

2. Regularization and renormalization of the equation of motion

1. How should one eliminate such additional divergences? First of all, because of the Haag theorem it is necessary to consider not the local quantum field theory interaction Hamiltonian of the type $H_1 = \int d\mathbf{x} \varphi^n(\mathbf{x})$ but its nonlocal analog (cf.[5]) $H_1^{\Lambda,L} = \int d\mathbf{x} g_L(\mathbf{x}) \varphi^n_\Lambda(\mathbf{x})$ expressed via the ultraviolet-cutoffed field $\varphi_\Lambda(\mathbf{x}) = \int d\mathbf{y} A_\Lambda(\mathbf{x}-\mathbf{y})\varphi(\mathbf{y})$. When the ultraviolet and infrared cutoffs Λ and L go to infinity, the functions g_L and A_Λ behave as $A_\Lambda(\mathbf{x}) \to \delta(\mathbf{x}), g_L(\mathbf{x}) \to 1$.

2. Consider the problem of renormalizing the Schrodinger equation

$$i\dot{\Psi}_{\Lambda,L} = \hat{H}^{\Lambda,L}\Psi_{\Lambda,L}. \tag{1}$$

First of all, the quantum Hamiltonian $\hat{H}^{\Lambda,L}$ should not have the simple form $H_0 + H_1^{\Lambda,L}$. It is necessary to add to this expression the counterterms in order to make the S-matrix finite. This means that one should impose the conditions on the dependence of the Hamiltonian on the cutoff parameters. This is the usual renormalization of the loop graphs [1].

3. But this renormalization procedure will not help us in eliminating the Stueckelberg divergences because they arise at the level of the tree Feynman graphs. One can note that the reason for the Stueckelberg divergences was that one considered "bare" electron instead of "physical" electron consisting of electron and virtual photons. However, the state of physical electron may singularily depend on the cutoff parameters within the Hamiltonian framework [5]. One can hope that there exists an unitary operator $T_{\Lambda,L}$ transforming bare particles to physical particles. This means that one must consider only the states $\Psi_{\Lambda,L}$ depending on the cutoff parameters Λ, L as follows, $\Psi_{\Lambda,L} = T_{\Lambda,L}\Phi$. All the singularities of the amplitude $\Psi_{\Lambda,L}$ are involved in the operator $T_{\Lambda,L}$, so that one can describe the physical state by the regular state vector Φ rather than by the singular vector $\Psi_{\Lambda,L}$. The Schrodinger equation for the regular vector Φ has the form:

$$i\dot{\Phi} = T^{+}_{\Lambda,L} H_{\Lambda,L} T_{\Lambda,L} \Phi. \tag{2}$$

This means that we are considering another, non-Fock representation of the canonical commutation relations (cf.[7]).

3. The Faddeev-type transformation

Let us try to choose the transformation $T_{\Lambda,L}$ in a suitable way to eliminate the Stueckelberg divergences. One can use the perturbative approach and seek for the transformation $T_{\Lambda,L}$ in the following form [6]:

$$T_{\Lambda,L} = \exp[gA^1_{\Lambda,L} + g^2 A^2_{\Lambda,L} + ...] \tag{3}$$

for the anti-Hermitian operators $A^1_{\Lambda,L}$, $A^2_{\Lambda,L}$, ... In the leading non-trivial order of perturbation theory, one obtains the following form of the operator entering to the right-hand side of eq.(2):

$$T^{+}_{\Lambda,L} H_{\Lambda,L} T_{\Lambda,L} = H_0 + g(H_1^{\Lambda,L} - [A^1_{\Lambda,L}, H_0]) + ...$$

instead of $H_{\Lambda,L} = H_0 + gH_1^{\Lambda,L} + ...$ This interaction Hamiltonian can be chosen to be regular. Of course, the choice of the operator $A^1_{\Lambda,L}$ is not unique. However, this ambiguity corresponds to the possibility of choice of different (equivalent) representations of the commutation canonical relations. All physical quantities such as S-matrix or decay rates are invariant under change of the operators $A^k_{\Lambda,L}$ entering to the formula for the Faddeev-type transformation (3).

4. The Bogoliubov S-matrix

There is also another approach to construct the transformation $T_{\Lambda,L}$ which is based on the Bogoliubov procedure of switching on the interaction [1]. Let us investigate for the simplicity the case of finite volume only. Consider the interaction Hamiltonian

$$H_1(t) = \xi(t)H_1, \tag{4}$$

instead of H_1, where $\xi(t)$ is a time-dependent intensity of the interaction. It is well-known that for smooth rapidly damping at $\pm\infty$ function $\xi(t)$ the corresponding S-matrix $S[\xi]$ can be renormalized [1].

Consider the function ξ_0 depicted on fig.1. This function corresponds to the interaction which is slowly swtched on and rapidly switched off. It happens that the transformation $T_{\Lambda,L}$ can be chosen as

$$T_{\Lambda,L} = S[\xi_0]. \tag{5}$$

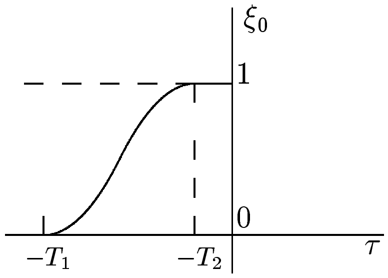

Fig.1

Let us show that the evolution operator corresponding to the Hamiltonian entering to eq.(2),

$$S^+[\xi_0]U_tS[\xi_0], \tag{6}$$

where U_t is the evolution operator in the original theory (1), is regular as $\Lambda, L \to \infty$.

Let us investigate some properties of the S-matrix.

1. For the intensity function $\xi_t(\tau) = \xi_0(\tau - t)$ one has $S[\xi_t] = e^{iH_0t}S[\xi_0]e^{-iH_0t}$.

2. For the function $\tilde{\xi}_t(\tau) = \begin{cases} \xi_0(\tau), \tau < 0 \\ 1, 0 < \tau < t \\ 0, \tau > t \end{cases}$ one has $S[\tilde{\xi}_t] = e^{iH_0 t} U_t S[\xi_0]$.

Consider the following functions of switching the interaction $\tilde{\xi}_1 = \tilde{\xi}_t$, $\tilde{\xi}_2 = \xi_t$ and corresponding smooth functions $\xi_1(\tau) = \begin{cases} \tilde{\xi}_1(\tau), \tau < t \\ \xi_0(t-\tau), t < \tau \end{cases}$ $\xi_2(\tau) = \begin{cases} \tilde{\xi}_2(\tau), \tau < t \\ \xi_0(t-\tau), t < \tau \end{cases}$ Since $\xi_1 = \xi_2$ at $\tau > t$, the product $S^+[\xi_2]S[\xi_1]$ being regular (because ξ_1, ξ_2 are smooth) is equal to $S^+[\tilde{\xi_2}]S[\tilde{\xi_1}]$. Making use of the properties of the Bogoliubov S-matrix, one obtains $S^+[\tilde{\xi_2}]S[\tilde{\xi_1}] = e^{iH_0 t} S^+[\xi_0] U_t S[\xi_0]$. We see that the operator (6) is expressed via the Bogoliubov S-matrices $S[\xi_1]$ and $S[\xi_2]$ corresponding to the smooth functions ξ_1 and ξ_2 and the evolution operator for the free field theory. This means that expression (6) is regular as $\Lambda, L \to \infty$, so that the operator $T_{\Lambda,L}$ can be chosen as (5).

5. Formal semiclassical theory

Consider the semiclassical approximation for the quantum field theory. For the simplicity, investigate the case of the scalar theory with the Lagrangian $\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial_\mu\varphi - \frac{1}{g}V(\sqrt{g}\varphi) - V_1(\sqrt{g}\varphi) - ...$, where the term $\frac{1}{g}V(\sqrt{g}\varphi)$ corresponds to the classical Lagrangian, while V_1 is a one-loop counterterm. From the Hamiltonian point of view, one can write the functional Schrodinger equation,

$$i\dot{\Psi}_t[\varphi(\cdot)] = \int d\mathbf{x} \left[-\frac{1}{2}\frac{\delta^2}{\delta\varphi(\mathbf{x})\delta\varphi(\mathbf{x})} + \frac{1}{2}(\nabla\varphi)^2(\mathbf{x}) + \frac{1}{g}V(\sqrt{g}\varphi) + V_1(\sqrt{g}\varphi) \right] \Psi_t[\varphi(\cdot)] \tag{7}$$

for the functional $\Psi[t,\varphi(\cdot)]$. After rescaling $\sqrt{g}\varphi = \tilde{\varphi}$ and multiplying by g eq.(7) is taken to the semiclassical form, since the coefficients of all derivation operators are of order $O(g)$, while the coefficients of the operator of multiplication be $\tilde{\varphi}$ are of order $O(1)$.

Let us apply the complex-WKB technique [3] to eq.(7). One should consider the following wave functional:

$$\Psi[t,\varphi(\cdot)] = e^{\frac{i}{g}S^t} e^{\frac{i}{\sqrt{g}}\int d\mathbf{x}\Pi^t(\mathbf{x})(\varphi(\mathbf{x}) - \frac{\Phi^t(\mathbf{x})}{\sqrt{g}})} f^t\left(\varphi(\cdot) - \frac{\Phi^t(\cdot)}{\sqrt{g}}\right). \tag{8}$$

Substituting expression (8) to eq.(7) and making equal the terms of order $O(1/g)$, one obtains the condition on the phase, $S^t = \int_0^t dt d\mathbf{x}\mathcal{L}$. The order

$O(g^{-1/2})$ gives us the classical equations of motion,

$$\dot{\Pi}^t = \Phi^t, \qquad \partial_\mu \partial_\mu \Phi^t + V'(\Phi^t) = 0,$$

while the order $O(1)$ corresponds to the Schrodinger-type equation on f with the quadratic Hamiltonian

$$i\dot{f} = H_2 f$$
$$H_2 = \int d\mathbf{x} \left[-\frac{1}{2} \frac{\delta^2}{\delta\phi(\mathbf{x})\delta\phi(\mathbf{x})} + \frac{1}{2}(\nabla\phi)^2(\mathbf{x}) + \frac{1}{2} V''(\Phi^t(\mathbf{x}))\phi^2(\mathbf{x}) + V_1(\Phi^t(\mathbf{x})) \right]. \tag{9}$$

One can notice that the complex-WKB method is equivalent to the procedure of extracting the classical field component, $\varphi = \frac{\Phi}{\sqrt{g}} + \phi$ and quantizing the obtained theory (cf. [9]).

One can construct then the exact solutions to eq.(9). The simplest way is to consider the Gaussian ansatz

$$f^t[\phi(\cdot)] = c^t \exp \left[\frac{i}{2} \int d\mathbf{x} d\mathbf{y} \phi(\mathbf{x}) \tilde{R}^t(\mathbf{x}, \mathbf{y}) \phi(\mathbf{y}) \right] \tag{10}$$

which obeys eq.(9) if

$$\begin{array}{c} \dot{\hat{R}}^t + \hat{R}^t \hat{R}^t + (-\Delta + V''(\Phi^t(\mathbf{x}))) = 0, \\ i\dot{c}^t = (-\frac{i}{2} Tr \hat{R}^t + \int d\mathbf{x} V_1(\Phi^t(\mathbf{x}))) c^t, \end{array} \tag{11}$$

where $\hat{R}^t$ is the operator with kernel $\tilde{R}^t$, $(\hat{R}^t f)(\mathbf{x}) = \int d\mathbf{y} \tilde{R}^t(\mathbf{x}, \mathbf{y}) f(\mathbf{y})$. We see that the Gaussian approximation considered as variational approximation in [10] is exact as $g \to 0$.

The more complicated ansatz for eq.(9) is the following,

$$\Lambda^+[\delta\Phi_1^t] ... \Lambda^+[\delta\Phi_k^t] f^t[\phi(\cdot)],$$
$$\Lambda^+[\delta\Phi] = \int d\mathbf{x} [\tfrac{d}{dt} \delta\Phi^t \phi(\mathbf{x}) - \delta\Phi^t(\mathbf{x}) \tfrac{1}{i} \tfrac{\delta}{\delta\phi(\mathbf{x})}]$$

where operators $\Lambda^+[\delta\Phi]$ commute with $i\frac{d}{dt} - H_2$ and transform a solution to eq.(9) to another solution if

$$\partial_\mu \partial_\mu \delta\Phi^t + V''(\Phi^t) \delta\Phi^t = 0.$$

We see that the function $\delta\Phi^t$ plays a role of a fluctuation about the classical solution.

The problem of renormalization of eq.(11) arises. To investigate this problem, it is convenient to use the operational technique [11].

6. Operational calculus

To analyse eq.(11), it is convenient to present all the operators via the differentiation operator $-i\frac{\partial}{\partial \mathbf{x}}$ and the operator of multiplication by $\mathbf{x}$. The function $A(\mathbf{x},\mathbf{k})$ such that $\hat{A} = A(\mathbf{x}, -i\frac{\partial}{\partial \mathbf{x}})$ is called as a symbol of the operator $\hat{A}$ [11]. The procedure of reproducing the operator $\hat{A}$ from the symbol A is called quantization. The main difficulty is that numbers x_i and k_i commute, while operators x_i and $-i\partial/\partial x_i$ do not commute. This means that one should specify the ordering of coordinate and momentum operators and obtain then different presciptions of quantization. For example, the qp-quantization is the following: one puts the differentiation operators to the right, the coordinate operators to the left, so that the matrix element $<\mathbf{x}|\hat{A}|\mathbf{k}>$ of the operator $\hat{A}$ between states $<\mathbf{x}|$ and $|\mathbf{k}>$ with given coordinate $\mathbf{x}$ and given momentum $\mathbf{k}$ correspondingly is expressed via its symbol

$$<\mathbf{x}|\hat{A}|\mathbf{k}>= A(\mathbf{x},\mathbf{k}) <\mathbf{x}|\mathbf{k}> . \tag{12}$$

Eq.(12) can be viewed as a definition of the qp-quantization,

$$\hat{A} = A(\overset{2}{\mathbf{x}}, -i\overset{1}{\frac{\partial}{\partial \mathbf{x}}}).$$

All operational calculus can be reformulated in terms of symbols. For example, let A and B be qp-symbols of the operators $\hat{A}$ and $\hat{B}$, $\hat{A} = A(\overset{2}{\mathbf{x}}, -i\overset{1}{\frac{\partial}{\partial \mathbf{x}}})$, $\hat{B} = B(\overset{2}{\mathbf{x}}, -i\overset{1}{\frac{\partial}{\partial \mathbf{x}}})$. Then the sum of these operators $\hat{A}+\hat{B}$ corresponds to the sum of the sumbols $A+B$, while the symbol of the product $\hat{A}\hat{B} = (A*B)(\overset{2}{\mathbf{x}}, -i\overset{1}{\frac{\partial}{\partial \mathbf{x}}})$ is

$$(A*B)(\mathbf{x},\mathbf{k}) = A(\overset{2}{\mathbf{x}}, \mathbf{k} - i\overset{1}{\frac{\partial}{\partial \mathbf{x}}})B(\mathbf{x},\mathbf{k}). \tag{13}$$

Eq.(13) can be viewed as a quantum "multiplication" of functions.

Eq.(13) is very suitable to analyze the asymptotic behaviour of the symbol of the operator $\hat{A}\hat{B}$ at large values of $|\mathbf{k}|$. Namely, expression (13) can be presented as a formal asymptotic series

$$(A * B)(\mathbf{x},\mathbf{k}) = A(\mathbf{x},\mathbf{k})B(\mathbf{x},\mathbf{k}) - i\frac{\partial A}{\partial \mathbf{k}}\frac{\partial B}{\partial \mathbf{x}} - \frac{1}{2}\frac{\partial^2 A}{\partial k_m \partial k_n}\frac{\partial^2 B}{\partial x_m \partial x_n} + ... \quad (14)$$

with the first term being a classical product. Eq.(14) is an expansion in $1/|\mathbf{k}|$ as $|\mathbf{k}| \to \infty$. Namely, if $A \sim |\mathbf{k}|^{-a}$, $B \sim |\mathbf{k}|^{-b}$ as $\mathbf{k} \to \infty$, the first term is of order $|\mathbf{k}|^{-a-b}$, the next being of order $|\mathbf{k}|^{-a-b-1}$ etc. It is very important that A and B should have no oscillating factors like $e^{i|\mathbf{k}|S}$ at large $|\mathbf{k}|$.

The trace of the operator can be also expressed via its symbol,

$$Tr\hat{A} = \frac{1}{(2\pi)^d}\int d\mathbf{x}d\mathbf{k}A(\mathbf{x},\mathbf{k}), \quad (15)$$

so that one can investigate the divergences in $Tr\hat{R}$ entering to eq.(11) with the help of eq.(15).

7. Possible Schrodinger functionals. Free theory

Not arbitrary operator $\hat{R}^t$ can enter to the Gaussian wave functional (10). First of all, let us investigate this problem for the simple case of free theory. The Gaussian functional (10) corresponds to the Gaussian Fock space vector $\exp[\frac{1}{2}\int d\mathbf{x}d\mathbf{y}a^+(\mathbf{x})\tilde{B}(\mathbf{x},\mathbf{y})a^+(\mathbf{y})]|0>$, where $a^{\pm}(\mathbf{x}) = \int \frac{d\mathbf{k}}{(2\pi)^{d/2}}a^{\pm}_{\mathbf{k}}e^{\mp i\mathbf{k}\mathbf{x}}$ are linear combinations of creation (annihilation) operators. It is well-known [8] that this expression corresponds to the Fock space state if $||\hat{B}|| < 1$, $Tr\hat{B}^+\hat{B} < \infty$. This means that the following conditions should be imposed on the asymptotical behaviour of the symbol of the operator $\hat{B}$, $B(\mathbf{x},\mathbf{k}) \sim \frac{1}{|\mathbf{k}|^{d/2+\varepsilon}}$, $|\mathbf{k}| \to \infty$. The Gaussian quadratic forms entering to the different representations for the state are associated each other, $\hat{R} = i\sqrt{\hat{\omega}}(1-\hat{B})(1+\hat{B})^{-1}\sqrt{\hat{\omega}}$, where $\hat{\omega}$ is the operator $\hat{\omega} = \sqrt{-\Delta + m^2}$ with the symbol $\omega_{\mathbf{k}}$. Thus, the symbol of the operator R should have the following asymptotical behaviour as $|\mathbf{k}| \to \infty$,

$$R(\mathbf{x},\mathbf{k}) = i\omega_{\mathbf{k}} + O(|\mathbf{k}|^{-(d/2+\varepsilon-1)}). \quad (16)$$

8. Possible Schrodinger functionals. General case

Let us impose the condition on the Gaussian functionals (10) [15]. First of all, notice that the condition (16) is not invariant under time evolution. Namely, consider the equation on the operator $\hat{R}$ (11) which can be presented as the equation on its *qp*-symbol R:

$$\dot{R}^t + R^t * R^t + \omega^2_{\mathbf{k}} + f_t = 0, \quad (17)$$

where $f_t(\mathbf{x},\mathbf{k}) = f_t(\mathbf{x}) = V''_{int}(\Phi^t(\mathbf{x})) = V''(\Phi^t(\mathbf{x})) - m^2$, $\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2+m^2}$. One can expand the symbol R^t in $1/|\mathbf{k}|$, $R = i\omega_{\mathbf{k}} + R_1 + ...$, where R_1 obeys the equation, $\dot{R}_1 + 2i\omega_{\mathbf{k}}R_1 + f_t(\mathbf{x}) = O(|\mathbf{k}|^{-1})$. the solution to this equation is expressed as $R_1^t(\mathbf{x},\mathbf{k}) = \frac{if_t(\mathbf{x})}{2\omega_{\mathbf{k}}} + \frac{\varphi_t(\mathbf{x},\mathbf{k}/\omega_{\mathbf{k}})}{\omega_{\mathbf{k}}}e^{-2i\omega_{\mathbf{k}}t}$ for some φ_t. We see that the condition (16) should be improved at $d+1 \geq 5$.

One can also notice that the conditions

$$\begin{gathered} R(\mathbf{x},\mathbf{k}) = i\omega_{\mathbf{k}} + \frac{i}{2\omega_{\mathbf{k}}}V''_{int}(\Phi(\mathbf{x})) + O(\frac{1}{|\mathbf{k}|^{d/2-1+\varepsilon}}), \quad d=5,6 \\ R(\mathbf{x},\mathbf{k}) = i\omega_{\mathbf{k}} + \frac{i}{2\omega_{\mathbf{k}}}V''_{int}(\Phi(\mathbf{x})) - \frac{1}{4}\left(\frac{\partial}{\partial t} + \frac{\mathbf{k}}{\omega_{\mathbf{k}}}\frac{\partial}{\partial \mathbf{x}}\right)\frac{V''_{int}(\Phi(\mathbf{x})}{\omega_{\mathbf{k}}^2} + O(\frac{1}{|\mathbf{k}|^{d/2-1+\varepsilon}}), \\ d=7,8 \end{gathered} \tag{18}$$

are invariant under time evolution.

These conditions can be interpretted from the point of view of the Bogoliubov procedure of switching the interaction. Let us consider the theory with interaction (4) with the function ξ_0 depicted on fig.1, One can apply the semiclassical complex-WKB technique to this theory and obtain the equation on R: $\dot{R}^t + R^t * R^t + [\omega_{\mathbf{k}}^2 + \xi_0(t)V''_{int}(\Phi(\mathbf{x}))] = 0$ instead of (17). One should also impose the conditions at $t < -T_1$: $R^{-T_1}(\mathbf{x},\mathbf{k}) = i\omega_{\mathbf{k}} + O(\frac{1}{|\mathbf{k}|^{d/2-1+\varepsilon}})$. Applying a regular perturbation theory in $1/|\mathbf{k}|$, one finds the conditions (18).

9. Conclusions

We conclude the talk by adding the following remarks.

1. Renormalization procedure is usually associated with loop Feynman graphs. We have seen that for time-dependent dynamical theory it is necessary to consider the divergences for the tree graphs, i.e. for the classical field theory. The fact that there exist the Stueckelberg divergences in the perturbation theory is shown to imply that the free condition on the Gaussian quadratic form, eq.(16), is not inveriant under time evolution. Non-triviality of initial conditions for the perturbation theory which is associated with Faddeev or Bogoliubov transformations leads to the nontriviality of the initial conditions on the Gaussian quadratic form.

2. The invariance of the conditions (18) under time evolution is not mathematically proved in this talk. Hovever, the mathematical formulation of this statement is presented in [12].

3. One can compare the obtained conditions on the quadratic form R, eqs.(18), with known conditions. For example, free initial condition is not invariant under time evolution. The conjecture that one should diagonalyse

the Hamiltonian at each moment of time [13] (this corresponds to the condition $\hat{R} \sim \sqrt{-\Delta + m^2 + V''_{int}(\Phi)}$) is also correct only for sufficiently small dimensions. For spatially homogeneous case, our condition is in agreement with [14].

4. One can investigate also the pre-exponential factor in the Schrodinger wave functional [15]. One can notice that some of the divergences are eliminated by the one-loop counterterm V_1, while other divergences should be involved into the condition on c. This means that the factor c may be singular as $\Lambda \to \infty$, so that the Schrodinger representation is singular for this case. One can understand this with the help of the following analogy. The expression $\exp(\frac{1}{2}\sum_k c_k (a_k^+)^2)|0>$ determines the Fock space vector if $c_k < 1$ for all k and $\sum_k |c_k|^2 < \infty$. However, the Schrodinger representation for this vector exists if $\sum_k |c_k| < \infty$, so that the Schrodinger representation may not exist even for the well-defined Fock vector. This may be analogous to the quantum field theory.

5. One can generalize the presented approach for the more complicated field systems. Generalization to the fermionic case is straightforward since it is necessary to extract the classical parts only from the bosonic degrees of freedom. To generalize the approach to the constrained systems, one can perform the gauge fixing procedure and apply this approach. One can also perform a direct investigation of the Hamiltonian formulation of the gauge theories by applying the theory of Lagrangian manifolds with complex germs [3]. The developed approach is also useful to investigate the large-N systems (see [16] for more details).

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