Puzzling Aspects of the Adiabatic Effective Action in Chern Simons Theories

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Abstract

The parity odd part of the adiabatic effective action in a 2+1 dimensional non-abelian Yang-Mills theory with a Chern-Simons term and scalar fields contains the Chern-Simons term and several other pretendors or "would be" Chern-Simons terms. We compute all of these terms, and we find that the renormalization of the coefficient of the Chern-Simons term is altered, whether or not the symmetry is spontaneously broken. The renormalization appears to be that which would correspond to the (putative) minimal residual unbroken gauge group. There remain two puzzles. Firstly, how can this renormalization be altered when the symmetry is not spontaneously broken? Secondly, are the Chern-Simons term and the set of pretenders linearly independent?

The Chern-Simons (CS) term is given by

$$L_{CS} = \frac{\mu}{2}\int d^3x \epsilon^{\mu\nu\lambda}(A^a_\mu \partial_\nu A^a_\lambda + \frac{1}{3} g f_{abc} A^a_\mu A^b_\nu A^c_\lambda) \tag{1}$$

where f_{abc} are the structure of the gauge group and g is the coupling constant. It is not gauge invariant but it can be thought of as a Wess-Zumino term which is manifestly gauge invariant[1]:

$$L_{CS} = \frac{\mu}{2}\int d^4x \frac{tr(F\tilde{F})}{32\pi^2}. \tag{2}$$

The integration is over a $3+1$ dimensional manifold with a boundary corresponding to the $2+1$ dimensional space-time. This expression is gauge invariant but it is not unique, it depends on the particular extension of the gauge fields that has been made into the additional dimension. To obtain a sensible theory in the case of such multi-valued functionals we have to quantize the coefficient of the term in order that its (discrete) ambiguity is only a harmless integer multiple of 2π. This yields the quantization condition

$$\frac{4\pi\mu}{g^2} = n \in Z. \tag{3}$$

Quantum corrections renormalize the value of n. It was found by Pisarski and Rao [2]that for an $SU(N)$ theory,

$$n \to n + N. \tag{4}$$

This correction comes from a one loop, and there can be no further contributions from higher loops. The case with symmetry breaking was studied for partial symmetry breaking, for the example $SU(3) \to SU(2)$ with a scalar field in the fundamental representation with $<\phi>= (0,0,v)$. Calculating the renormalization of the coefficient of the residual $SU(2)$ sub-group yields $n \to n+2$ and not the usual renormalization appropriate for the $SU(3)$ original symmetry $n \to n+3$[4, 5]. Hence there seems to be a correction to this renormalization in equation (4) due to the scalar fields. For the broken generators, the correction is a complicated function of the vacuum expectation value[6], which should be explainable in terms of the effective action for the gauge fields, by terms which mix with the Chern-Simons terms.

Consider a simpler model, with an $SU(2)$ spontaneously broken by a fundamental scalar doublet ϕ. Terms in the effective action[6], which we will call "would be" Chern-Simons terms,

$$\begin{aligned} & \alpha(\phi^\dagger\phi)\epsilon^{\mu\nu\lambda}(\phi^\dagger T^a \overleftrightarrow{D}_\mu \phi F^a_{\nu\lambda}) \\ + \quad & \beta(\phi^\dagger\phi)\epsilon^{\mu\nu\lambda}\epsilon_{abc}(\phi^\dagger T^a \overleftrightarrow{D}_\mu \phi\phi^\dagger T^a \overleftrightarrow{D}_\mu \phi\phi^\dagger T^a \overleftrightarrow{D}_\mu \phi) \end{aligned} \tag{5}$$

become

$$\alpha(\phi^\dagger\phi)\epsilon^{\mu\nu\lambda}v^2 A^a_\mu F^a_{\nu\lambda} + \beta(\phi^\dagger\phi)\epsilon^{\mu\nu\lambda}\epsilon_{abc}v^6 A^a_\mu A^b_\nu A^c_\lambda. \tag{6}$$

These mix with the Chern-Simons term and hide its true renormalization. The problem is to disentangle the contributions of such terms from the true renormalization. The background field effective action is the ideal way to proceed.

The explicit Lagrangian that we consider is

$$\begin{aligned}\mathcal{L}(\phi, A^a_\mu) &= -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\mu}{2}\epsilon^{\mu\nu\lambda}(A^a_\mu\partial_\nu A^a_\lambda + \frac{1}{3}g\epsilon_{abc}A^a_\mu A^b_\nu A^c_\lambda) \\ &+ (D_\mu\phi)^\dagger D^\mu\phi + m^2\phi^\dagger\phi - \frac{\lambda}{4}(\phi^\dagger\phi)^2 + \text{gauge fixing}\end{aligned} \tag{7}$$

where a, b, c take values from 1 to 3, and ϕ is a doublet. The effective action computed in an adiabatic expansion, considering only the parity odd part, has three possible terms, equation (1) and the two "would be" CS terms in equation (5), that are relevant for the renormalization of the CS term. If we define the coefficients $-C_1, C_2$ and C_3 respectively for the three terms, then for $\varphi = (0, v/\sqrt{2})$ we get

$$\left(-C_1 + \frac{gv^2}{2}C_2\left(\frac{v^2}{2}\right)\right)\epsilon^{\mu\nu\lambda}A^a_\mu\partial_\nu A^a_\lambda \tag{8}$$

while the three gauge field term is

$$\left(-\frac{C_1}{3} + \frac{gv^2}{4}C_2\left(\frac{v^2}{2}\right) + \frac{g^2v^6}{64}C_3\left(\frac{v^2}{2}\right)\right)\epsilon^{\mu\nu\lambda}\epsilon_{abc}gA^a_\mu A^b_\nu A^c_\lambda. \tag{9}$$

Hence and independent calculation of C_2 and C_3 needs to be done in order to isolate the true renormalization of the CS term, which is contained in C_1. For example, computing the coefficient of the term

$$\epsilon^{\mu\nu\lambda}(\varphi^\dagger iT^a\vec{\partial}_\mu\phi)(\partial_\nu A^a_\lambda - \partial_\lambda A^a_\nu) \tag{10}$$

will give C_2. The calculation has been done for this case, of $SU(2)$ completely broken by a scalar doublet. We find that all corrections to the CS term are absorbed into the "would be" CS terms. This means that in fact there is no renormalization of the coefficient of the CS term here.

$$n \to n\ (\neq n+2) \tag{11}$$

It seems that each gauge boson that gets a Higg's mass does not contribute to the renormalization. Correspondingly in the case considered in the beginning $SU(3) \rightarrow SU(2)$ we expect that the renormalization is

$$n \rightarrow n + 2 \ (\neq n + 3) \tag{12}$$

The first puzzling aspect of the result is that it seems to be independent of whether or not the symmetry is broken. Since we shift the scalar field in the background field method, we give all putatively massive gauge bosons a Higg's mass, even though we may remove this at the end if the symmetry is not broken, ie. if the minimum of the effective potential is at vanishing shift. Putting the shift to zero will not matter in the calculation of the renormalization of the coefficient of the CS term. If, as in our case, we find no renormalization, $n \rightarrow n$, this result will not change if we evaluate it at zero shift of the scalar field. It does not depend on the scalar field, it is just a constant (integer valued) function. A priori we expect that without symmetry breaking, the scalar fields cannot modify the renormalization of the CS term in concordance with the Coleman-Hill theorem (which actually only applies for abelian theories). This lack of commutativity, between computing the renormalization of the coefficient of the CS term with a shifted scalar field followed by letting the shift vanish or computing it directly without shifting the scalar field, warrants further investigation.

The second puzzling aspect comes from the almost linear dependence of the Chern-Simons term and the appropriate parts of the "would be" terms. The "would be" terms are constructed out of the current

$$J^a_\mu = \phi^\dagger i \overset{\leftrightarrow}{D}_\mu T^a \phi. \tag{13}$$

If $\phi \neq 0$, we can write

$$\phi = \mathcal{U}\bar{\phi} \tag{14}$$

where

$$\bar{\phi} = \begin{pmatrix} 0 \\ v(x) \end{pmatrix} \tag{15}$$

with $v(x)$ real. Furthermore if we write

$$A_\mu = \left(\bar{A}_\mu\right)^{\mathcal{U}} \tag{16}$$

Then

$$J^a_\mu(\bar{A},\bar{\phi}) = g\frac{v^2}{2}\bar{A}^a_\mu, \tag{17}$$

so that the "would be" terms (which are of course absolutely gauge invariant) become, for example

$$\epsilon^{\mu\nu\lambda}J^a_\mu(\bar{A},\bar{\phi})F^a_{\nu\lambda}(A) = g\frac{v^2}{2}\bar{A}^a_\mu F^a_{\nu\lambda}(\bar{A}). \tag{18}$$

Thus the part in this "would be" term that is proportional to $1/v^2$ is not independent of the part of the Chern-Simons term

$$\epsilon^{\mu\nu\lambda}\bar{A}^a_\mu F^a_{\nu\lambda}(\bar{A}). \tag{19}$$

Correspondingly the term

$$\epsilon^{\mu\nu\lambda}\epsilon_{abc}J^a_\mu(\bar{A},\bar{\phi})J^b_\nu(\bar{A},\bar{\phi})J^c_\lambda(\bar{A},\bar{\phi}) = \left(g\frac{v^2}{2}\right)^3\epsilon^{\mu\nu\lambda}\epsilon_{abc}\bar{A}^a_\mu\bar{A}^b_\nu\bar{A}^c_\lambda \tag{20}$$

which does not seem to be independent of the other part of the Chern-Simons term. Hence it seems that the relevant parts of the "would be " terms are not linearly independent of the Chern-Simons term.

This would be catastrophic, and it is probably not true. First we needed that $\phi \neq 0$ which is not the case in general. Moreover since under the field redefinition in equation(16) the Chern-Simons term transforms as

$$C.S.(A) \to C.S.(\bar{A}) + \frac{4\pi\mu}{g^2}\frac{2\pi}{24\pi^2}\int tr(\mathcal{U}^\dagger d\mathcal{U}), \tag{21}$$

that is

$$\delta C.S. = 2\pi n \times \text{Winding}\# \quad (\mathcal{U}), \tag{22}$$

hence the Chern-Simons term and the "would be" terms are not linearly independent, but only by a topological winding number.

Thus an independent confirmation of the previous calculation is possible. We should simply calculate the variation of the effective action under a topologically non-trivial gauge transformation. The change under such a gauge transformation is only sensitive to the Chern-Simons terms, the others being explicitly gauge invariant. Such a claculation has been done for the pure gauge theory, but remains to be done with the presence of the scalar fields.

Acknowledgements

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