Complex Monopoles in YM + Chern-Simons Theory in 3 Dimensions

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Abstract

In this talk, we give a brief discussion of complex monopole solutions in the three dimensional Georgi-Glashow model with a Chern-Simons term. We find that there exist complex monopole solutions of finite action. They dominate the path integral and disorder the Higgs vacuum, but electric charges are not confined. Subtleties in the model and issues related to Gribov copies are also noted.

1. Introduction

Monopole solutions can exist in gauge theories in which there is an unbroken compact U(1)group. The three dimensional SO(3) Yang-Mills theory with an adjoint Higgs field which breaks the $SO(3)$ to $U(1)$ (also known as the Georgi-Glashow model) was shown by Polyakov [1] to have monopole solutions which lead to the linear confinement of electric charges. The monopole contribution causes the Higgs vacuum to be 'disordered', *i.e.,* $\langle h^a \rangle = 0$ while $\langle h_a h^a \rangle = v^2 \neq 0$. In other words, long-range order in the Higgs vacuum is destroyed by the existence of monopoles. If now one adds the Chern-Simons (CS) term, it can be readily seen that [2] the gauge fields acquire topological mass. the unbroken $U(1)$ gauge field also becomes massive. As a consequence, the linear confinement disappears in the presence of the CS term: there is no long-range force in the Georgi-Glashow-Chern-Simons (GGCS) model to start with. The electric flux is not conserved. It does not matter for the issue of the confinement whether or not monopole configurations dominate in the functional integral.

Recall that the action for the GGCS model is given by:

$$S = S_{\rm YM} + S_{\rm CS} + S_{\rm H} \qquad \text{(Euclidean)} \tag{1.1}$$

where

$$\begin{aligned}
S_{\rm YM} &= -\frac{1}{2g^2}\int d^3x \,{\rm tr}\, F^2 \\
S_{\rm CS} &= -\frac{i\kappa}{g^2}\int d^3x \epsilon^{\mu\nu\lambda}\,{\rm tr}(A_\mu\partial_\nu A_\lambda + \frac{2}{3}A_\mu A_\nu A_\lambda) \\
S_{\rm H} &= \frac{1}{g^2}\int d^3x[\tfrac{1}{2}\left(D_\mu h^a\right)^2 + \frac{\lambda}{4}\left(h_a h^a - v^2\right)^2]
\end{aligned}$$

Notice that $S_{\rm SC}$ is pure imaginary, *i.e.,* the action is complex. Therefore it is natural to seek complex solutions that extremize the complex action. In the following, we shall explore the complex monopole solutions and address the following issues:

• Are there finite action complex monopole solutions?

• See the connection with gauge (Gribov) copies (i.e., do they cancel or wipe out the effect of monopoles, as in the real monopole case?

• What happens to the long-range order in Higgs vacuum?

We shall address these questions within a saddle point approximation. Few details are presented below, but they could be found in ref. [8]. However, before discussing the complex monopoles, it is worthwhile to recall a few facts of life about the model and the scenario of the real monopole solutions in this model. Many authors have shown that there is no real monopole-type field configuration of a finite action which solves the equations of motion [3]-[6]. This fact has been interpreted as indicating the irrelevance of (real) monopole configurations in the model.

THE MODEL

The standard monopole ansatz [7] for the action given by eq. (1) is given by:

$$\begin{aligned}
&h^a(\vec{x}) = \hat{x}^a h(r) \\
&A_{a\mu}(\vec{x}) = \frac{1}{r}\left[\epsilon_{a\mu\nu}\hat{x}_\nu(1-\phi_1) + (\delta_{a\mu} - \hat{x}_a\hat{x}_\mu)\phi_2 + rA\hat{x}_a\hat{x}_\mu\right]
\end{aligned} \tag{1.2}$$

with boundary conditions:

$$\begin{array}{lllll} h=0 & , & \phi_1=1 & , & \phi_2=0 \quad \text{at } r=0 \\ h=v & , & \phi_1=0 & , & \phi_2=0 \quad \text{at } r=\infty \end{array}$$

Recall that with this ansatz the 't Hooft-Polyakov monopole has $h, \phi_1 \neq 0$, and $\phi_2 = A = 0$. The residual $U(1)$ symmetry in the model after the breaking $SO(3) \to U(1)$ is parameterized using again the standard abelian projection as follows:

$$\Omega = \exp\left(\frac{i}{2} f(r) \hat{x}_a \tau^a\right). \tag{1.3}$$

Under this symmetry the components of the gauge field (2) transform as:,

$$\begin{array}{rcl} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} & \to & \begin{pmatrix} \cos f & \sin f \\ -\sin f & \cos f \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \\ A & \to & A - f'. \end{array}$$

The equations of motion are invariant under this transformation but the action (1.1) is not. Under a more general gauge transformation $A \to \Omega A \Omega^{-1} + g^{-1} \Omega d \Omega^{-1}$,

$$\delta_g S_{CS} = \frac{i\kappa}{g^2} \int \mathrm{tr}\ d(A \wedge d\Omega^{-1}\Omega) + \frac{i\kappa}{3g^2} \int \mathrm{tr}\ d\Omega^{-1}\Omega \wedge d\Omega^{-1}\Omega \wedge d\Omega^{-1}\Omega \ . \tag{1.4}$$

If the theory is defined on S^3, the first term vanishes. The second term is the winding number of the mapping Ω. This leads to the quantization condition $4\pi\kappa/g^2 = n =$ an integer [2].

On R^3, however, the first term does not vanish for monopole configurations. Under the transformation (1.3) the first and last terms on the r.h.s. of eq. (1.4) are $(4\pi i\kappa/g^2)\sin f(\infty)$ and $(4\pi i\kappa/g^2)(f(\infty) - \sin f(\infty))$, respectively. Hence

$$S_{CS} \to S_{CS} + \frac{4\pi i\kappa}{g^2} f(\infty) \quad . \tag{1.5}$$

This raises a puzzle regarding the quantization of this theory. The usual Faddeew-Popov procedure

$$\mathcal{Z} \sim \int [\mathcal{D}A_\mu][\mathcal{D}h] \Delta_F(A_\mu) \delta[\mathcal{F}(A_\mu)] e^{-S[A_\mu, h]}$$

works fine if $\delta_g S = 0$, but not for a non-invariant action. For a more detailed discussion (but not a resolution!) of this puzzle, see ref. [8]. Let us just remark that at this stage, to our knowledge, there is no consistent formulation of this model beyond semi-classical level.

(Real) Monopole Solutions

As mentioned earlier, the real monopole solutions of the GGCS model have been the subject of several previous studies, with the similar conclusion that the contribution of real monopoles in this model vanishes in the partition function. Pisarski [3] showed that real monopoles give rise to a linearly divergent action, which makes (real) monopoles irrelevant, and furthermore, the linearly divergent action leads to the interpretation that the monopole-antimonopole pairs are confined. In ref. [5], the authors reach a similar conclusion. There, in the gauge transformation of the full action, what we have referred to as $f(\infty)$ in: $S_{\rm CS} \to S_{\rm CS} + inf(\infty)$ is interpreted as the collective coordinate associated with the monopole solution, and is restricted to a 2π interval. They argue that, therefore, integrating over this collective coordinate[1] wipes out the monopole contribution in the path integral:

$$\mathcal{Z} \propto \int df(\infty) e^{inf(\infty)} = 0.$$

We will re-examine these considerations for complex monopoles shortly. We do this by considering the complex monopole solutions for different gauge choices.

2. 'Axial' gauge: $\hat{x}_\mu A_\mu = 0$

In this gauge, the component $A = 0$ (see eq. (1.2)) and the remaining component fields satisfy the following equations of motion:

$$\begin{cases} \left(\dfrac{d^2}{dr^2} - \dfrac{1}{r^r}(\phi_1^2 + \phi_2^2 - 1) - h^2\right)\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} + i\kappa \dfrac{d}{dr}\begin{pmatrix} \phi_2 \\ -\phi_1 \end{pmatrix} = 0 \\ h'' + \dfrac{2}{r}h' - \lambda(h^2 - v^2) + \dfrac{2}{r}(\phi_1^2 + \phi_2^2)h = 0. \end{cases} \tag{2.1}$$

[1] We shall instead interpret $f(\infty)$ as the parameter associated with the gauge copies (or Gribov copies) which is also summed over in the partition function.

In this gauge, complex solutions exist, which satisfy:

$$\frac{\phi_2}{\phi_1} = i \tanh\left(\frac{\kappa r}{2}\right)$$

where $\kappa = \dfrac{ng^2}{4\pi}$ and boundary conditions:

$$\begin{cases} \phi_1(0) = 1, & \phi_1(\infty) = 0 \\ \phi_2(0) = 0, & \phi_2(\infty) = 0 \end{cases}$$

Let us simply outline how the solutions in th axial gauge are obtained. First we choose the coordinates:

$$\begin{aligned} \eta &= (\phi_1 + i\phi_2)e^{-i\int^r A(r)dr} \\ \zeta &= (\phi_1 - i\phi_2)e^{+i\int^r A(r)dr} \end{aligned}$$

in which the action is written as follows:

$$\begin{aligned} S &= \frac{4\pi}{g^2}\int_0^\infty dr[\zeta'\eta' + \frac{1}{2r^2}(1-\eta\zeta)^2 - \frac{\kappa}{2}(\eta'\zeta - \eta\zeta') \\ &+ \frac{r^2}{2}(h')^2 + h^2\eta\zeta + \frac{\lambda r^2}{4}(h^2 - v^2)^2 + i\kappa A]. \end{aligned} \tag{2.2}$$

We then let $A = 0$ (*i.e.,* fixing the axial gauge). Notice that the action in this gauge becomes real. The BC's and the equations of motion result in the relation: $\zeta = \eta \exp \kappa r$. In the limit of no CS term ($\kappa = 0 \quad \Rightarrow \zeta = \eta$) the equations of motion reduce to:

$$\begin{cases} \zeta'' + \frac{\zeta}{r^2}(1-\zeta^2) - h^2\zeta = 0 \\ (r^2h')' - (2\zeta^2 - \mu^2r^2 + \lambda r^2h^2)h = 0 \end{cases}$$

In the limit $\lambda = 0$, the exact solutions for these equations are the well known BPS solutions:

$$\begin{cases} h(r) = v\coth(vr) - \dfrac{1}{r} \\ \\ \zeta(r) = \dfrac{vr}{\sinh(vr)} \end{cases} \tag{2.3}$$

We use these solutions, and numerically generate the solutions for $\lambda \neq 0$ and $\kappa \neq 0$. These solutions are shown in Fig. 1, These complex solutions have the following features which are characteristic of monopoles: They minimize the <u>real</u> action (in axial gauge), they have mass $\propto 1/g^2$ and the $U(1)$ field strength: $F_{\mu\nu} = -\frac{1}{r^2}\hat{x}_a\epsilon_{a\mu\nu}$. Notice also that in this gauge there are no Gribov copies.

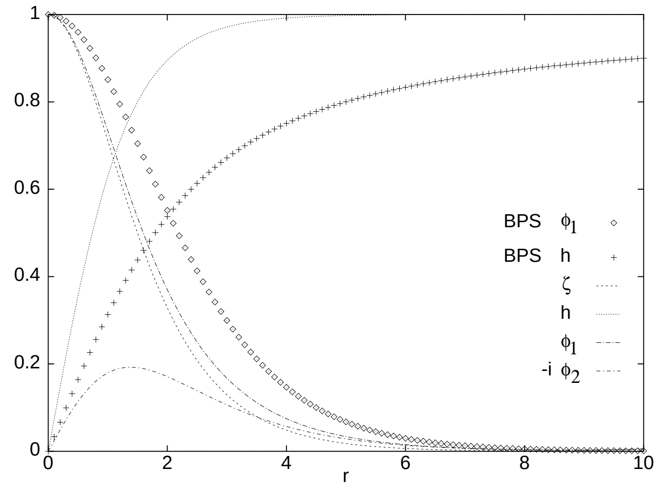

Figure 1: Axial gauge: $\frac{\kappa}{2v} = 0.25$ and $\lambda = 0.5$.

3. Radiation gauge $(D_\mu A^a_\mu = 0)$

In terms of the ansatz (eq. (1.2)), but again with ϕ_1, ϕ_2, and A complex, the gauge condition gives rise to a relations between ϕ_2 and A, namely:

$$A' + \frac{2}{r}A - \frac{2}{r^2}\phi_2 = 0 \ \text{ or } \ A(r) = \frac{2}{r^2}\int_0^r dr\phi_2(r) \qquad (3.1)$$

plus the equations of motion for ϕ_1, ϕ_2, and h, (resp.) in this gauge:

$$(\phi_1' + A\phi_2)' + A(\phi_2' - A\phi_1) + \frac{1}{r^2}(1 - \phi_1^2 - \phi_2^2)\phi_1 + i\kappa(\phi_2' - A\phi_1) - h^2\phi_1 = 0, \qquad (3.2)$$

$$(\phi_2' - A\phi_1)' - A(\phi_1' + A\phi_2) + \frac{1}{r^2}(1 - \phi_1^2 - \phi_2^2)\phi_2 - i\kappa(\phi_1' + A\phi_2) - h^2\phi_2$$

$$= 2\int_r^\infty du \frac{1}{u^2}[\phi_2\phi_1' - \phi_1\phi_2' + A(\phi_1^2 + \phi_2^2) + \frac{i\kappa}{2}(\phi_1^2 + \phi_2^2 - 1)], \tag{3.3}$$

$$(r^2h')' - \lambda r^2(h^2 + v^2)h - 2(\phi_1^2 + \phi_2^2)h = 0. \tag{3.4}$$

Exact numerical solutions of these equations for ϕ_1 and ϕ_2 is rather difficult, and will be determined in the future, but our preliminary numerical analysis indicates that the solutions exist. For the purposes of studying the question of gauge copies which we turn to next, we use lump-like ansatz for these solutions that are consistent with the asymptotic equations.

Recall the residual $U(1)$ with: $\Omega = \exp(\frac{i}{2} f(r)\hat{x}_a\tau^a)$. In this gauge however, there is a nontrivial condition on $f(r)$ [9]:

$$f'' + \frac{2}{r}f' - \frac{2}{r^2}(\phi_1 \sin f + \phi_2(1 - \cos f)) = 0. \tag{3.5}$$

Also recall the non-invariance of the CS term:

$$S_{CS} \longrightarrow S_{CS} + inf(\infty).$$

We now ask the following question: in real monopole case, summing over the gauge copies (or alternatively integrating over the collective coordinate of the monopole [5]) leads to cancellation. Is this true for the complex monopoles as well? In other words,

$$\sum_{\text{Gribov copies}} e^{inf(\infty)} \overset{??}{=} 0.$$

GRIBOV COPIES

We now examine the question of Gribov copies by looking at the solutions $f(\infty)$ of the Gribov equation (3.5) with the appropriate initial conditions at $r = 0$. First we consider the trivial case of vacuum solutions ($A^a_\mu = 0$) i.e., $\phi_1 = 1$, $\phi_2 = 0$:

$$f'' + \frac{2}{r}f' - \frac{2}{r^2}\sin f = 0.$$

This implies the following solutions:

$$f(0) = 0 \Longrightarrow \begin{cases} (i)\ \ f(\infty) = 0 & \leftrightarrow\ \ A^a_\mu = 0 \\ (ii), (iii)\ \ f(\infty) = \pm\pi & \leftrightarrow\ \ \text{Gribov Copies} \end{cases}$$

One of the Gribov copies ($+\pi$) is shown in Fig. 2. For various (positive) $f'(0)$, the value of $f(\infty)$ approached is unique – no surprise in the case of the vacuum solution.

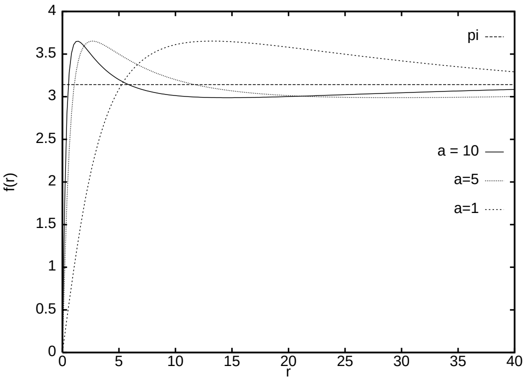

Figure 2: $f(r)$ for vacuum configuration. There are similar curves (not shown) approaching $-\pi$ for negative initial values of $f'(0)$.

It is interesting to consider the 't Hooft-Polyakov monopole $\phi_2 = 0$ but $\phi_1 \neq 0$ ($\phi_1(0) = 1$ and $\phi_1 \to 0$ as $r \to \infty$).[2] In terms of the Gribov equation, this case corresponds to a particle moving in a time-dependent potential. As the potential becomes exponentially small for large t, there can be a continuous family of solutions parameterized by the value of $f(\infty)$. The asymptotic value $|f(\infty)|$ depends of the initial slope $f'(0)$. In the BPS limit it ranges from 0 to 3.98 (see Fig. 3). For $|f'(0)| \ll 1$, $|f(r)|$ remains small. For $|f(r)| \gg 1$, $f(r)$ approaches an asymptotic value before ϕ_1 and ϕ_2 make sizable changes, *i.e.,* $f(r)$ behaves as in the vacuum case. The maximum value for $|f(\infty)|$ is attained for $f'(0) = \pm 2.62$.

We see that even for the BPS solutions, The values of $f(\infty)$ are <u>not</u> restricted to $[-\pi, \pi]$, nor are they completely arbitrary. The plot in Fig. 4 shows the range of $f(\infty)$ as a function of $f'(0)$ with the latter varying over five orders of magnitude. What we find is that $0 \leq f(\infty) < 3.98$, and so integration over all possible values of $f(\infty)$ is not expected to cancel (cf. ref. [5]).

[2] Although this is clearly not a solution to the theory including the CS term, this exercise illustrates the possibility that summing of the Gribov copies may not lead to the cancellation of the monopole-type contribution.

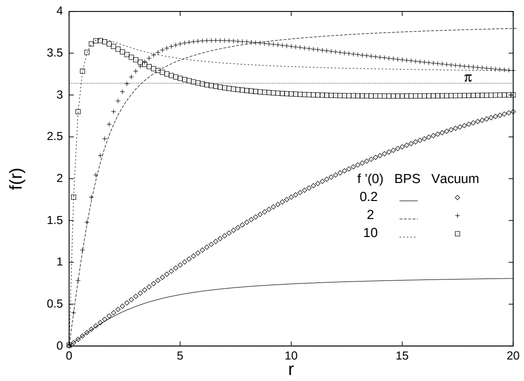

Figure 3: $f(r)$ for BPS monopole configuration. Solid lines correspond to monopoles (generally $\not\lesssim \pi$), other lines are the vacuum solutions (all converging to π for positive initial slopes, a).

Now let us consider the full theory with the Chern-Simons term. For complex ϕ_1 and ϕ_2, solutions of eq. (3.5) are necessarily complex. The lump-like ansatz that are used are such that ϕ_1 is real and ϕ_2 is pure imaginary. Therefore A is also pure imaginary (see eq. (3.1)) We explore "complex" Gribov copies of the solution. There is no "real" Gribov copy. With $f = f_R + i f_I$ eq. (3.5) becomes

$$
\begin{aligned}
&f_R'' + \frac{2}{r} f_R' - \frac{2}{r^2}\Big\{\phi_1 \sin f_R \cosh f_I - \bar{\phi}_2 \sin f_R \sinh f_I\Big\} = 0 \\
&f_I'' + \frac{2}{r} f_I' - \frac{2}{r^2}\Big\{\phi_1 \cos f_R \sinh f_I + \bar{\phi}_2 (1 - \cos f_R \cosh f_I)\Big\} = 0 \ . \qquad (3.6)
\end{aligned}
$$

Boundary conditions are $f_R(0) = f_I(0) = 0$ and $f_R(\infty), f_I(\infty) =$ finite. Although the meaning of these solutions is not clear for $f_I(r) \neq 0$, we point out that solutions satisfying $f_I(\infty) = 0$ might have special role in the path integral. In view of (1.4), such copies carry the additional oscillatory factor $(4\pi i\kappa/g^2) f_R(\infty)$ in the path integral. Examination of Eq. (3.6) with representative ϕ_1 and $\bar{\phi}_2$ shows that such a solution is uniquely found with given $f_R'(0)$. $f_R(\infty)$ is determined as a function of $f_R'(0)$. The range of $f_R(\infty)$ is not restricted to $[-\pi, \pi]$ etc. We expect no cancellation in the path integral from these copies. Therefore the monopole contribution to the path integral does not appear to be destroyed.

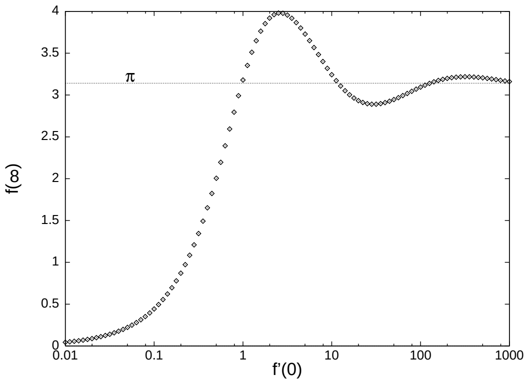

Figure 4: $f(\infty)$ for BPS monopole vs. initial slope ($f'(0)$).

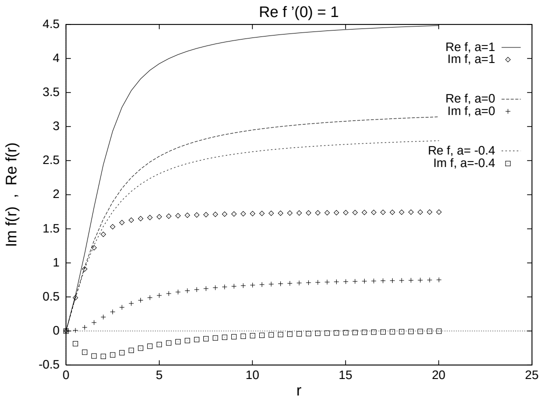

Figure 5: Complex $f(r)$ - YM-Chern-Simons.

4. Conclusions and Discussion

We have seen that in the GGCS theory, complex monopoles exist, and that they have a non-vanishing contribution to the path integral. As we have shown, the cleanest way to see this is in the radial gauge. The action is minimized by complex solutions, and is real and finite. Furthermore, the solutions have the usual characteristics of monopoles. They have $U(1)$ field strength given by $F_{\mu\nu} = -\epsilon_{\mu\nu}\hat{x}^a/r^2$ and mass $\sim 1/g^2$. As a consequence, the long range order in the Higgs vacuum is destroyed. However, we must recognize that we are far from understanding this theory at quantum level, or beyond semi-classical approximation. The understanding of the quantum theory is obscured by the gauge non-invariance of the CS term. There are other murky areas in this model some of which are related to the puzzles about the gauge invariance and quantization.

We started with a theory with compact U(1) symmetry, where by definition the gauge transformation parameter (which we called f) is originally a real valued function. However, in discussing the Gribov copy problem in the radiation gauge, we have looked for complex field configurations which are related to each other by complex f.

Curiously enough, the gauge invariant part of the action (*i.e.,* everything other than the CS term) is still invariant under the transformation with complex f. There are many saddle points in the complex field configuration space which are related by these complex f's. Nevertheless, the physical interpretation of f being complex is not obvious at all.

If we had restricted the gauge parameter f in this theory to be real and had not allowed complex f, then the absence of real solutions to the Gribov equation in the radiation gauge would have led to no Gribov copies. We have adopted the view that complex solutions to Gribov's equation correspond to generalized Gribov copies of complex saddle points. We understand that this is a question not completely settled, and warrants further investigation. Meanwhile we conclude that within our semi-classical approximation, it appears that summation over the Gribov copies (or integrating over the collective coordinates) of the complex monopole solutions does not lead to the cancellation of the monopole contribution. What if quantum corrections to the Jacobian of the Gribov copies somehow cancel the effect after all? This is one of the questions which we could not answer in this model unless we learn how to go beyond the semi-classical approximation. We remark that one could raise the same objection for the real monopole case where it has been argued in the literature that the integral cancels. Recall that the issues of the non-invariance of the CS term, and the problems associated with the quantization are irrespective of whether the monopole is real or complex.

Let us remark that complex gauge field configurations have been studied before in the literature. In particular Wu and Yang [10] have given a prescription of how complex gauge fields in $SU(2)$ theory can be converted to real gauge fields for the group $SL(2,C)$. Witten [11] shows that Chern-Simons theory with the group $SL(2,C)$ is equivalent to the 2+1 dimensional Einstein-Hilbert gravity. Inclusion of matter in this theory has not yet been resolved. Nevertheless, it would be interesting to see the role of the complex monopoles in connection with such theories. Making contact between 3 dimensional gravity and the present work is the subject of a separate study

[12].

Another interesting future direction is the connection with Josephson junction. The three-dimensional compact QED is related to the Josephson junction system by an electro-magnetic dual transformation [13] . The $U(1)$ field strengths (E_1, E_2, B) in the Georgi-Glashow model correspond to (B_1, B_2, E_3) in the Josephson junction. Electric charges in the Georgi-Glashow model are magnetic charges inserted in the barrier in the Josephson junction. At the moment we haven't understood what kind of an additional interaction in the Josephson junction system corresponds to the Chern-Simons term in the Georgi-Glashow model. It could be a $\theta F_{\mu\nu}\tilde{F}^{\mu\nu}$ term in the superconductors on both sides. Normally a $\theta F_{\mu\nu}\tilde{F}^{\mu\nu}$ term is irrelevant in QED. However, if the values of θ on the left and right sides are different, this term may result in a physical consequence, which may mimic the effect of the Chern-Simons term in the Georgi-Glashow model.

If monopoles are irrelevant in the presence of the Chern-Simons term, it would imply that suppercurrents cease to flow across the barrier in the corresponding Josephson junction. Although we have not found the precise analogue in the Josephson system yet, and therefore we cannot say anything definite by analogy, we feel that it is very puzzling if suppercurrents suddenly stop to flow. Monopoles should remain important even in the presence of the Chern-Simons term. It is clear that further investigation is necessary for a better understanding of this subject.

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