Influence of Multiple Scattering on Particle Decays in a Medium

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Abstract

Impact of scattering in matter on particle decays at high energy is studied. The consistent method of the calculation of the decay probability of the particles scattered multiply elastically in an equilibrium medium is developed. The decay probability of such particles is obtained. The Dalitz decay of a pion in the matter is studied.

PACS : 25.15.-g; 12.39.Fe; 12.38.Mh; 12.40.Vv.

1 Introduction.

The need to investigate of influence of scattering in a medium on particle decays arises in the study of states of the nuclear matter produced in the result of collisions of high energy heavy ions [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Photons and leptons produced in the medium are very important source of the information about the state of the nuclear matter because ones are electro-weak interacting particles only and therefore they carry information about the primordial state of the nuclear matter studied. In this way one of the most important channel of production of such particles is decays in the matter.

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Impact of a medium on damping rates is studied in the papers [2, 5, 9, 11, 12, 13, 14, 15, 16]. On the basis of the resummation method of infinite sets of diagrams which has been developed by E.Braaten and R.D.Pisarski [17, 18], in-matter polarization effects (renormalization of quark and pion masses) in QCD [5, 18] is studied. In the papers [11, 12, 13, 14, 15, 16] the method of the hard thermal loops [17, 18] has been used and developed for the calculation of self energy due to electron photon interaction in QED. In the papers [2, 19] impact of the occupancy of final states in the photon bath on decoupling constants has been investigated.

But the influence of multiple scattering of particles on their decay are not studied anywhere in the papers above mentioned. In this way if energy fluctuations of the decaying particle because of multiple scattering of this particle in matter is of the order of the medium temperature impact of multiple scattering in the matter on the particle decay is strong. This means that in-medium effects of multiple scattering should be taken into account in calculations of decay damping widths of the particles in the matter in this case.

We study decays of the particles undergoing multiple elastic collisions in an equilibrium medium in this paper. The model-independent method of calculation of the decay of such particles is developed. Obtained decay probability of particles depends strongly just as on the matter temperature, so on the parameters characterizing particle scattering in the medium. On the basis of the developed method the Dalitz decay of a pion in an equilibrium pion gas is studied.

2 Decay probability of a particle in a scattering medium.

Let us consider in-mater particles with the spin s and the mass M. We suppose that the particles undergo multiple elastic collisions in the medium which is in thermodynamic equilibrium. We consider the decay of the particle according to the processes $A \to A_1 + \ldots + A_N$).

In the quasi-classical case when the wave length of a particle less than the mean distance between them, the decoupling constant $\gamma(x,p)$ is given by the well known expression (see, for example [20]) :

$$\gamma(x,p) = -2Im\Big\{Tr\big\{\Sigma^{ret}\big\}\Big\}, \tag{1}$$

where Σ^{ret} is the retarded self energy in the Keldysh diagram technique [21], $x = (t, \vec{r})$ and $= (p^0, \vec{p})$ are a four-coordinate and a four-momentum.

Let us suppose that the influence of particle scattering on the decay vertex and on the interaction of decay products in the final state is negligible. Then we can write

$$\Sigma^{ret} = \pm\Sigma^{\alpha\beta}{}_{vac}(p) G^{-+}_{\alpha\beta}(x,p) \prod_{j=1}^{N}(1 \pm n_j), \tag{2}$$

where $\Sigma^{\alpha\beta}_{vac}(p)$ is the self energy of the decaying particle in vacuum; $G^{-+}_{\alpha\beta}(x,p)$ is the Green's function of the particle in the matter; n_j are the occupancy number of the decay products (plus and minus signs correspond to the Boson and Fermi statistics, respectively); α and β are the spin variables.

To obtain the decay damping width Γ, which would be observed, we should average expression (1) over four-coordinates and four-momentums. Using the relation between $G^{-+}_{\alpha\beta}(x,p)$ and particle density $n(t, \vec{r}, \vec{p})$ at the thermodynamics equilibrium [20, 22] and integrating over d^4x and d^4p in the formula (1), we obtain following expression for the damping width of an on-shell particle :

$$\Gamma = \frac{2\pi}{t_0}\prod_{j=1}^{N}(1 \pm n_j) Im\Bigg\{\int_0^{t_0} dt \int \frac{d^3\vec{p}}{\varepsilon} Tr\big\{\Sigma^{\alpha\beta}_{vac}(p)\varrho_{\alpha\beta}\big\}\int d^3\vec{r}\cdot n(t,\vec{r},\vec{p})\Bigg\} \tag{3}$$

where $\varrho_{\alpha\beta}$ is the polarization density matrix of the decaying particle; t_0 is the time during which the decay is observed (the observation time); $p^0 \equiv \varepsilon = \sqrt{\vec{p}^2 + M^2}$. We set the normalization volume $V = 1$.

Let us suppose that times of relaxation in the matter $\tau \ll min\{t_0, \Gamma^{-1}\}$. This means that the static approximation [23] for the description of the decaying particle movement in the medium can be used. In this case the density $n(t, \vec{r}, \vec{p})$ satisfies the standard transport equation [24]. We note that the function $n(t, \vec{r}, \vec{p})$ appears in Eq.(3) should be integrated over the particle coordinates $\vec{r}$. Then, integrating transport equation [24] over $d^3\vec{r}$, we find the following equation for the function $f(\varepsilon, \vec{p}, t)$ being the integral of the density $n(t, \vec{r}, \vec{p})$ over the particle coordinate :

$$\frac{\partial f(p;t)}{\partial t} = \int d^3\vec{p}_1 d\sigma(p-p';p;p_1) v f_{eq}(p_1) \Big\{ f(p';t) - f(p;t) \Big\}, \quad (4)$$

where $d\sigma(p-p';p;p_1)$ is the cross section of a single scattering of the particle in the medium; $f_{eq}(p_1)$ is an equilibrium distribution function of the particles in the matter; $v = \sqrt{s(s-4m^2)}/2\varepsilon\varepsilon_1; p = (\varepsilon, \vec{p}); p_1 = (\varepsilon_1, \vec{p_1}); p' = (\varepsilon', \vec{p}'); s = (p+p_1)^2$.

Let us note that in the case of homogeneous and isotropic matter the expression $Tr\Big\{\Sigma^{\alpha\beta}_{vac}(p)\varrho_{\alpha\beta}\Big\}$ appears in the Eq.(3) depends only on the particle energy but does not depend on the momentum of the decaying particle. This circumstance allows us to rewrite the formula (3) as follows :

$$\Gamma = \frac{1}{t_0}\int\limits_0^{t_0} dt \int\limits_0^{+\infty} p^2 dp \Gamma_{vac}(\varepsilon) F(\varepsilon, t) \quad (5)$$

Here we introduce

$$\Gamma_{vac}(p) = \frac{2\pi}{\varepsilon}\prod_{j=1}^{N}(1 \pm n_j) Im\Big\{ Tr\Big\{\Sigma^{\alpha\beta}_{vac}(p)\varrho_{\alpha\beta}\Big\}\Big\} \quad (6)$$

$$F(\varepsilon, t) = \int d\Omega_{\vec{p}} f(\varepsilon, \vec{p}), \quad (7)$$

where $d\Omega_{\vec{p}}$ is the solid angle in the direction of the vector $\vec{p}$. Function $F(\varepsilon, t)$ satisfies following equation obtained by means of the integration of the expression (4) over all directions of the vector $\vec{p}$:

$$\frac{\partial F(\varepsilon;t)}{\partial t} = \int\limits_{\varepsilon - E_{max}}^{\varepsilon + E_{max}} d\varepsilon' \int d^3\vec{p}_1 \int d\Omega_{\vec{p}} \frac{d\sigma(\varepsilon - \varepsilon';\varepsilon;p_1)}{d\varepsilon' d\Omega_{\vec{p}}} v f_{eq}(p_1) \Big\{ F(\varepsilon';t) - F(\varepsilon;t) \Big\}, \quad (8)$$

where E_{max} is the maximum energy imparted in the result of the individual elastic collision of two particles of the matter. The energy E_{max} depends on the energies ε and ε' of the colliding particles.

In the case when the matter is an equilibrium one and when elastic collisions occur only the energy transmitted is small as compared with the matter temperature. Then we can expand the collision integral in right hand of the last expression over small parameter $|\varepsilon - \varepsilon'|/\varepsilon \ll 1$. In the result we get :

$$\frac{\partial F(\varepsilon;t)}{\partial t} = \alpha \frac{\partial^2 F(\varepsilon;t)}{\partial \varepsilon^2}, \tag{9}$$

where α is the energy imparted to particle per unit time period. When obtaining the last equation we use the fact that the cross section is the even function of the transmitted energy. The parameter α is defined by the following expression :

$$\alpha = \int d^3\vec{p}_1 d\Omega_{\vec{p}} \int\limits_0^{E_{max}} d\varepsilon' \frac{d\sigma(\varepsilon - \varepsilon'; \varepsilon; p_1)}{d\varepsilon' d\Omega_{\vec{p}}} v f_{eq}(p_1) E^2 \tag{10}$$

To solve the Eq. (4) we should specify the initial condition for the function $F(\varepsilon;t)$. We suppose that some equilibrium distribution of the particle is given at time $t = 0$:

$$F(\varepsilon; t = 0) = f_{eq}(\varepsilon)\eta(\varepsilon), \tag{11}$$

where $\eta(\varepsilon)$ is the standard unit function ($\eta(\varepsilon) = 1$, when $\varepsilon \geq 0$, but $\eta(\varepsilon) = 0$ at $\varepsilon < 0$); $\varepsilon = \sqrt{M^2 + \vec{p}^2}$.

The parameter α determined by the Eq.(10) depends on the energy particle ε. Except that in the case of an equilibrium matter the average value of the particle energy is constant but the energy imparted in the result of an individual collision of particles is small as compared with the matter temperature. Therefore we can set $\varepsilon = \varepsilon_1$ in the formula (10) and think that α is constant approximately. Then, solving the equation (9) with the initial condition (11) we get :

$$F(\varepsilon;t)=\frac{\eta(\varepsilon)}{\sqrt{4\pi\alpha t}}\int\limits_0^{+\infty}d\varepsilon'\Big\{\exp\big\{-\frac{(\varepsilon-\varepsilon')^2}{4\alpha t}\big\}+\exp\big\{-\frac{(\varepsilon+\varepsilon')^2}{4\alpha t}\big\}\Big\}f_{eq}(\varepsilon') \quad (12)$$

Substituting obtained function $F(\varepsilon;t)$ into the Eq.(5) we find probability W of the particle decay in the scattering matter :

$$W=\Gamma\cdot t_0=\int\limits_0^{t_0}\frac{dt}{\sqrt{4\pi\alpha}}\int\limits_0^{+\infty}p^2dp\Gamma_{vac}(\varepsilon(p))\int\limits_0^{+\infty}d\varepsilon'$$
$$\Big\{\exp\big\{-\frac{(\varepsilon-\varepsilon')^2}{4\alpha t}\big\}+\exp\big\{-\frac{(\varepsilon+\varepsilon')^2}{4\alpha t}\big\}\Big\}f_{eq}(\varepsilon') \quad (13)$$

The last formula determines the decay probability of the particle undergoing multiple elastic collisions in an equilibrium matter. The obtained expression for the decay probability does not depend on the interaction model of the particle in the scattering medium at all. The influence of the matter on the probability W is only due to the observable parameters such as the matter temperature (by means of the equilibrium distribution function f_{eq}), the mean square energy imparted by the particle in the result of an individual collision α and the observation time t_0.

We should note that the density of the states of the particle in an equilibrium matter has a sharp maximum at the energy $\varepsilon\sim T$ where T is the matter temperature. On the other hand it follows from the formula (13) that the width of this maximum is determined by the relation between the value of the energy fluctuations of the particle due to the multiple scattering and the matter temperature. In this way the energy fluctuation during the observation time t_0 is given by the magnitude of the parameter αt_0. If the $\alpha t_0\geq T$ influence of the multiple scattering the particle decay is strong. In opposite case $\alpha t_0\ll T$ the impact is negligible.

Let us next consider applications of the developed method for the calculation of decay probability of the particles in a scattering medium.

3 Dalitz decay of a pion in a scattering medium.

The decay of a pion $\pi^0\to e^+e^-\gamma$ (Dalitz decay) is of interest in the connection of the study of dilepton pair production in the nuclear matter arising in the result of collisions of high energy heavy ions [7, 8, 25]. Contribution of this decay into the rate of dilepton pair production in the absence of multiple scattering of decaying particle in matter is studied in the paper [26]. As this electromagnetic decay is also strong one virtually we use the quark model for the calculation of its probability. In this way the calculation result depends on both the quark and the strong interaction constant. We note that presently the same combination of these parameters appears in the formula for the width $\Gamma_{\pi^0\to 2\gamma}$ of the decay $\pi^0 \to 2\gamma$. Therefore we change above mentioned parameters by the expression for the width $\Gamma_{\pi^0\to 2\gamma}$ in the final equation for the decay probability of the pion in matter.

Ignoring the distinction between masses of u and d quarks in the pion quark model we write the following relativistic invariant expression for the matrix element of the decay $\pi^0 \to e^+e^-\gamma$:

$$\mathcal{M} = \frac{4\pi g e^{3/2}}{q^2 {m_q}^2}\epsilon_{ijsr}(v^*)^i(k)k^j q^s \bar{u}(-p_+)\gamma^r u(p_-),$$
$$q = p_+ + p_- \tag{14}$$

where $p_- = (E_-, \vec{p}_-), p_+ = (E_+, \vec{p}_+), k = (\omega, \vec{k})$ are 4-momentum of an electron, a positron and a photon, respectively; $u(p)$ and γ^r are the Dirac spinor and the Dirac matrixes ; $v^j(k)$ is the photon polarization vector; ϵ_{ijsr} is the antisymmetric tensor; e is the electron charge; m_q is a quark mass; g is the constant of strong interaction which corresponds to the vertex of the pion quark loop.

Calculating the decay rate according to the standard rules we find the following expression for $\Gamma_{vac}(p)$ being inserted in the Eq.(5) for the decay probability in matter :

$$\frac{d\Gamma_{vac}}{dM_l} = \frac{4e^2}{3\pi\varepsilon {M_l}^4 M^5}(M^2 - {M_l}^2)^3({M_l}^2 + 2m^2)({M_l}^2 - 4m^2)^{1/2}\Gamma_{\pi^0\to 2\gamma}$$
$$\Gamma_{\pi^0\to 2\gamma} = \frac{27\pi g^2 e^2 M^3}{{m_q}^2}, \tag{15}$$

where $\Gamma_{\pi^0\to 2\gamma} = 8eV$ is the width of the decay of a rest pion [27], M and m are the masses of a pion and an electron, respectively; ε is the pion energy. $M_l = |p_+ + p_-|$ is the invariant mass of a dilepton pair. When we have obtained the last expression we suppose that the occupation numbers of decay products are equal to zero. We note that in the case of nonrelativistic pions the last formula coincides with expression for the Dalitz decay rate [26].

Let us consider the Dalitz pion decay in an equilibrium pion gas. In the real situation the temperature T of the pion gas arising in the result of collisions of high energy heavy ions is about $T \sim 200 - 300 MeV$ [3, 25]. This means that we can think approximately that the pions are ultrarelativistic ones. In this case the energy imparted in the result of an individual elastic collision of the particles is small as compared with the particle energy which is of the order of the matter temperature. Estimating from the conservation laws we find that the maximum energy imparted E_{max} is of the order of $E_{max} \sim T(M/T)^2 \ll T$.

Then, substituting obtained expression for $d\Gamma_{vac}/dM_l$ into the general Eq. (5) for the decay probability in matter and assuming that distribution $f_{eq}(\varepsilon)$ is Boltzmann one we obtain

$$\frac{dW}{dM_l} = \frac{4e^2}{3\pi^{3/2} T {M_l}^4 M^5} (M^2 - {M_l}^2)^3 ({M_l}^2 + 2m^2)({M_l}^2 - 4m^2)^{1/2} \Gamma_{\pi^0 \to 2\gamma}$$
$$\left\{ \frac{2\alpha^{1/2} {t_0}^{3/2}}{3T} + \frac{T^2}{\alpha} \cdot \left[\exp(\frac{\alpha t_0}{T^2}) \cdot \int\limits_{\frac{\sqrt{\alpha t_0}}{T}}^{+\infty} dx \exp(-x^2) - \frac{\sqrt{\pi}}{2} + \frac{\sqrt{\alpha t_0}}{T} \right] \right\}, \qquad (16)$$

where T is the matter temperature, t_0 is the observation time which can be estimated as time life of the equilibrium state of the matter which is of the order of $\sim 10 Fm$ [10].

Let us study the obtained expression for the Dalitz decay of the pion in the equilibrium hadronic matter.

When the energy fluctuation due to elastic scattering is small as compared with the matter temperature, the parameter $(\sqrt{\alpha t_0}/T) \ll 1$. Then, calculating the integral at small lower limit in Eq.(16) we find:

$$\frac{dW}{dM_l} = \frac{2e^2 t_0}{3\pi T {M_l}^4 M^5} (M^2 - {M_l}^2)^3 ({M_l}^2 + 2m^2)({M_l}^2 - 4m^2)^{1/2} \Gamma_{\pi^0 \to 2\gamma} \times$$
$$\left\{ 1 + \frac{\alpha t_0}{2T^2} + O((\alpha t_0/T^2)^{3/2}) \right\}, \qquad (17)$$

It follows from the last formula that the small energy fluctuations due to multiple elastic collisions lead to the insignificant increasing of the decay probability as compared with the situation of the particle decay in the matter in the absence of scattering.

In opposite limit case $(\sqrt{\alpha t_0}/T) \geq 1$ when the energy fluctuations are large we obtain from Eq. (16):

$$\frac{dW}{dM_l} = \frac{4e^2 t_0}{3\pi^{3/2} T {M_l}^4 M^5} (M^2 - {M_l}^2)^3 ({M_l}^2 + 2m^2)({M_l}^2 - 4m^2)^{1/2} \Gamma_{\pi^0 \to 2\gamma} \times \left\{ \frac{2\sqrt{\alpha t_0}}{3T} + O(T/\sqrt{\alpha t_0}) \right\}, \tag{18}$$

The last means that the strong energy fluctuation results in the considerable increasing of the decay probability by the factor $(\sqrt{\alpha t_0}/T) \gg 1$.

Let us discuss obtained results from the point of view of the possibility of the observation of increasing of the Dalitz pion decay in the equilibrium matter due to multiple elastic scattering. According to the Eq. (10) the parameter α being the mean square fluctuation of the particle energy, can be estimated as the product of the collision frequency ν and the square of the maximum energy ${E_{max}}^2$ imparted in the result of an individual collision of the particles. Because $E_{max} \sim T(M/T)^2$ the parameter $(\sqrt{\alpha t_0}/T) \sim \sqrt{\nu t_0}(M/T)^2 = (M/T)^2 \cdot \sqrt{N}$, where $N \gg 1$ is the number of collisions of the particle in the matter during life time of the equilibrium state of the medium. The last means when number of the particle collisions is more than the ratio $(T/M)^2 \gg 1$ the considerable increasing of the pion decay probability occurs. In the real situation [3, 10, 25] $T \sim 200 - 300 MeV; \nu \sim 1.33 - 4Fm^{-1}, respectively; t_0 \sim 10Fm$ that is $\sqrt{\nu t_0}(M/T)^2 \sim 1.38 - 1.79$. This means that the multiple elastic scattering have to influence strongly on the pion Dalitz in the matter. In conclusion we should note that as the dielectron yield due to the Dalitz decay of pions are not suppressed by any other processes of the dielectron production at low invariant mass of a dielectron pair [25]. That is we can say about the possibility to observe the increasing of the pion decay probability due to multiple scattering of the pions in hadron matter.

4 Conclusion.

The decay of particles undergoing multiple elastic collisions in an equilibrium scattering medium is studied in this paper. The decay probability of such particles is calculated. The developed method of the calculation of the decay probability does not depend on the dynamic model of the particle scattering in the medium. In this way the impact of the medium on the decay probability is determined by the energy fluctuation of the decaying particle due to multiple scattering and the matter temperature. It is shown when the energy fluctuation is of the order of the medium temperature multiple scattering results in considerable changes of the decay probability as compared with the case of scattering lack.

We study the Dalitz decay of pions undergoing multiple elastic collisions in an equilibrium pion gas. We shown that the multiple scattering leads to the increasing of the decay probability in any cases. In this way when the energy fluctuations are of the order of the magnitude of the matter temperature multiple scattering leads to the strong broadening of the in-matter pion in the Dalitz decay channel. We discuss the possibility to observe the increasing of the Dalitz decay probability due to the multiple scattering of a pion in matter arising in the result of collisions of heavy ions of high energy. According to the estimations made impact of multiple scattering may be considerable in the equilibrium pion gas at temperature $T \sim 200 - 300 MeV$.

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