The Electroweak Phase Transition in the MSSM

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Abstract

The construction of an effective 3D theory at high temperatures for the MSSM as a model of electroweak baryogenesis is discussed. The analysis for a single light scalar field shows, that given the experimental constraints, there is no value of the Higgs mass for which a sufficiently strong first-order phase transition is obtained. A precise determination of the 3D parameters of the effective theory for the case of a light right-handed stop allows us to obtain an upper bound on the masses of the lightest Higgs and right handed stop using the two-loop effective potential. A two-stage phase transition persists for a small range of values of $m_{\tilde{t}_R}$.

1 Introduction

The study of the electroweak phase transition is motivated in part by the idea of generating the baryon asymmetry of the Universe (BAU) at the electroweak scale. As Sakharov [1] pointed out, the necessary ingredients for baryogenesis are: CP violation, non-equilibrium, and baryon-number violation. The Standard Model has all of the necessary features for the production of the BAU. In the Standard Model baryon number is violated non-perturbatively through sphaleron processes. At zero temperature the tunnelling rate of the anomalous processes is strongly suppressed. However, at

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high temperatures the situation changes drastically; comparing the rate of sphaleron processes to the expansion rate of the Universe, Kuzmin et al. [2] showed that these processes are in equilibrium for temperatures above the electroweak phase transition. Thus, unless a net B-L asymmetry is produced above the electroweak scale, sphaleron processes will wash out the B asymmetry. We can consider alternatively that the production of the baryon asymmetry takes place at the electroweak scale precisely because of the anomalous electroweak processes. The generation of the baryon asymmetry is a non-trivial problem, which involves non-equilibrium dynamics. Here we focus instead on avoiding the elimination of the produced baryon asymmetry immediately after the electroweak phase transition. At the electroweak scale, typical weak reaction rates are faster that the expansion rate of the Universe. This implies that another mechanism for a departure from thermal equilibrium is needed. The mechanism that has been commonly employed relies on a sufficiently strong first-order phase transition at the electroweak scale. In addition, as weak-scale physics is involved, experimental constraints can indicate whether, within a specific model, the requirement of preserving the asymmetry is satisfied. The rate of sphaleron transitions in the broken phase is proportional to

$$\Gamma \propto T e^{E_{sph}/T}, \tag{1}$$

where $E_{sph} = \frac{m_W}{\alpha_W}$, is the sphaleron energy and $m_W = \frac{1}{2} g\phi$ is the gauge-boson mass. For the BAU to survive, baryon-number violation must be turned off after the phase transition. This implies that the exponent in eq. (1) must be large just after the phase transition[1]

$$\frac{E_{sph}}{T} > 45 \qquad \rightarrow \frac{v(T_c)}{T_c} \gtrsim 1, \tag{2}$$

where $v(T_c)$ is the expectation value of the Higgs field at the critical temperature.

The effective potential is the quantity used in the analysis of the phase transition to determine the order of the transition, critical temperature, latent heat, etc. For a precise determination of the effective potential the contributions from all of the particles that receive mass terms from the scalar

[1] In our analysis we will be working in the case of a single light-scalar doublet at the phase transition and so it is appropriate to use the constraint from eq. (2).

Higgs field should be included. A generic expression for the effective potential, using the high-temperature expansion, assuming a single light-scalar field at the phase transition is given by

$$V(\phi, T) = D(T^2 - T_o^2)\phi^2 - ET\phi^3 + \frac{\lambda_T}{4}\phi^4. \tag{3}$$

The quantities D, E, T_o and λ_T must be determined for each specific model. The presence of the cubic term ensures a first-order phase transition. A minimum occurs, for a non-zero value of ϕ, at the phase transition when all three terms in the effective potential are roughly of the same order of magnitude, that is $\phi \sim \frac{E}{\lambda}T$. Thus, roughly a first constraint of the value of the Higgs mass arises

$$\frac{E_{sph}}{T} \sim \frac{m_W}{g^2 T} \sim \frac{\phi}{gT} \sim \frac{g^2}{\lambda} \sim \frac{m_W^2}{m_H^2}, \tag{4}$$

where we have included only gauge bosons contributions to the effective potential so that $E \sim g^3$. This implies that the electroweak phase transition will be sufficiently of first order only for small enough values of m_H given the constraint of eq. (2). In order to perform a more detailed calculation, we must include the effect of all of the particles, and also higher-order corrections can be important. However, the analysis is constrained by the appearance of infrared divergences due to the massless gauge bosons in the symmetric phase. Resummation has been used to incorporate leading higher-order effects to one and two-loops in models for electroweak baryogenesis [3, 4]. The net effect of resummation in the Standard Model is to weaken the strength of the phase transition. In addition, even with resummation the results of 1-loop calculations are unreliable for values of the Higgs field $\phi \lesssim gT$. In order to tackle this problem non-perturbative methods have been put forward. An interesting approach was introduced by Kajantie et al. [5, 6, 7], in which the main idea is to separate the perturbative and non-perturbative aspects of the theory. The fact that there exists, at finite temperature, a hierarchy of mass scales can be used to construct effective 3D theories for a given model. The perturbative component of the calculation, called dimensional reduction, consists of constructing an effective 3D theory by matching 3D and 4D Green's functions of the light degrees of freedom up to a certain order in the loop expansion. At finite temperature the non-zero Matsubara modes have masses $\sim \pi T$, and a further reduction can be performed generically, noting that some of the static modes in the theory, as a result of integrating out the non-static modes, have acquired thermal masses proportional to a gauge coupling multiplied by the temperature $\sim g_{W(s)}T$. The final effective theory contains a single light-scalar and transverse-gauge bosons. We emphasize that the whole procedure of dimensional reduction is perturbative and free from infrared divergences. The Lagrangian of the resulting purely bosonic theory is

$$L = \frac{1}{4}F_{ij}F_{ij} + \frac{1}{2}m_3^2\phi^2 + \lambda_3\phi^4, \tag{5}$$

which facilitates lattice simulations. The lattice analysis, which incorporates non-perturbative effects, translates the constraint for a sufficiently strong first-order phase transition into an upper bound on the ratio of the 3D Higgs coupling and the square 3D gauge coupling

$$x_c = \frac{\lambda_3}{g_3^2} < 0.03 \cdots 0.04, \tag{6}$$

at the critical temperature. The result of the analysis for the Standard Model, given the experimental constraints, is that there is no value of the Higgs mass that gives a sufficiently strong phase transition [6].

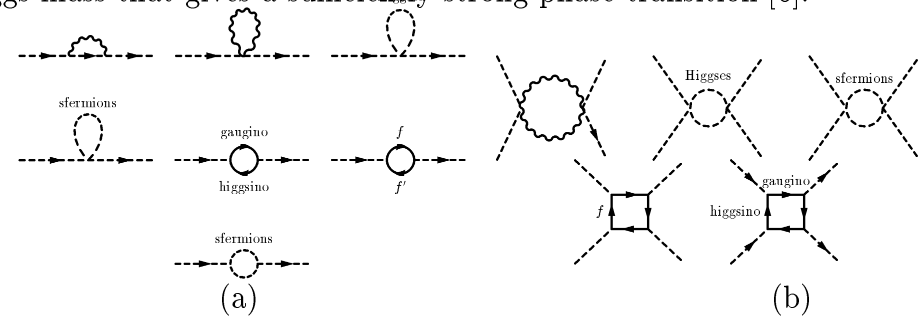

Figure 1: (a) Feynman diagrams contributing to the mass of the scalar Higgses and to wave-function renormalization. (b) Diagrams contributing to the quartic Higgs couplings.

2 Dimensional Reduction in the MSSM

In the Minimal Supersymmetric Standard Model (MSSM), it is also generic to have a single light scalar at the electroweak phase transition. Thus the effective theory is also described by eq. (5). Initially, perturbative 4D calculations of the one-loop effective potential were used to analyse the strength of the phase transition in this model [8, 9]. The region of parameter space that was consistent with a sufficiently strong phase transition was found to favour low values of the ratio of the vacuum expectation values of the Higgs doublets $\tan\beta = \frac{v_2}{v_1}$, and large values of the pseudoscalar mass m_A.

The procedure of dimensional reduction can be applied to the MSSM and other extensions of the Standard Model [10, 11, 12]. For the MSSM the first stage of reduction will integrate out all of the fermions (quarks, higgsinos, gauginos) and all non-zero Matusbara modes of the bosonic fields (gauge bosons, sfermions, higgses). Figures 1a and 1b show some of the diagrams that must be calculated for a one-loop dimensional reduction for a two-point function and four-point function, respectively.

The general structure of the 3D couplings in terms of the 4D parameters is of the form

$$\lambda_3 = T[\lambda - \beta_\lambda^b L_b - \beta_\lambda^f L_f + \text{const.}]. \tag{7}$$

where $\beta_\lambda^{b(f)}$ are the corresponding bosonic(fermionic) β-functions. Similarly, for the mass terms we have the one-loop relation

$$\overline{m}_3^2 = m^2 + \eta_m(\mu)T^2 - \beta_m^b L_b - \beta_m^f L_f, \tag{8}$$

where $\eta_m \sim g^2_{W(s)}(\mu)$. The 3D scalar and gauge couplings are renormalization-group invariant. This implies that a one-loop matching of the 3D coupling constants to the physical parameters and the temperature suffices to determine the strength of the phase transition, using the constraint given by eq. (6). In order to complete the matching of the 3D parameters to 4D physical parameters, the zero-temperature theory must be properly renormalized. We insist that the value of the critical temperature does depend on a precise determination of the 3D mass parameter, which does get renormalized in the effective theory, see below; a two-loop calculation (in 4D) must be performed even in the case of single light scalar at the phase transition. However, as x_c has only a weak dependence on the temperature for values close to the critical temperature of the phase transition for the allowed range of values of the Higgs mass [13], a two-loop calculation is not necessary for the adequate suppression of the sphaleron rate. The analysis of the dimensionally reduced theory at one loop confirmed that, for the large-m_A region of parameter space, a sufficiently strong first order phase transition occurs, with an upper bound for the Higgs mass of $m_h \lesssim 80$ GeV. However, given the current experimental limits on the Higgs mass, this region is excluded. Two possible ways of strengthening the phase transition, which can enlarge the allowed region of parameter space, have been observed:

a) a fine-tuned scenario with a light right-handed stop [14][2]. At one-loop the effect consists of enhancing the coefficient of the cubic term in eq. (3), thus making the phase transition stronger:

$$E \simeq E_{SM} + \frac{h_t^3 \sin^3\beta}{2\pi}\left(1 - \frac{X_t^2}{m_Q^2}\right)^{3/2}, \tag{9}$$

where h_t, X_t, m_Q are the top quarks Yukawa coupling, the stop mixing parameter, and the soft SUSY-breaking mass of the third-generation left squark doublet, respectively. This scenario requires a soft SUSY-breaking mass for the right-handed stop $m_U^2 < 0$, which tends to cancels the positive temperature corrections to the thermal mass. However, this may lead to physically unacceptable solutions at zero-temperature. In order to avoid colour-breaking minima at zero temperature the following constraint must be imposed [14]

$$-m_U^2 \geq \left(\frac{m_h^2 v^2 g_s^2}{12}\right)^{1/2}, \tag{10}$$

which ensures that the physical minimum is deeper that the colour-breaking minimum. This scenario could not be studied with the previous dimensional reduction, as the perturbative expansion breaks down for values of the right-handed stop mass $m_{t_R} \lesssim 190$ GeV [15].

b) Higher-order QCD corrections from stops were also shown to be relevant by affecting the value of the scalar field at the phase transition [16, 17].

We now direct our attention to the analysis of the phase transition with a light right-handed stop. The dimensional-reduction procedure has to be redone as the perturbative expansion starts to break down for smaller values of $m_{\tilde{t}_R}$. In addition new numerical simulations must also be performed; they will have to take into account gluonic fields as they are not decoupled from the squarks in the 3D theory. However, it is not so simple to obtain a constraint similar to that of eq. (6) for the case of two light-scalar fields [18]. A

[2]Previous studies had shown that light stops strengthen the transition, but it is not possible to make both of the stop fields light from the constraints at zero temperature on the ρ parameter [9].

surprising result of the first perturbative analysis at two loops was the appearance of a two-stage phase transition, where the Universe would reach the physical vacuum having first gone through an intermediate colour-breaking phase [19]. The effective potential in the 3D theory, which reproduces the 4D results, can be studied in order to compare with non-perturbative results as well[3].

The final expression for the tree-level 3D potential is given by

$$\begin{aligned} V_{3D} &= \overline{m}^2_{H_3} H^\dagger H + \overline{\lambda}_{H_3}(H^\dagger H)^2 + \overline{m}^2_{U_3} U^\dagger U \\ &+ \overline{\lambda}_{U_3}(U^\dagger U)^2 + \overline{\gamma}_3 (H^\dagger H)(U^\dagger U); \end{aligned} \tag{11}$$

the explicit expressions for the masses and couplings can be found in ref. [15]. A precise determination of the critical temperatures is necessary to ensure the existence of a two-stage phase transition. The most relevant quantities that define the critical temperatures are the 3D mass parameters for the Higgs doublet and the right-handed stop

$$\overline{m}^2_{H_3}(\mu) = \overline{m}^2_{H_3} + \frac{1}{(16\pi^2)} f_{2m_H} \log\frac{\Lambda_{H_3}}{\mu}, \tag{12}$$

$$\overline{m}^2_{U_3}(\mu) = \overline{m}^2_{U_3} + \frac{1}{(16\pi^2)} f_{2m_U} \log\frac{\Lambda_{U_3}}{\mu}, \tag{13}$$

where $\overline{m}^2_{H_3(U_3)}$ have the form of eq. (8).

The expressions for the two-loop beta functions f_{2m_H}, f_{2m_U} for the mass parameters have been given in ref. [19]. As mentioned there, in order to fix the values of the parameters Λ_{H_3} and Λ_{U_3}, we must employ the two-loop effective potential of the 4D theory [4]. In addition two-loop effects can be numerically important because the scale of the couplings in the thermal contributions to eq. (8) can only be fixed at the two-loop level. The strategy we employ follows that of ref. [7]. The idea is to use the 4D two-loop effective potential in order to fix the scales in the 3D theory, and to use the 3D effective potential expressions for the Higgs and stop fields given in ref. [19]

[3]The first lattice analysis seems to indicate that the results of the perturbative calculations are conservative in the bounds that are placed on the physical masses.

[4]In refs. [19, 18] an estimate of $\Lambda_{H_3} \sim \Lambda_{U_3} \sim 7T$ was used.

to analyse the phase transition We calculate the unresummed two-loop effective potential in order to include all 4D corrections to the mass parameters; resummation is automatically included in the calculation of the two-loop effective potential in the 3D theory. We must also include the contributions to the two-loop effective potential of the static modes, which have been integrated out at the second stage (includes the effects of resummation of the heavy fields)[5].

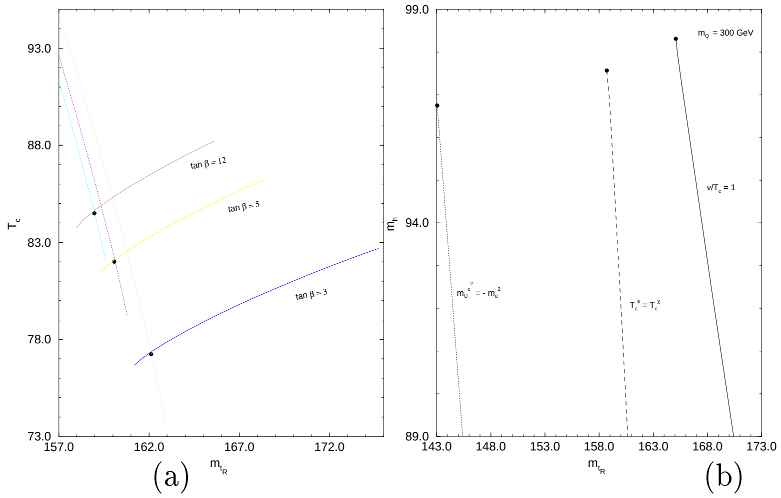

Figure 2: (a) Critical temperatures in the ϕ (solid) and χ (dotted) directions as functions of $m_{\tilde{t}_R}$ for $\tan\beta = 3, 5, 12$ and $m_Q = 300$ GeV. (b)Allowed region in m_h–$m_{\tilde{t}_R}$ plane for $m_Q = 300$ GeV. To the left of the solid line there is a sufficiently strong first-order phase transition, to the right of the dotted line the physical vacuum is absolutely stable. The dashed line separates the region for which a two-stage phase transition can occur.

Having done this we can now analyse the phase transition. In fig. 2a we show the critical temperatures for the transitions in the ϕ- and χ-directions as a function of the right-handed stop pole mass $m_{\tilde{t}_R}$, for $\tan\beta = 3, 5, 12$. We find that, for $m_Q \sim 300$ GeV, there still is a region in which a two-stage

[5]We derive the effective potential using the background fields ϕ and $\chi = \tilde{t}_{R\alpha} u^\alpha$, where we have chosen the unit vector in colour space $u^\alpha = (1, 0, 0)$. The details of the calculation are found in ref. [15].

phase transition can occur. This region is to the left of the crossing points of the curves. With respect to the work of ref. [19] the structure of the phase diagram is preserved, although it is slightly shifted towards higher values of the right-handed stop mass. The total effect does not substantially increase or decrease the range of values of the right-handed stop mass for which a two-stage phase transition can occur. However, the exact location of this small range in the value of $m_{\tilde{t}_R}$ depends on the value of the third-generation left-handed squark doublet mass. We note that the strength of the phase transition has a weak dependence on the values of the scales that have been fixed in our calculation, and only slight differences are observed with respect to previous analyses.

The allowed region in parameter space is shown in fig. 2b, given the current experimental limits on the Higgs mass [20]. The region on the left of the solid line indicates when a sufficiently strong first-order phase transition occurs. The dotted line gives the condition for absolute stability of the physical vacuum. As explained above, to the left of this line the colour-breaking minimum is lower than the physical one at zero-temperature. The dashed line is obtained when the critical temperatures of the transitions in the ϕ- and χ-directions are the same. A two-stage phase transition occurs to the left of the dashed line.

3 Conclusions

We have performed a full two-loop dimensional reduction of 4D MSSM parameters to the 3D couplings and masses of the effective theory. In this way, we have fixed the scales appearing in the 3D mass terms that are due to the thermal polarizations and the super-renormalizability of the 3D theory. The values of the parameters Λ_{H_3} and Λ_{U_3} can vary significantly for different values of the input parameters and the particle content of the theory, thus modifying the critical temperatures of the transitions. This effective theory can now be used for lattice simulations. We conclude that the allowed range of masses is $m_h \lesssim 110$ GeV and $m_{\tilde{t}_R} \lesssim m_t$, in complete agreement with previous results. We find that the phase structure diagram still allows a possible two-stage phase transition for a small range of values of $m_{\tilde{t}_R}$. This range of values is shifted with respect to previous results. However, whether or not the transition actually occurs must be explicitly checked. Initial lattice analysis suggests that the second stage of the transition is extremely strong and thus this transition might not have taken place on cosmological time scales. Consequently, this region of parameter space for electroweak baryogenesis would be excluded.

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