The Determination of $\alpha_s(M_Z)$, Higher Twists and xF_3 Parameterization from CCFR Data at the NNLO and Beyond

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Abstract

The improved next-to-next-to-leading order (NNLO) QCD analysis of the experimental data of the CCFR collaboration for the xF_3 structure function is made. Theoretical ambiguities of the NNLO fits are estimated by means of the Padé resummation technique. The NNLO and the new N^{3}LO $\alpha_s(Q^2)$ $\overline{MS}$-matching conditions are used. In the process of the fits we are taking into account the the twist-4 $1/Q^2$-terms. Our NNLO results for $\alpha_s(M_Z)$ values, extracted from the CCFR xF_3 data, are $\alpha_s(M_Z) = 0.118 \pm 0.002(stat) \pm 0.005(syst) \pm 0.004(theory)$ provided the twist-4 contributions are fixed through the infrared renormalon model and $\alpha_s(M_Z) = 0.116^{+0.006}_{-0.007}(stat) \pm 0.005(syst) \pm 0.004(theory)$ provided the twist-4 terms are considered as the free parameters. It is shown that the extracted at the NNLO order x-shape of the twist-4 correction is almost unchanged after application of the Padé resummation approximant.

1. Deep-inelastic lepton-nucleon scattering (DIS) belongs to the classical and continuously studying processes in the modern particle physics. It should be stressed that the program of getting the information about the behavior of the xF_3 structure function (SF) of νN DIS is among the aims of the experimental program of Fermilab Tevatron and CCFR/NuTeV collaboration in particular. The most recent CCFR data for this SF was distributed among the potential users in the beginning of 1997 [1], while the final results of the original CCFR DGLAP [2] NLO analysis of this data was presented in the journal publication of Ref.[1].

The subsequent steps of performing NLO and first NNLO analysis of the CCFR data with the help of the Jacobi polynomial - Mellin moments version of the DGLAP method were made in Refs.[3]-[7] (the definite stages in the development of this formalism are described in Refs.[8]-[9]).

In the process of the analysis of Refs.[4]-[7] the authors used the information about the NNLO corrections to the coefficient functions [10] and the available analytical expressions for the NNLO corrections to the anomalous dimensions of the NS moments with $n = 2, 4, 6, 8, 10$ [11], supplemented with the given in Ref.[4] $n = 3, 5, 7, 9$ similar numbers, obtained using the smooth interpolation procedure, which was previously proposed in Ref.[12]. Moreover, the attempts to extract the shape of the twist-4 contributions and study the predictive abilities of the IR-renormalon (IRR) model of Ref.[13] were performed (for the definite details of modeling the effects of the power-suppressed contributions to measurable physical quantities using the IRR language see Ref.[14] and the reviews of Ref.[15]).

However, the important question of the estimation of theoretical uncertainties of the NNLO analysis of the CCFR data of Ref.[6] was still non-analyzed in detail. These uncertainties are determined by

1) the differences in the definitions of $\alpha_s(Q^2)$ matching conditions (see e.g. [16, 17, 18]) which are responsible for penetrating into the energy region, characteristic for $f = 5$ numbers of flavours, where the pole of the Z^0-boson is manifesting itself;

2) the incorporation into the condition of Ref.[16] recalculated NNLO QCD corrections [19] and the newly calculated N^3LO corrections, namely the 4-loop coefficient of the QCD β-function [20] and the N^3LO-term [21] in the matching condition of Ref.[16];

3) the consideration of theoretical uncertainties due to other non-calculated N^3LO contributions into the coefficient functions and the anomalous dimensions function.

This work is devoted to the analysis of the important problems outlined above and to the more detailed extraction of the values of $\alpha_s(M_Z)$ and the x-shape of the twist-4 power-suppressed term at available orders of perturbative QCD with taking into account the effects enumerated above. We are supplementing the NNLO fits of Ref.[6] by the $\mathrm{N^3LO}$ analysis, which is based on the application of the Padé resummation technique developed in QCD in the definite form in Refs.[22, 23] using the scheme invariant foundations of Ref.[24] and considered previously as the possible method of fixing theoretical uncertainties in the analysis of DIS data in Ref.[25]. Our analysis could be considered as the attempt to estimate perturbative QCD uncertainties beyond the NNLO level. Moreover, it could give us the hint whether the outcomes of the NNLO fits, related to perturbative and non-perturbative sectors, stay stable after the inclusion of the explicitly calculated and estimated $\mathrm{N^3LO}$ QCD corrections.

2. Let us define the Mellin moments for the NS SF $xF_3(x,Q^2)$: $M_n^{NS}(Q^2) = \int_0^1 x^{n-1}F_3(x,Q^2)dx$ where $n = 2,3,4,....$ The theoretical expression for these moments obey the standard renormalization group equation The renormalization group functions are defined as $\mu\frac{\partial A_s}{\partial\mu} = \beta(A_s) = -2\sum_{i\geq 0}\beta_i A_s^{i+2}$, $\mu\frac{\partial ln Z_n^{NS}}{\partial\mu} = \gamma_{NS}^{(n)}(A_s) = \sum_{i\geq 0}\gamma_{NS}^{(i)}(n)A_s^{i+1}$ where Z_n^{NS} are the renormalization constants of the corresponding NS operators. The solution of the renormalization group equation can be presented in the following form :

$$\frac{M_n^{NS}(Q^2)}{M_n^{NS}(Q_0^2)} = exp\Big[-\int_{A_s(Q_0^2)}^{A_s(Q^2)}\frac{\gamma_{NS}^{(n)}(x)}{\beta(x)}dx\Big]\frac{C_{NS}^{(n)}(A_s(Q^2))}{C_{NS}^{(n)}(A_s(Q_0^2))} \qquad (1)$$

where $M_n^{NS}(Q_0^2)$ is the phenomenological quantity related to the factorization scale dependent factor. It can be parametrized through the parton distributions at fixed momentum transfer Q_0^2 as

$$M_n^{NS}(Q_0^2) = \int_0^1 x^{n-2}A(Q_0^2)x^{b(Q_0^2)}(1-x)^{c(Q_0^2)}(1+\gamma(Q_0^2)x)dx.$$

At the $\mathrm{N^3LO}$ the expression for the coefficient function $C_{NS}^{(n)}$ can be presented as $C_{NS}^{(n)}(A_s) = 1 + C^{(1)}(n)A_s + C^{(2)}(n)A_s^2 + C^{(3)}(n)A_s^3$ while the corresponding expansion of the anomalous dimensions term is

$$exp\Big[-\int^{A_s(Q^2)}\frac{\gamma_{NS}^{(n)}(x)}{\beta(x)}dx\Big] = (A_s(Q^2))^{\gamma_{NS}^{(0)}(n)/2\beta_0}\times AD(n_s) \qquad (2)$$

where $AD(n_s) = [1 + p(n)A_s(Q^2) + q(n)A_s(Q^2)^2 + r(n)A_s(Q^2)^3]$ and $p(n)$, $q(n)$ and $r(n)$ have the following form:

$$p(n) = \frac{1}{2}\Big(\frac{\gamma_{NS}^{(1)}(n)}{\beta_1} - \frac{\gamma_{NS}^{(0)}(n)}{\beta_0}\Big)\frac{\beta_1}{\beta_0} \tag{3}$$

$$q(n) = \frac{1}{4}\Big(2p(n)^2 + \frac{\gamma_{NS}^{(2)}(n)}{\beta_0} + \gamma_{NS}^{(0)}(n)\frac{(\beta_1^2 - \beta_2\beta_0)}{\beta_0^3} - \gamma_{NS}^{(1)}(n)\frac{\beta_1}{\beta_0^2}\Big) \tag{4}$$

$$r(n) = \frac{1}{6}\Big(p(n)^3 + 6p(n)q(n) + \frac{\gamma_{NS}^{(3)}(n)}{\beta_0} - \frac{\beta_1\gamma_{NS}^{(2)}(n)}{\beta_0^2} \tag{5}$$
$$-\frac{\beta_2\gamma_{NS}^{(1)}(n)}{\beta_0^2} + \frac{\beta_1^2\gamma_{NS}^{(1)}(n)}{\beta_0^3} - \frac{\beta_1^3\gamma_{NS}^{(0)}(n)}{\beta_0^4} - \frac{\beta_3\gamma_{NS}^{(0)}(n)}{\beta_0^2} + \frac{2\beta_1\beta_2\gamma_{NS}^{(0)}(n)}{\beta_0^3}\Big)$$

The coupling constant $A_s(Q^2)$ can be expressed in terms of the inverse powers of $L = \ln(Q^2/\Lambda^2_{\overline{MS}})$ as $A_s^{NLO} = A_s^{LO} + \Delta A_s^{NLO}$, $A_s^{NNLO} = A_s^{NLO} + \Delta A_s^{NNLO}$ and $A_s^{N^3LO} = A_s^{NNLO} + \Delta A_s^{N^3LO}$, where $A_s^{LO} = 1/\beta_0 L$, $\Delta A_s^{NLO} = -\beta_1 ln(L)/\beta_0^3 L^2$ $\Delta A_s^{NNLO} = [\beta_1^2 ln^2(L) - \beta_1^2 ln(L) + \beta_2\beta_0 - \beta_1^2]/\beta_0^5 L^3$ $\Delta A_s^{N^3LO} = [\beta_1^3(-ln^3(L) + \frac{5}{2}ln^2(L) + 2ln(L) - \frac{1}{2}) - 3\beta_0\beta_1\beta_2 ln(L) + \beta_0^2\beta_3/2]/\beta_0^7 L^4$.

Notice that in our normalization the numerical expressions for β_0, β_1, β_2 and β_3 read $\beta_0 = 11 - 0.6667f$, $\beta_1 = 102 - 12.6667f$, $\beta_2 = 1428.50 - 279.611f + 6.01852f^2$, $\beta_3 = 29243.0 - 6946.30f + 405.089f^2 + 1.49931f^3$ where the expression for β_3 was obtained in Ref.[20]. The inverse-log expansion for $\Delta A_s^{N^3LO}$, which incorporates the information about the coefficient β_3, was presented in Ref.[21].

Let us now describe the procedure of fixing other theoretical uncertainties. After the work of Ref.[22] it became rather popular to model the effects of the higher order terms of perturbative series in QCD using the expanded Padé approximants.

In the framework of this technique the values of the terms $C^{(3)}(n)$ and $r(n)$ could be expressed as

$$Pade\ [1/1]\ :\quad C^{(3)}(n) = [C^{(2)}(n)]^2/C^{(1)}(n) \tag{6}$$
$$r(n) = q(n)^2/p(n) \tag{7}$$
$$Pade\ [0/2]\ :\quad C^{(3)}(n) = 2C^{(1)}(n)C^{(2)}(n) - [C^{(1)}(n)]^3 \tag{8}$$
$$r(n) = 2p(n)q(n) - [p(n)]^3 \tag{9}$$

Another possibility is to use the idea of Ref.[25] and consider the results of applications of non-expanded Padé approximants in the process of the analysis of the DIS data.

Let us remind that the corresponding non-expanded [1/1] Padé approximants can be defined as

$$AD(n)|_{[1/1]} = \frac{1 + a_1^{(n)} A_s}{1 + b_1^{(n)} A_s} \tag{10}$$

$$C_{NS}^{(n)}(A_s)|_{[1/1]} = \frac{1 + c_1^{(n)} A_s}{1 + d_1^{(n)} A_s} \tag{11}$$

where

$$a_1^{(n)} = \Big([p(n)]^2 - q(n)\Big)/p(n), \quad b_1^{(n)} = -q(n)/p(n)$$

and

$$C_1^{(n)} = \Big([C^{(1)}(n)]^2 - C^{(2)}(n)\Big)/C^{(1)}(n), \quad d_1^{(n)} = -C^{(2)}(n)/C^{(1)}(n)$$

.

The explicit expressions for the non-expanded [0/2] Padé approximants read:

$$AD(n)|_{[0/2]} = \frac{1}{1 + b_1^{(n)} A_s + b_2^{(n)} A_s^2} \tag{12}$$

$$C_{NS}^{(n)}(A_s)|_{[0/2]} = \frac{1}{1 + d_1^{(n)} A_s + d_2^{(n)} A_s^2} \tag{13}$$

where $b_1^{(n)} = -p(n)$, $b_2^{(n)} = p(n)^2 - q(n)$, $d_1^{(n)} = -C^{(1)}(n)$ and $d_2^{(n)} = [C^{(1)}(n)]^2 - C^{(2)}(n)$. Since we consider the applications of both [1/1] and [0/2] Padé approximants as the attempts to model the behavior of the perturbative series for the NS Mellin moments beyond the NNLO level, we will use in Eqs.(10)-(13) the N^{3}LO expression for the coupling constant A_s. It is worth to mention here, that quite recently the expanded and non-expanded Padé approximants were successfully used for the study of the N^{3}LO approximation of the ground state energy in quantum mechanics [26].

The next step is the reconstruction of the structure function $xF_3(x, Q^2)$ with taking into account both target mass corrections and twist-4 terms. The reconstructed SF can be expressed as:

$$xF_3^{N_{max}}(x,Q^2) = xF_3^{PQCD}(x,Q^2) + \frac{h(x)}{Q^2} \tag{14}$$

where the perturbative part $xF_3^{PQCD}(x,Q^2)$ can be reconstructed from the Mellin moments using the Jacobi polynomial method of Refs.[8,9].

In order to take into account the target mass corrections we used the expanded up to $O(M_{nucl}^2/Q^2)$-terms Nachtamnn moments where M_{nucl} is the mass of a nucleon.

The form of the twist-4 contributions $h(x)$ in Eq.(14) was first fixed using the IRR model of Ref.[13] for the Mellin moments: $M_{n,xF_3}^{IRR}(Q^2) = \tilde{C}(n)M_{n,xF_3}^{NS}(Q^2)A_2^{'} + O(1/Q^2)$ wsith $A_2^{'}$ taken as the free parameter. The results of the LO, NLO, NNLO and N^{3}LO fits are presented at Table 1 for the case of $f=4$ active flavours.

It is also rather instructive to consider the function $h(x)$ as the free parameters of the fits, not related to IRR-model estimates.

We will estimate the uncertainties of the values of $\Lambda_{\overline{MS}}^{(4)}$, $\alpha_s(M_Z)$ IRR-model parameter $A_2^{'}$ and the twist-4 function $h(x)$ due to the inclusion of the definite explicitly calculated N^{3}LO QCD corrections in the fits and modeling other ones with the help of the Padé resummation technique. Our aim will be also the verification of the stability of the results of Ref.[6] and the study of the possible reason of the peculiar behavior of $h(x)$ for xF_3 SF, discovered in Ref.[6] at the NNLO level.

3 (a). Let us discuss the influence of the Padé-motivated estimates of the N^{3}LO expressions for anomalous dimensions and corresponding coefficient function (written down both in the expanded and non-expanded forms) to the results of the fits , which are resulting in the extractions of the parameter $\Lambda_{\overline{MS}}^{(4)}$ (and thus $\alpha_s(M_Z)$), and of the common factor $A_2^{'}$ of the IRR model.

It should be stressed that the N^{3}LO [1/1] Padé approximant description of the CCFR'97 experimental data turned out to be not acceptable in our case, since it produces rather high value of χ^2: $\chi^2/nep > 2$ (where $nep = 86$ is the number of the experimental points, taken into account in the case of the cut $Q^2 > 5\ GeV^2$). However, the application of [0/2] Padé approximants produced reasonable results. We think that the nonapplicability of the [1/1]

$Q^2 >$	$\Lambda^{(4)}_{\overline{MS}}$	A	b	c	γ	A'_2/GeV^2	χ^2
5 GeV^2							
LO	266±37	5.13±0.46	0.72±0.03	3.87±0.05	1.42±0.33	–	113.2/86
	436±56	4.73±0.39	0.68±0.03	3.79±0.05	1.82±0.31	-0.33±0.06	82.8/86
NLO	341±41	4.05±0.38	0.65±0.03	3.71±0.06	1.96±0.36	–	87.1/86
	371±31	3.94±0.12	0.64±0.01	3.69±0.04	2.05±0.12	-0.12±0.05	81.8/86
NNLO	293±29	4.25±0.38	0.66±0.03	3.56±0.07	1.33±0.33	–	78.4/86
	293±29	4.25±0.37	0.66±0.03	3.56±0.07	1.33±0.32	-0.01±0.05	78.4/86
N^3LO	293±28	4.80±0.35	0.69±0.03	3.40±0.09	0.69±0.26	–	79.0/86
	294±28	4.80±0.35	0.69±0.03	3.40±0.09	0.68±0.26	-0.02±0.05	78.8/86
N^3LO	305±32	4.79±0.35	0.70±0.03	3.43±0.09	0.77±0.27	–	79.0/86
(n.e.)	308±32	4.80±0.36	0.69±0.03	3.43±0.09	0.76±0.27	-0.03±0.05	78.5/86
10 GeV^2							
LO	287±37	6.52±0.20	0.79±0.01	3.73±0.04	0.60±0.10	–	77.7/63
	531±74	6.48±0.65	0.79±0.04	3.68±0.05	0.94±0.26	-0.52±0.11	58.0/63
NLO	350±36	5.29±0.21	0.74±0.01	3.64±0.06	1.06±0.17	–	64.4/63
	439±55	4.92±1.06	0.73±0.07	3.61±0.08	1.42±0.75	-0.24±0.10	58.6/63
NNLO	308±34	4.65±0.53	0.70±0.04	3.52±0.08	1.06±0.38	–	58.7/63
	313±37	4.64±0.52	0.70±0.04	3.52±0.08	1.07±0.38	-0.03±0.09	58.6/63
N^3LO	295±30	5.20±0.48	0.72±0.04	3.35±0.11	0.45±0.31	–	59.9/63
	297±33	5.21±0.48	0.72±0.04	3.35±0.11	0.45±0.31	-0.01±0.09	59.8/63
N^3LO	303±33	5.28±0.53	0.73±0.04	3.39±0.11	0.50±0.33	–	60.4/63
(n.e.)	309±36	5.29±0.51	0.73±0.04	3.38±0.11	0.49±0.32	-0.03±0.09	60.3/63
15 GeV^2							
LO	319±41	7.58±0.24	0.84±0.01	3.61±0.05	0.16±0.09	–	58.8/50
	531±57	7.76±0.26	0.86±0.02	3.61±0.04	0.45±0.09	-0.57±0.13	50.2/50
NLO	365±40	6.28±0.21	0.80±0.02	3.60±0.04	0.58±0.09	–	53.0/50
	441±44	5.98±0.19	0.80±0.02	3.57±0.05	0.87±0.10	-0.25±0.13	50.9/50
NNLO	314±37	4.68±0.75	0.70±0.05	3.53±0.08	1.08±0.51	–	50.9/50
	308±45	4.69±0.72	0.70±0.05	3.53±0.08	1.07±0.50	0.03±0.14	50.8/50
N^3LO	296±33	5.25±0.65	0.73±0.05	3.36±0.12	0.45±0.38	–	52.3/50
	286±38	5.25±0.69	0.73±0.05	3.37±0.12	0.45±0.40	0.07±0.14	52.0/50
N^3LO	304±36	5.32±0.71	0.73±0.05	3.40±0.12	0.50±0.41	–	52.8/50
(n.e.)	297±43	5.32±0.77	0.73±0.06	3.41±0.12	0.49±0.44	0.05±0.14	52.7/50

Table 1: The results of the fits of xF_3 (νFe) data from CCFR (revised 1997). The value of $\Lambda^{(4)}_{\overline{MS}}$ are given in MeV. The abriviation (n.e.) marks the results of non-expanded Padé fits and chi^2 is devised by the number of experimental points

Padé method in the process of fitting CCFR xF_3 data using the Jacobi polynomial approach can be related to the manifestation of rather large value of the ratio $[C^{(3)}(2)/C^{(2)}(2)]|_{[1/1]}$ in the expression for NS moment M_{2,xF_3}^{NS}.

The similar effect of the preference of the [0/2] Padé approximant analysis over the [1/1] one was found in Ref.[23] in the case of the comparison of the QCD theoretical predictions for the polarized Bjorken sum rule (which are closely related to the QCD predictions for the first moment of the xF_3 SF, namely for the Gross-Llewellyn Smith sum rule) with the available experimental data.

Using the LO, NLO, NNLO and N^3LO variants of the rigorous $\overline{MS}$-scheme matching conditions, derived in Ref.[21] following the lines of Ref.[16], we transform $\Lambda_{\overline{MS}}^{(4)}$ values through the threshold of the production of the fifth flavour $M_5 = m_b \approx 4.6\ GeV$ (where m_b is the b-quark pole mass) and obtain the related values of $\Lambda_{\overline{MS}}^{(5)}$.

It should be stressed that we are considering the outcomes of our N^3LO approximated fits as the rate of theoretical uncertainties of the NNLO results in the same manner like the results of the NNLO analysis will be considered as the measure of theoretical uncertainties of the NLO results. In particular, we will introduce the characteristic deviations $\Delta^{NNLO}= |(\Lambda_{\overline{MS}}^{(4)})^{N^3LO} - (\Lambda_{\overline{MS}}^{(4)})^{NNLO}|$, $\Delta^{NLO}= |(\Lambda_{\overline{MS}}^{(4)})^{NNLO} - (\Lambda_{\overline{MS}}^{(4)})^{NLO}|$.

It is worth to emphasize that the results for the Padé motivated N^3LO results for $\Lambda_{\overline{MS}}^{(4)}$ and thus $\alpha_s(M_Z)$ became closer to the NNLO ones (provided the statistical error bars are taken into account, see Table 1). Moreover, the difference $\Delta^{NNLO}= |(\Lambda_{\overline{MS}}^{(4)})^{N^3LO} - (\Lambda_{\overline{MS}}^{(4)})^{NNLO}|$, is drastically smaller then the NLO correction term $\Delta^{NLO}= |(\Lambda_{\overline{MS}}^{(4)})^{NNLO} - (\Lambda_{\overline{MS}}^{(4)})^{NLO}|$. The similar tendency $\Delta^{NNLO} << \Delta^{NLO}$ is taking place in the case of the fits without twist-4 corrections. These observed properties indicate the reduction of the theoretical errors due to cutting the analyzed perturbation series at the different orders.

We are presenting now the values of $\alpha_s(M_Z)$, extracted from the fits of the CCFR'97 experimental data for the xF_3 SF, obtained with fixing twist-4 contribution through the IRR model of Ref.[13]:

$$\begin{aligned} NLO\ HT\ of [13]\ \ \alpha_s(M_Z) &= 0.121 \pm 0.002(stat) \\ &\pm 0.005(syst) \pm 0.005(theory) \end{aligned} \tag{15}$$

$$NNLO\ HT\ of[13]\ \alpha_s(M_Z) = 0.118 \pm 0.002(stat) \pm 0.005(syst) \pm 0.003(theory)\ . \tag{16}$$

The results of the extractions of $\alpha_s(M_Z)$, with twist-4 contribution, considered as the additional free parameters of the fit, have the following form:

$$NLO\ HT\ free\ \alpha_s(M_Z) = 0.124^{+0.007}_{-0.009}(stat) \pm 0.005(syst) \pm 0.008(theory) \tag{17}$$

$$NNLO\ HT\ free\ \alpha_s(M_Z) = 0.116^{+0.006}_{-0.007}(stat) \pm 0.005(syst) \pm 0.004(theory) \tag{18}$$

where the systematic uncertainties are taken from the CCFR experimental analysis, presented in the first work of Ref.[1], and the theoretical uncertainties in the results of Eqs.(15),(16) [Eqs.(17),(18)] are estimated by the diffferences between the central values of the outcomes of the NNLO and NLO [N^{3}LO and NNLO] fits, presented in Table 1, plus the arbitrariness in the application of smoothing procedure of the $\overline{MS}$-scheme matching condition (which following the considerations of Ref.[18] was estimated as $\Delta\alpha_s(M_Z) = \pm 0.001$). In estimating the theoretical errors of the inclusion of the N^{3}LO corrections we take into account the diffferences between the applications of the expanded and non-expanded Padé approximants.

3 (b). Apart of the perturbative QCD contributions the expressions for DIS structure functions should contain the power-suppressed high-twist terms, which reflect the possible non-perturbative QCD effects. In this section we are discussing the results of more refined analysis of the behavior of the twist-4 contributions to xF_3 SF at the LO, NLO, NNLO and beyond.

Note, that the parameter $A_2^{'}$ was introduced in the IRR model of Ref.[13], and fixed there as $A_2^{'} \approx -0.2\ GeV^2$, which is necessary for the description of the fitted twist-4 results of Ref.[28] for F_2 within IRR language. We found that the value of this parameter extracted at the LO and NLO, is negative, differ from zero for about one standard deviation and qualitatively in agreement with the IRR-motivated guess of Ref.[13]. Moreover, the results of our LO and NLO fits are also in agreement with the made in Ref.[27] earlier extraction from the old DIS neutrino data of the value $h = -0.38 \pm 0.06\ GeV^2$ of the different model of the twist-4 contribution to xF_3, namely $xF_3(x,Q^2)h/((1-x)Q^2)$.

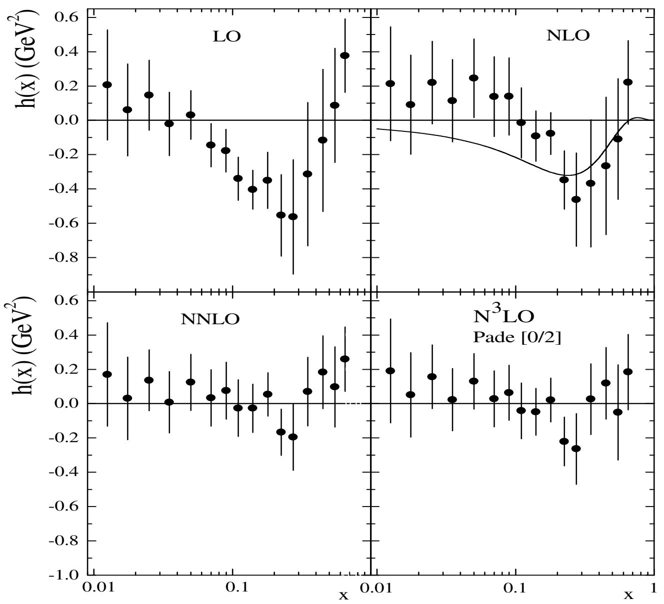

Fig. 1.

We also observed that in the process of multiloop extraction of $A_2^{'}$ the tendency $\chi^2_{LO} > \chi^2_{NLO} > \chi^2_{NNLO} \sim \chi^2_{N^3LO}$ takes place. It should be stressed that in opposite to the results of the LO and NLO extraction the NNLO value of $A_2^{'}$ is compatible with zero and is stable to the inclusion of the $\mathrm{N^3LO}$ contribution to A_s through the Padé approximants for the NS Mellin moments.

We are now turning to the pure phenomenological extraction of the twist-4 contribution $h(x)$ to xF_3, which is motivated by the work of Ref.[28] for F_2 data. In the framework of this approach the x-shape of $h(x)$ is parametrized by the additional parameters $h_i = h(x_i)$, where x_i are the points of the experimental data bining. The results of the multiloop extractions of these parameters are presented at the curves of Fig.1.

Looking carefully on Fig.1 we observe the following features:

1. The x-shape of the twist-4 parameter is not inconsistent with the ex-

pected rise of $h(x)$ for $x \to 1$ [29] in all orders of perturbation theory;

2. In the LO and NLO our results are in qualitative agreement with the IRR prediction of Ref.[13] (for discussions see Ref.[15]);

3. However, at the NNLO we observe the partial nullification of $h(x)$ within statistics error bars. Thus we conclude that the inclusion of the NNLO corrections into the game is shadowing the effects of the power suppressed terms, or that the effects of the twist-4 corrections are non-detectable at the NNLO;

4. We checked the reliability of the foundation of partial nullification of $h(x)$ at the NNLO by going beyond this perturbative approximation using the methods of Padé approximants. The result of this analysis reveals the stability of the NNLO results for $h(x)$ and its partial nullification;

5. The property of partial nullification of $h(x)$ at the NNLO and N^{3}LO is identical to the effect of nullification of the IRR model parameter $A_2^{'}$ at the NNLO and N^{3}LO (see Table 1);

Conclusion In this work we presented the results of the extractions of $\alpha_s(M_Z)$ and twist-4 terms from the QCD analysis of the CCFR data with taking into account definite QCD corrections at the NNLO and beyond. Within experimental and theoretical errors our results for $\alpha_s(M_Z)$ are in agreement with other extractions of this fundamental parameter, including its world average value $\alpha_s(M_Z) = 0.118 \pm 0.005$.

Our estimate of the NNLO theoretical uncertainties is based on application of the [0/2] Padé approach at the N^{3}LO level. The uncertainties of our NNLO analysis can be decreased after explicit NNLO calculations of the NS Altarelli-Parisi kernel.

As to the twist-4 terms, we found that despite the qualitative agreement with the IRR model prediction, at the NNLO level they have the tendency to decrease and are stable to the application of the [0/2] Padé motivated N^{3}LO analysis.

This feature can be related to the fact that the analysis of the CCFR data can not distinguish the twist-4 $1/Q^2$ terms from the NNLO perurbative

QCD approximations of the Mellin moments. This possible explanation lies in the lines of the results of the LO analysis of the old less precise neutrino DIS data, made by the authors of Ref.[30], who were unable to distinguish between LO logarithmic and $1/Q^2$-behavior of the QCD contributions to Mellin moments of xF_3. The achieved in our days increase of experimental precision might move this effect to the NNLO.

Another related explanation is that the observed by us NNLO effect is manifesting itself in view of the fact that the detected by us twist-4 terms come from the partial summation of the definite terms of the asymptotic perturbative QCD series and thus the increase of the order of perturbative QCD analysis effectively suppresses the remaining sum of the perturbative QCD contribution. Unfortunately, at the NNLO level we can not detect the true twist-4 terms, which reflect the non-trivial non-perturbative nature of QCD vacuum. One can hope that the future experiments of NuTeV collaboration will allow to get the new experimental data at the precision level, necessary for extracting more detailed information about higher twist contributions to SF.

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