Two-loop renormalization group profile of the standard model and a new generation

Yu. F. Pirogov and O. V. Zenin

Institute for High Energy Physics,
Protvino, Moscow Region, Russia

Moscow Institute of Physics and Technology,
Dolgoprudny, Moscow Region, Russia

Abstract

The two-loop renormalization group global profile of the Standard Model (SM) in its full parameter space is investigated. Restrictions on the Higgs boson mass as a function of a cutoff scale are obtained from the stability of electroweak vacuum and from the perturbative validity both in the Higgs and Yukawa sectors. The cutoff equal to the Planck scale requires the Higgs mass to be $M_H = (161.3 \pm 20.6)^{+4}_{-10}$ GeV and $M_H \geq 140.7^{+10}_{-10}$ GeV, where the M_H corridor is the theoretical one and the errors are due to the top mass uncertainty. Modification of the two-loop global profile of the SM extended by one new chiral generation is studied, and bounds on the masses of the generation are derived. Under the precision experiment restriction $M_H \leq 200$ GeV, the fourth chiral generation, taken alone, is excluded. Nevertheless a pair of the chiral generations constituting the vector-like one could exist.

The renormalization group (RG) study of a field theory (for the review see, e.g., Refs. [1, 2]) enables one to understand in grosso the structure of the theory as a function of a characteristic energy scale. Of special interest are the cases when self-consistency of the theory is under danger of violation. They may signal either the breakdown of the perturbative validity or/and the onset of a “new physics”.

There are two problems of the kind in the Standard Model (SM). First, it encounters when some of the running couplings tend to blow up at the finite scales violating thus the perturbativity. In the case of the ϕ^4 scalar theory, the problem is known for a long time as the triviality problem. Second, the problem occurs when a running coupling leaves the physical region at some finite scale. In the SM, this happens when the Higgs quartic effective coupling becomes negative, indicating the absence of a ground state in the quantum theory. It is the so-called electroweak vacuum stability problem. It is a real problem of the quantum field theory because this phenomenon takes place in the realm of the perturbative validity. In the framework of the SM, the light Higgs bosons resulting in the unstable electroweak vacuum should be forbidden. On the other hand, if this happens some new physics beyond the SM will be required to stabilize the vacuum.

The SM self-consistency study in the framework of the one-loop RG and restrictions thereof on the SM heavy particles, the Higgs boson and the top quark, was undertaken in Refs. [3, 4, 5]. A generalization to the two-loop level was given in Refs. [6]–[8]. The one-loop RG restrictions on a new heavy chiral family were studied in Ref. [9], and that on the vector-like family were investigated in Ref. [10]. In Ref. [11] the RG study of the SM extended by the fourth chiral family was generalized to the two-loop level. This required a generalization of the SM two-loop β functions to the massive neutrino case which was presented. As a by-product, the two-loop RG global profile of the SM in its parameter space at all conceivable scales was investigated. In particular the self-consistency restrictions on the Higgs mass were refined. More generally, the problem of what principally new the fourth heavy chiral family brings in the RG global profile of the SM was considered. A short summary of these results is given in the report.

1. Renormalization group profile of the SM

The SM two-loop β functions in the $\overline{\mathrm{MS}}$ renormalization scheme are well-known in the literature [12]–[14] (compact summaries can be found in Refs. [15, 16]). In what follows we put just the generic structure of the emerging one- and two-loop RG differential equations. So let g_i, $i = 1, 2, 3$, y_f, λ, v and μ be the SM gauge couplings, the Yukawa couplings for fermions f, the Higgs self-interaction coupling, the vacuum expectation value (VEV) and the renormalization scale, respectively. We neglected for simplicity by the mixing of the Yukawa couplings and, thus, by the CP violating phase.

The following essential features of the SM RG system are readily ascertained. At one-loop order, one has a kind of the three-level up-down hierarchy among the SM couplings, so that the differential equations for g_i, y_f and λ disentangle. One can first find $g_i(\mu)$, then insert them into $\beta_{y_f}^{(1)}$ and find $y_f(\mu)$, and finally put $g_i(\mu)$, $y_f(\mu)$ into the equation for λ and integrate it. The solution to the equation for v is determined completely by those for the first three equations, both in one and two loops.

In two loops, the RG equations partially entangle with each other due to a down-up feedback to the neighbour level: from λ to y_f and from y_f to g_i. But there is no direct influence of λ on g_i. It emerges only in three loops. Hence to completely entangle the RG system one needs the three-loop SM β functions, which are unknown at present. Thus we have to restrict ourselves to the two-loop order. On the other hand, the two- and higher-loop contributions to β functions, even the sign including, are known to depend in a multi-coupling theory on the renormalization scheme [2]. Hence the physical meaning of the running couplings becomes ambiguous, and it is impossible to improve the perturbative RG analysis of the SM in the scheme-independent way beyond one loop.

We integrated the SM RG system numerically for $\mu \geq M_Z$ by the first-order Runge-Cutta method with the initial conditions at the scale M_Z taken as $\alpha_1(M_Z) = 0.0102$, $\alpha_2(M_Z) = 0.0338$ and $\alpha_3(M_Z) = 0.123$ in accordance with $\alpha(M_Z) = 1/127.90$ and $\sin^2\theta_W(M_Z) = 0.2315$ [17]. Our normalizations of the gauge couplings are as follows: $g_1 = (5/3)^{1/2}g'$, $g_2 \equiv g$ and $g_3 \equiv g_S$, with g', g and g_S being the conventional SM couplings. We choose also the relations $m_f = y_f v$ and $m_H = \lambda^{1/2}v$ as the definition of normalization for the Higgs and Yukawa couplings, with $v = (\sqrt{2}\,G_F)^{-1/2} = 246.22$ GeV being the Higgs VEV. Because the evolution of $v(\mu)$ is gauge dependent we use in what follows only the gauge independent observable $v \equiv v(M_Z)$. Besides we used at $\mu = M_Z$ the one-loop matching condition for the physical and running fermion masses M_f and $m_f(\mu) \equiv y_f(\mu)\,v$, respectively. We have got at $M_H = 150$ GeV: $m_\tau(M_Z) = 1.764$ GeV, $m_b(M_Z) = (4.47 \pm 0.50)$ GeV and $m_t(M_Z) = (171.8^{+4.6}_{-4.7})$ GeV. The last two values correspond in turn to the physical bottom and top masses $M_b = (4.5 \pm 0.5)$ GeV [17] and $M_t = (175 \pm 5)$ GeV [18], respectively. Only errors in the top mass are left as the main source of the subsequent uncertainties.

As a field theory, SM is legitimate to be pulled to its inner ultimate limits. This may help to understand better its structure in the physically reasonable region $\mu < M_{Pl}$, $M_{Pl} \simeq 10^{19}$ GeV, which is to be considered more seriously. So all the numerical results were obtained at all allowed μ with the exact two-loop β functions. Let us discuss the results in turn for the gauge, Yukawa and Higgs sectors of the SM.

(i) **Gauge sector**

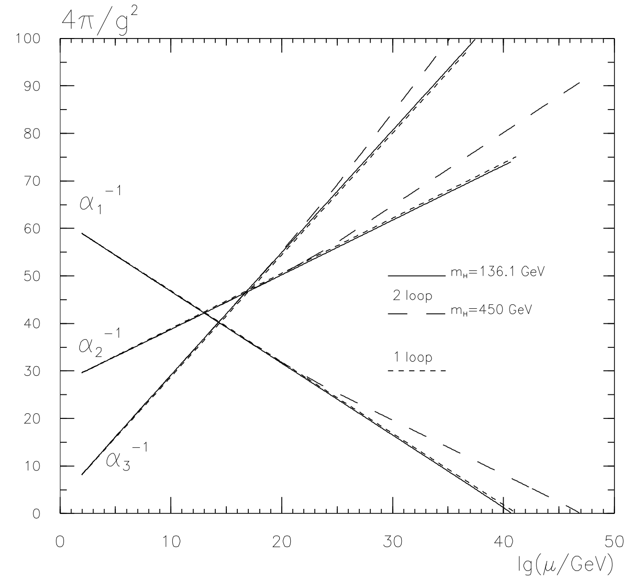

Figure 1: Running of the inverse gauge couplings squared α_i^{-1}, $i = 1, 2, 3$. Number of generations is $n_g = 3$. Represented Higgs masses are those corresponding to the typical heavy Higgs and to the lower critical Higgs curve shown in bold in Fig. 3.

Fig. 1 shows the running with μ of the inverse gauge couplings squared. It is seen that the coupling g_1 develops a pole singularity at Λ_{g_1}, $\log \Lambda_{g_1} \simeq 41$. Validity of the perturbation theory in g_1 restricts $\alpha_1 \leq 4\pi$ and hence $\log \mu \leq 40$, which is in the logarithmic scale twice as large as the Planck scale. We should assume this restriction on the physical grounds in what follows. Nevertheless, taken at its face value the two-loop RG has the meaning by itself. So, to understand better the mathematical structure of its solutions we extend them up to the singularity point Λ_{g_1}. The actual influence of $y_f(\lambda(\mu))$ on g_1 in two loops is somewhat sizable only for the heavy Higgs.

It diminishes the slope of $g_1(\mu)$ at μ beyond the Planck scale, where y_f are large, and shifts the singularity position $\Lambda^{(2)}_{g_1}$ upwards to $\log \Lambda^{(2)}_{g_1} = 47$ for the heavy Higgs ($m_H(M_Z) = 450$ GeV). The critical value $m_H(M_Z) = 136.1$ GeV corresponds to the maximal lower bound of the vacuum stability.

(ii) Yukawa sector