# Photon Splitting in a Strong Magnetic Field

M.V. Chistyakov, A.V. Kuznetsov and N.V. Mikheev

*Division of Theoretical Physics, Department of Physics,*
*Yaroslavl State University, Sovietskaya 14,*
*150000 Yaroslavl, Russian Federation*

**Abstract**

The process of photon splitting $\gamma \to \gamma\gamma$ in a strong magnetic field is investigated both below and above the pair creation threshold.The partial amplitudes and the splitting probabilities are calculated taking account of the photon dispersion and large radiative corrections near the resonance.

The non-linear electromagnetic process of the photon splitting in external field is one of the most exiting effect of the quantum electrodynamics. It is interesting to note that possible manifestations of this process belong to the two completely oposite fields of our physical world. On one hand this effect could manifest itself in the microcosm, in the electric field of a heavy atoms. On the other hand, possible manifistations of this process in macrocosm are discussed during a long time, namely, in the astrophysical objects, where strong magnetic fields could exist.

In this talk we consider the procees of the photon splitting in the strong magnetic field. Theoretical study of this process has a rather long history [1, 2, 3]. Adler was the first who considers the photon splitting as a possible mechanism of the production linearly polarized photons in a pulsar magnetic field. In his work the comprehensive investigation of this process was carried out for the magnetic field below the critical value, $B_e = m^2/e \simeq 4.41 \cdot 10^{13}$ G (m is the electron mass, $e > 0$ is the elementary charge). There were found the selection rules connected whith the defenite polarization states of the initial and final photons. These selection rules are based on the collinear kinematics of photon splitting in the case of the subcritical magnetic field.

The recent progress in astrophysics has drawn attention again to the photon splitting induced by a magnetic field. It has found application in the study of the spectral formation of gamma ray burst (GRB) from neutron stars and has also been used in models of soft gamma ray repeaters (SGR), where it soften the photon spectrum. It is supposed for these objects to have very strong magnetic field which can exceed essentialy the critical value. The good review on this topic could be found in [4].

Therefore the study of the photon splitting has been continued. In some recent papers [5, 6] this process was studied in the magnetic field much greater than the critical value. In our opinion there are some facts which are not taken into account in this works.

- The limit of the collinear kinematics is not aplicable in the strong magnetic field ($B \gg B_e$). Therefore ones cannot use Adler's selection rules.
- The radiative corrections can give a large contribution near the pair creation threshold.

In this talk, photon splitting in a strong magnetic field is investigated both below and above the pair-creation threshold, with taking account of the noncollinearity of the kinematics and large radiative corrections.

# 1 The kinematic of photon splitting.

The kinematic of the photon splitting in magnetic field is defined by the vacuum polarization. The dispersion relations for the photons of different polarizations could be found from the following equations

$$\omega_\lambda^2(k) - \mathbf{k}^2 - \mathcal{P}^\lambda = 0, \tag{1}$$

where $\mathcal{P}^\lambda$ are the eigenvalues of the photon polarization operator. Only two of these equations correspond to the real photons with the polarization vectors

$$\varepsilon_\alpha^{(\parallel)} = \frac{(k\varphi)_\alpha}{\sqrt{k_\perp^2}}, \qquad \varepsilon_\alpha^{(\perp)} = \frac{(k\tilde{\varphi})_\alpha}{\sqrt{k_\parallel^2}}, \tag{2}$$

These are the so-called parallel and perpendicular modes in Adler's notation. Here $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$, $\tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\mu\nu}\varphi_{\mu\nu}$ are the dimensionless tensor of the external magnetic field and the dual tensor, $k_\parallel^2 = \omega^2 - k_3^2, \qquad k_\perp^2 = k_1^2 + k_2^2$. Magnetic field is directed along the third axes, $\mathbf{B} = (0, 0, B)$.

The corresponding eigenvalues $\mathcal{P}^\lambda$ are singular at the points of the so-called cyclotron resonances. In the limit of the strong magnetic field ($B \gg B_e$) the lowest of them is only essential and corresponds to the pair creation threshold, $k_\parallel^2 = 4m^2$. Therefore we shall further analyse the kinematical region $k_\parallel^2 < (m + \sqrt{m^2 + 2eB})^2$. In this region $\mathcal{P}^\lambda$ take the following one-loop approximate form

$$\mathcal{P}^\parallel \simeq -\frac{\alpha}{3\pi} k_\parallel^2, \qquad \mathcal{P}^\perp \simeq -\frac{2\alpha}{\pi}\, eB\, H\!\left(\frac{4m^2}{k_\parallel^2}\right) \tag{3}$$

where

$$H(z) = \frac{z}{\sqrt{z-1}} \arctan \frac{1}{\sqrt{z-1}} - 1, \; z > 1,$$
$$H(z) = -\frac{1}{2}\left(\frac{z}{\sqrt{1-z}} \ln \frac{1+\sqrt{1-z}}{1-\sqrt{1-z}} + 2 + i\pi \frac{z}{\sqrt{1-z}}\right), \; z < 1.$$

One can consider the eigenvalues of the polarization operator as a photon effective masses squared, $m_{eff}^2$. There is the effective mass of photon which defines the kinematics of the photon splitting. This process is kinematically allowed only when one of the final photons of the $\perp$-mode is created in the kinematic region $k_\parallel^2 < 4m^2$. Realy, if all photons are in the region $k_\parallel^2 > 4m^2$ the limit of the collinear kinematic is realized because of the relatively small ($|m_{eff}^2| \ll \omega^2$) effective masses of the photons in this region. The photon of the $\perp$-mode aquares positive and the photon of the $\|$-mode negative effective mass squared. It means, that only $\perp$-mode could split. On the other hand it is known, see [2], that only transition $\| \to \perp\perp$ is allowed in the collinear kinematic. Therefore the photon splitting is forbidden when $k_\parallel^2 > 4m^2$ for all photons in the procces. If the final photon of the $\perp$-mode is created in the region $k_\parallel^2 < 4m^2$ the kinematic of photon splitting could be far from collinearity. This is due to the fact that the photon of this mode aquares significant effective mass, $m_{eff}^2 < 0$, $|m_{eff}^2| \sim \omega^2$, in this region. Moreover, the large value of the polarization operator near the resonance leads to the neccessity of taking acount a large radiative corrections which reduce to renormalization of photon wave-function.

$$\varepsilon_\alpha^{(\lambda)} \to \varepsilon_\alpha^{(\lambda)} \sqrt{Z_\lambda}, \quad Z_\lambda^{-1} = 1 - \frac{\partial \mathcal{P}^\lambda}{\partial k_\parallel^2}, \quad \lambda = \parallel, \perp \tag{4}$$

## 2 The Amplitude of the Process $\gamma \to \gamma\gamma$ in the Strong Magnetic Field.

The process of the photon splitting in the magnetic field is depicted by two Feinman diagrams, see the Fig.1. The amplitude of this process can be written in the following form

$$\begin{aligned} \mathcal{M} &= e^3 \int d^4X \, d^4Y \, Sp\{\hat{\varepsilon}(k)\hat{S}(Y)\hat{\varepsilon}(k'')\hat{S}(-X-Y)\hat{\varepsilon}(k')\hat{S}(X)\} \times \\ &\times \; e^{-ie\,(XFY)/2} \, e^{i(k'X - k''Y)} + (\varepsilon(k'), k' \leftrightarrow \varepsilon(k''), k''), \\ X &= z - x, \, Y = x - y \end{aligned} \tag{5}$$

where $k_\alpha = (\omega, \mathbf{k})$ is the 4-vector of the momentum of initial photon with the polarization vector $\varepsilon_\alpha$, $k'$ and $k''$ are the 4-momenta of final photons, S(X) is the electron propagator in the magnetic field [7]. In the case of the strong field it is advantegeous to use the asymptotic expression of the electron propagator which could be presented in the following form

$$\hat{S}(X) \simeq S_a(X) = \frac{ieB}{2\pi} \exp(-\frac{eBX_\perp^2}{4}) \int \frac{d^2p}{(2\pi)^2} \frac{(p\gamma)_\parallel + m}{p_\parallel^2 - m^2} \Pi_- e^{-i(pX)_\parallel}, \tag{6}$$

$$d^2p = p_0 p_3, \quad \Pi_- = \frac{1}{2}(1 - i\gamma_1\gamma_2).$$

Substituting this in Eq. (5) and integrating, one would expect to obtain the amplitude which depends linearly on the field strength, namely, as $B^3/B^2$, where $B^2$ in the denominator arises from the integration over $d^2X_\perp d^2Y_\perp$. However, two parts of the amplitude (5) cancel each other exactly. Thus, the asymptotic form of the electron propagator (6) only shows that the linear-on-field part of the amplitude is zero and provides no way of extracting the next term of expansion over the field strength.

As the analysis shows, that could be done by the insertion of two asymptotic (6) and one exact propagator $\hat{S}(x)$ into the amplitude (5), with all interchanges. It is worthwhile now to turn from the general amplitude (5) to the partial amplitudes corresponding to definite photon modes, $\|$ and $\perp$, which are just the stationary photon states with definite dispersion relations in a magnetic field. There are 6 independent amplitudes and only two of them are of physical interest. We have obtained the following expressions, to the terms of order $1/B$

$$\mathcal{M}_{\| \to \| \perp} = i4\pi \left(\frac{\alpha}{\pi}\right)^{3/2} \frac{(k'\varphi k'')(k'\tilde{\varphi} k'')}{[(k')^2_{\|}(k'')^2_{\perp} k^2_{\perp}]^{1/2}} H\left(\frac{4m^2}{(k')^2_{\|}}\right), \tag{7}$$

$$\begin{aligned}\mathcal{M}_{\| \to \perp \perp} &= i4\pi \left(\frac{\alpha}{\pi}\right)^{3/2} \frac{(k'\widetilde{\Lambda} k'')}{[(k')^2_{\|}(k'')^2_{\|} k^2_{\perp}]^{1/2}} \left\{(k\Lambda k'')H\left(\frac{4m^2}{(k')^2_{\|}}\right)\right. \\ &+ \left.(k\Lambda k')H\left(\frac{4m^2}{(k'')^2_{\|}}\right)\right\},\end{aligned} \tag{8}$$

As for remaining amplitudes, we note that $\mathcal{M}_{\| \to \| \|}$ is equal to zero in this approximation. On the other hand, the photon of the $\perp$ mode due to its dispersion can split into two photons only in the kinematic region $k^2_{\|} > 4m^2$ where the tree-channel $\gamma_\perp \to e^+ e^-$ [8] strongly dominates.

## 3 The Probability of the Photon Splitting.

Although the process involves three particles, its amplitude is not a constant, because it contains the external field tensor in addition to the photon 4-momenta. The general expression for the splitting probability can be written in the form

$$\begin{aligned}W_{\lambda \to \lambda' \lambda''} &= \frac{g}{32\pi^2 \omega} \int |\mathcal{M}_{\lambda \to \lambda' \lambda''}|^2 Z_\lambda Z_{\lambda'} Z_{\lambda''} \times \\ &\times \delta(\omega_\lambda(\mathbf{k}) - \omega_{\lambda'}(\mathbf{k}') - \omega_{\lambda''}(\mathbf{k} - \mathbf{k}')) \frac{d^3k'}{\omega_{\lambda'}\omega_{\lambda''}},\end{aligned} \tag{9}$$

where the factor $g = 1 - \frac{1}{2}\delta_{\lambda'\lambda''}$ is inserted to account for possible identity of the final photons. The factors $Z_\lambda$ account for the large radiative corrections which reduce to the wave-function renormalization of a real photon with definite dispersion $\omega = \omega_\lambda(\mathbf{k})$. The integration over phase space of two final photons in Eq. (9) has to be performed using the photon energy dependence on the momenta, $\omega = \omega_\lambda(\mathbf{k})$, which can be found from the dispersion equations (1). A calculation of the splitting probability (9) is rather complicated in the general case. In the limit $m^2 \ll \omega^2 \sin^2\theta \ll eB$, where $\theta$ is an angle between the initial photon momentum $\mathbf{k}$ and the magnetic field direction, we derive the following analytical expression for the probability of the channel $\gamma_\| \to \gamma_\| \gamma_\perp$:

$$
\begin{aligned}
W_{\|\to\|\perp} &\simeq \frac{\alpha^3 \omega \sin^2\theta}{16}(1-x)[1 - x + 2x^2 + 2(1-x)(1+x)^2 \ln(1+x) - \\
&- 2x^2 \frac{2-x^2}{1-x} \ln\frac{1}{x}], \qquad x = \frac{2m}{\omega \sin\theta} \ll 1. \qquad (10)
\end{aligned}
$$

Within the same approximation we obtain the spectrum of final photons in the frame where the initial photon momentum is orthogonal to the field direction:

$$
\begin{aligned}
&\frac{dW_{\|\to\|\perp}}{d\omega'} \simeq \frac{\alpha^3}{2} \cdot \frac{\sqrt{(\omega-\omega')^2 - 4m^2}}{\omega' + \sqrt{(\omega'-\omega)^2 - 4m^2}}, \qquad (11)\\
&\frac{\omega}{2} - \frac{2m^2}{\omega} < \omega' < \omega - 2m,
\end{aligned}
$$

where $\omega, \omega'$ are the energies of the initial and final photons of the $\|$ mode.

We have made numerical calculations of the process probabilities for both channels, which are valid in the limit $\omega^2 \sin^2\theta \ll eB$. Our results are represented in Figs. 2,3. The photon splitting probabilities below and near the pair-creation threshold are depicted in Fig. 2. In this region the channel $\gamma_\| \to \gamma_\perp \gamma_\perp$ (allowed in the collinear limit) is seen to dominate the channel $\gamma_\| \to \gamma_\| \gamma_\perp$ (forbidden in this limit). For comparison we show here the probability obtained without considering the noncollinearity of the kinematics and radiative corrections (the dotted line *3*) which is seen to be inadequate. For example, this probability becomes infinite just above the threshold. As is seen from Fig. 3, both channels give essential contributions to the probability at high photon energies, with the "forbidden" channel dominating. It should be stressed that taking account of the photon polarization leads to the essential dependence of the splitting probabilities on the magnetic field, while the amplitudes (7), (8) do not depend on the field strength value.

# 4 Conclusions

- The collinear limit is inadequate approximation for the photon splitting $\gamma \to \gamma\gamma$ in a strong magnetic field ($B \gg B_e$), because of the significant deviation of the photon dispersion in the strong field from the vacuum dispersion.

- The "allowed" channel $\gamma_{\parallel} \to \gamma_{\perp}\gamma_{\perp}$ is not comprehensive for the splitting in the strong field. The "forbiden" channel $\gamma_{\parallel} \to \gamma_{\parallel}\gamma_{\perp}$ is also essential, moreover, it dominates at high energies of the initial photon.

- The partial amplitudes and the splitting probabilities are calculated in the strong field limit for both channels. The amplitudes do not depend on the field strength value, while the probabilities do depend essentially, due to the photon polarization in the strong magnetic field.

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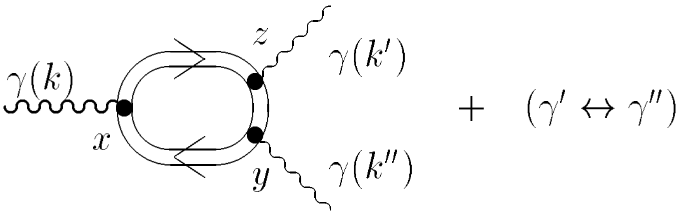


Figure 1: The Feynman diagram for photon splitting in a magnetic field. The double line corresponds to the exact propagator of an electron in an external field.

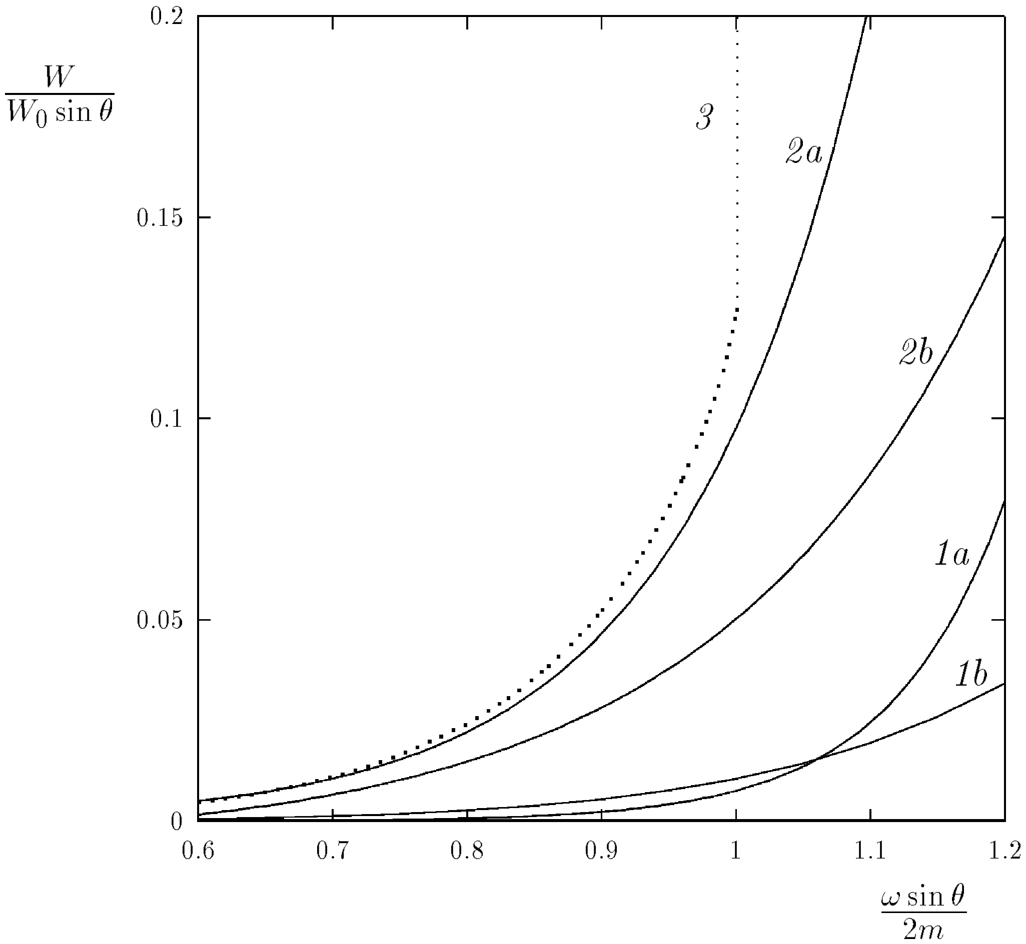


Figure 2: The dependence of the probability of photon splitting $\gamma \to \gamma\gamma$ on energy, below and near the pair-creation threshold: *1a*, *1b* – for the "forbidden" channel $\gamma_{\|} \to \gamma_{\|}\gamma_{\perp}$ and for the magnetic field strength $B = 10^2\, B_e$ and $10^3\, B_e$, correspondingly; *2a*, *2b* – for the "allowed" channel $\gamma_{\|} \to \gamma_{\perp}\gamma_{\perp}$ for the field strength $B = 10^2\, B_e$ and $10^3\, B_e$; *3* – for the channel $\gamma_{\|} \to \gamma_{\perp}\gamma_{\perp}$ in the collinear limit without taking account of large radiative corrections. Here $W_0 = (\alpha/\pi)^3\, m$.

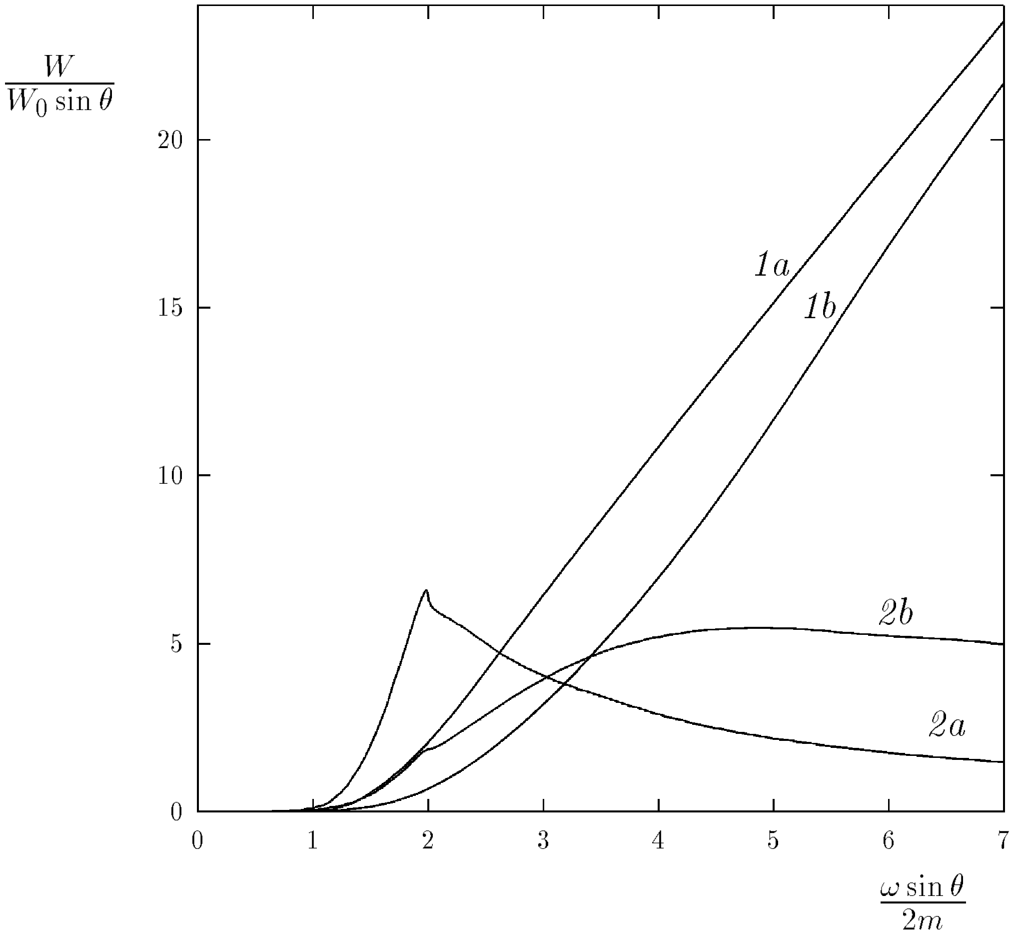


Figure 3: The probability of photon splitting above the pair-creation threshold: *1a*, *1b* – for the "forbidden" channel $\gamma_\parallel \to \gamma_\parallel \gamma_\perp$ and for the magnetic field strength $B = 10^2\, B_e$ and $10^3\, B_e$, correspondingly; *2a*, *2b* – for the "allowed" channel $\gamma_\parallel \to \gamma_\perp \gamma_\perp$ for the field strength $B = 10^2\, B_e$ and $10^3\, B_e$.