Localised p-branes in Supergravities and M-Theory

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Abstract

Recent results concerning a special class of p-brane solutions in eleven-dimensional and ten-dimensional supergravities are reviewed. Delocalised as well as localised intersections of p-branes are discussed. The new mechanism of "blackening" localised branes is presented.

p-brane solutions of eleven- and ten-dimensional supergravities are relevant to different aspects of superstring theory, general relativity and supersymmetric gauge theories. p-branes appeared to be low-energy (supergravity) manifestation of non-perturbative string theory objects called D-branes [1]. This determined their special role in the recent development of string theory. p-branes being supergravity solutions could be investigated by classical methods so they provide a way of obtaining a non-perturbative information about quantum string theory via classical calculations.

p-branes are also important for existence of the net of dualities which connect different superstring theories and support the conjecture that five known superstrings are different points of the unique theory called M-theory [2]. Dualities could be reliably checked only on the classical level and different connections between p-branes provide tools for it.

It is difficult to overestimate the role of p-branes in the black hole physics. One of the most important recent achievements here is a microscopic interpretation of the black hole entropy [3]. Black holes are considered as configurations of wrapped branes and counting of brane microstates allows to derive the Bekenstein-Hawking formula.

Recently a new application of p-branes was found. They appeared to be important for interpretation of non-perturbative results in supersymmetric gauge theories via M-theory [4]. In this way different aspects of the non-perturbative dynamics have got a clear geometrical explanation in terms of brane configurations.

Let us start with considering p-branes in eleven-dimensional supergravity. The bosonic sector of this theory consists of a metric and a three-form potential, the corresponding part of the action is [5]

$$S = \frac{1}{2\kappa^2}\int d^{11}x\sqrt{-G}(R - \frac{1}{2\cdot 4!}F^2_{MNPQ}) - \frac{1}{12\kappa^2}\int A\wedge F\wedge F. \qquad (1)$$

p-branes are purely bosonic solutions, so the fermionic sector of the theory is not important here.

Eleven-dimensional supergravity admits two basic solutions, which correspond to M2- and M5-branes of M-theory. The M2-brane solution has the following form [6]

$$ds^2 = H^{1/3}(H^{-1}(-dt^2 + dy_1^2 + dy_2^2) + (dx_1^2 + \ldots + dx_8^2)), \qquad (2)$$

$$A_{ty_1y_2} = \epsilon_{ty_1y_2}H^{-1}, \qquad (3)$$

where H is harmonic function, which depends on x-coordinates

$$H = 1 + \frac{Q}{r^6}, \qquad r = |\vec{x}|. \qquad (4)$$

This solution describes a single membrane with the world-volume oriented along the t, y_1, y_2 directions and located at the point $r = 0$ in the tranversal space. M2-brane carries the electric four-form charge which is defined as the integral of the dual seven-form around a seven sphere that surrounds the brane. M2-brane is a BPS state, it preserves $1/2$ of the initial supersymmetry. It leads to the extremality of the brane: it's charge is equal to it's ADM mass.

Another important solution of eleven-dimensional supergravity is M5-brane [7]

$$ds^2 = H^{2/3}(H^{-1}(-dt^2 + dy_1^2 + \ldots dy_5^2) + (dx_1^2 + \ldots + dx_5^2)), \qquad (5)$$

$$F_{\alpha_1 \ldots \alpha_4} = \epsilon_{\alpha_1 \ldots \alpha_5} \partial_{\alpha_5} H, \qquad H = 1 + \frac{Q}{r^4}. \tag{6}$$

The M5-brane carries magnetic four-form charge which can be obtained by integrating F around a four-sphere that surrounds the M5-brane. As M2-brane M5-brane also preserves one half of supersymmetry. Note that harmonic functions in those solutions can depend not on all transverse coordinates but on the part of them. A brane solution whose harmonic function is independent of a number of transverse coordinates is sometimes said to be "delocalised" or "smeared" over those directions. It means that one can't determine the position of the brane in some directions, the uncertainty of it's location appears.

The presented solutions are building blocks for construction of more complicated configurations which correspond to intersecting branes. Intersections could be localised or delocalised. As an example of delocalised intersection let me consider a configuration (3|M5,M5) of two M5-branes intersecting in a 3-brane [8, 9]

$$ds^2 = H_1^{2/3}(x) H_2^{2/3}(x) (H_1^{-1}(x) H_2^{-1}(x)(-dt^2 + du_1^2 + du_2^2 + du_3^2)$$

$$+ H_1^{-1}(x)(dy_1^2 + dy_2^2) + H_2^{-1}(x)(dz_1^2 + dz_2^2) + dx_\alpha dx^\alpha), \tag{7}$$

$$F_{z_1 z_2 \alpha\beta} = \epsilon_{\alpha\beta\gamma} \partial_\gamma H_1(x), \qquad F_{y_1 y_2 \alpha\beta} = \epsilon_{\alpha\beta\gamma} \partial_\gamma H_2(x). \tag{8}$$

The first brane is located in t, u, y-directions, the second in t, u, z-directions. The coordinates y and z are called relative transverse ones. The important feature of this solution is that the harmonic functions $H_1(x)$ and $H_2(x)$ depend on overall transverse coordinates which are orthogonal to both branes. It means that each brane is delocalised in it's relative transverse directions.

Except this configuration there are other delocalised pair intersections involving M2- and M5-branes, such as [9, 10]

(0|M2,M2)- M2-branes intersecting in a point,

(1|M2,M5)- M2-brane intersects M5-brane along 1-brane.

Note that the intersection of branes is controlled by the general intersection rule [9, 10]. This intersection rule is consistent with dualities [11].

Now let us turn to localised intersections of M-branes. There are only two configurations of this type. One of them describes two M5-branes intersecting in a string (1|M5,M5), the other can be obtained by adding the third M2-brane (1|M5,M5,M2) [12]. The line element and the non-trivial components of the differential form for localised intersection of M5-branes are

$$ds^2 = H_1^{2/3}(y)H_2^{2/3}(x)(H_1^{-1}(y)H_2^{-1}(x)(-dt^2 + du^2) \\ + H_1^{-1}(y)d\vec{x}^2 + H_2^{-1}(x)d\vec{y}^2 + dw^2), \tag{9}$$

$$F_{y_i y_k y_l w} = \epsilon_{y_i y_k y_l y_n}\partial_{y_n} H_1(y), \qquad F_{x_i x_k x_l w} = \epsilon_{x_i x_k x_l x_m}\partial_{x_m} H_2(x). \tag{10}$$

This solution describes M5-brane which is oriented along t, u, y directions and intersects the other M5-brane located in t, u, x directions. There is a key difference between this solution and those already discussed: the harmonic functions are now independent of the single overall transverse direction and only depend on the relative transverse directions. That is, the M5-brane are now localised inside the directions tangent to the other M5-brane but are delocalised in the overall transverse direction that separates them. Note that there is also the difference in intersection rules between delocalised and localised intersections of M-branes [12, 13, 14].

All solutions discussed above are BPS-saturated. Besides them one can consider non-extremal solutions, which can be interpreted as deformations of the extremal ones. The procedure of obtaining non-extremal solutions from extremal is called "blackening". In the case of one brane or delocalised intersecting branes this procedure implies the following replacements in the D-dimensional overall transverse space of the line element as well as in the time part of it [15, 16]

$$dx_\alpha dx^\alpha \longrightarrow f^{-1}(r)dr^2 + r^2 d\Omega_{D-1}, \tag{11}$$

$$dt^2 \longrightarrow f(r)dt^2, \tag{12}$$

where

$$f(r) = 1 - \frac{\mu}{r^{D-2}}. \tag{13}$$

Also one scales the form field

$$A \longrightarrow \sqrt{1 + \frac{\mu}{Q}}. \tag{14}$$

As an example let us present the delocalised intersection of non-extremal M5-branes obtained in this way

$$ds^2 = H_1^{2/3}(x)H_2^{2/3}(x)(H_1^{-1}(x)H_2^{-1}(x)(-f(r)dt^2 + du_1^2 + du_2^2 + du_3^2)$$

$$+H_1^{-1}(x)(dy_1^2+dy_2^2)+H_2^{-1}(x)(dz_1^2+dz_2^2)+f^{-1}(r)dr^2+r^2d\Omega_2). \qquad (15)$$

Note that in the limit $\mu \longrightarrow 0$ one gets the extremal configuration already discussed. Another limit $Q \longrightarrow 0$ gives a multidimensional analog of the Schwarzschild solution. Unfortunately this procedure doesn't work properly for localised intersections of branes. It's application non-extremizes only one brane, not both. Thus one needs another approach which is a generalisation of this one. It was proposed [14] that the new "blackening" procedure should be based on deformations which in the vacuum limit give not Schwarzschild but more complicated solutions. This procedure applied to the localised intersection of M5-branes yield the following configuration

$$ds^2 = H_x^{2/3}H_y^{2/3}(H_x^{-1}H_y^{-1}(-f_xf_ydt^2+du^2)+H_y^{-1}f_x^{1/3}(f_x^{-1}dr_x^2+r_x^2d\Omega_3)$$

$$+H_x^{-1}f_x^{1/3}(f_y^{-1}dr_y^2+r_y^2d\Omega_3)+f_x^{-2/3}dw^2), \qquad (16)$$

$$H_x = 1+\frac{Q_x}{r_x^2},\ H_y = 1+\frac{Q_y}{r_y^2},\ f_x = 1-\frac{\mu_x}{r_x^2},\ f_y = 1-\frac{\mu_y}{r_y^2}. \qquad (17)$$

The limit $\mu_x,\ \mu_y \longrightarrow 0$ returns back to extremal configuration, the limit $Q_x,\ Q_y \longrightarrow 0$ leads to new vacuum solution of eleven-dimensional supergravity, which differs from Schwarzshild solution.

Now let us discuss briefly type IIA supergravity. Ten-dimensional type IIA supergravity action can be obtained by dimensional reduction on a circle of eleven-dimensional supergravity

$$S = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-G}(R-\frac{1}{2}(\partial\phi)^2-\frac{1}{2\cdot 3!}e^{-\phi}F_3^2)$$

$$\frac{1}{2\cdot 2!}e^{\frac{3\phi}{2}}F_2^2-\frac{1}{2\cdot 4!}e^{\frac{\phi}{2}}F_4^{'2})+(\text{Chern-Simons})+(\text{fermions}), \qquad (18)$$

$$F_4^{'} = dA_3+A_1\wedge F_3. \qquad (19)$$

The bosonic part of the action contains the fields come from NS-NS and R-R sectors of the superstring. Dilaton ϕ, metric G_{MN} and 2-form A_2 are NS-NS fields, 1-form A_1 and 3-form A_3 are R-R fields. Multidimensional objects which carry R-R charge are called D-branes. Also we have NS-branes, which carry NS-charge. In accordance with the field content we have the following branes in the theory

NS-branes: NS1 (fundamental string), NS5 (solitonic five-brane),

D-branes: D0, D2, D4, D6.
Just as in the case of M-branes one can construct intersecting configurations of these branes as well as consider non-extremal deformations of them. As example let me present the configuration of two non-extremal localised NS5-branes intersecting in a string [14]

$$ds^2 = H_x^{3/4}H_y^{3/4}(H_x^{-1}H_y^{-1}f_x^{-3/4}(-f_xf_ydt^2+du^2)+H_y^{-1}f_x^{1/4}(f_x^{-1}dr_x^2+r_x^2d\Omega_3)$$
$$+H_x^{-1}f_x^{1/4}(f_y^{-1}dr_y^2+r_y^2d\Omega_3)), \tag{20}$$
$$e^{-\phi} = H_x^{1/2}H_y^{1/2}f_x^{-1/2}, \tag{21}$$
$$H_x = 1+\frac{Q_x}{r_x^2},\ H_y = 1+\frac{Q_y}{r_y^2},\ f_x = 1-\frac{\mu_x}{r_x^2},\ f_y = 1-\frac{\mu_y}{r_y^2}. \tag{22}$$

In conclusion let us discuss some problems and open questions concerning p-brane solutions in various supergravities. First of all note that up to now the intersections of *completely* localised p-branes are not constructed. Moreover it seems that such configurations do not exist at all (the contradiction with the equations of motion appears) [17].

As it was mentioned there are localised intersections of three extremal branes (M5, M5 and M2 for example). However in the case of non-extremal branes only pair localised intersections are known. It should be interesting to understand whether it is possible to add the third non-extremal brane which is localised in the relative transverse directions.

One more unsolved problem is to find supergravity solutions representing branes ending on other branes (like in Hanany-Witten configuration [4]). The importance of such solutions is determined by the possibility to interpret some results in supersymmetric gauge theories via p-branes. It might be that studying these configurations will lead us to deeper understanding of non-perturbative phenomena in supersymmetric as well as in non-supersymmetric field theories.

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