On the role of Lorentz-covariance in hybrid chiral quark bag models

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Abstract

The structure of surface terms for hybrid chiral quark bag models in (1+1)D is studied by means of the method of covariant group variables. The analytical connection between fields from both sides of the bag boundary following from the specific structure of invariant dynamics is considered. It is shown that in general case such connection imposes essential constraints on the possible field dynamics.

1 Introduction

In this paper we study the structure of surface terms in the (1+1)D 2-phase field theories with boundaries within the framework of the covariant generalization of the Bogoliubov's collective coordinate method. The total action

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of a hybrid quark bag model [1, 2] contains three terms

$$S_{total} = S_{in} + S_{out} + S_{on} \ . \tag{1}$$

Here $S_{in} = \int dx \ \mathcal{L}_F(x)\theta(V)$ is the action of the fermions confined inside the bag volume V (the phase of asymptotic freedom), $S_{out} = \int dx \ \mathcal{L}_B(x)\theta(\bar{V})$ is the effective non-linear theory (e.g. the Skyrme model, [3]) which describes the meson cloud around the bag (the hadronization phase). Besides this, one needs to introduce an additional surface action $S_{on} = \int dx \ \mathcal{L}_{FB}(x)\delta(\sigma)$, which describes the chiral quark-meson coupling on the boundary surface σ of the bag, while in the interior as well as in the exterior of the bag the phases are decoupled [1, 2].

Although hybrid models show up a remarkable success in describing the low-energy hadron physics [2], they suffer from the principal problem of explicit breaking of the space-time symmetry. Namely, the total action (1) contains explicit $\theta-$ and $\delta-$ functions of space-time variables, which divide the bag space into the interior and exterior parts.

In this paper we'll show that the careful account of the relativistic covariance from the very beginning leads to a number of conclusions concerning the general treatment of hybrid models, that cannot be obtained in any finite order of perturbative expansion [4]. The main tool is to introduce the boundary between phases in the center-of-mass system of the bag, rather than in the laboratory system as the standard approach prescribes. Such a covariant division allows us to take correctly into account any non-trivial classical object which has to be included in the quantized theory. The specific finite-difference structure of the dynamics in the invariant c.m.s. space [5] leads to surface terms and corresponding surface action in the form of contour integrals of fields, considered as analytical functions of their complex spatial argument. In turn, this circumstance implies, that the boundary conditions on fields should be imposed on these contours in the complex plane, rather than on the physical real boundary between the phases. The most important point here is that due to the requirement of analyticity of fields these conditions are actually nothing else, but the condition of analytical continuation between the phases. The direct consequence of such analytical connection is the existence of chirally invariant relation for the dynamics of fields in different phases, which is consistent with bosonization procedure, but in more general situation implies some strict predictions for the form of possible interaction terms in both sides of the boundary.

2 Lorentz-covariance in two-phase theories

To avoid unnecessary complications we study the bag model in (1+1)D, the generalization to higher dimensions can be performed directly. The nontrivial classical objects which commute with any unitary transformation of the Poincare group are the bag surface, which is an explicit c-number function of the space-time variables, and the external soliton-like solution. For a consistent treatment of such objects in the quantum theory we apply the method of covariant group variables [5] using the QFT analogies of the relativistic Newton-Wigner center-of-mass coordinate operators [6]. These operators enable the bag with the surface and meson cloud to move as a whole. Let the (1+1)D Poincare algebra be

$$[H,P]=0, \quad [L,P]=i\hbar H, \quad [L,H]=i\hbar P \ , \tag{2}$$

where $\hbar$ is the effective Plank constant, then the NW coordinate is given by

$$q=\frac{1}{2}\{H^{-1},L\}_{+} \ . \tag{3}$$

By definition the operator q plays the role of the c.m.s. coordinate and so enters the fields arguments only in the combination $x-q(t)$ [7]. It is convenient to use the following substitution

$$x \to \xi(x,t) \ , \tag{4}$$

where

$$\xi(x,t)=Hx-Pt-L=H^{1/2}(x-q(t))H^{1/2} \ . \tag{5}$$

Recall, that the replacement (4) follows also from the direct evaluation of the secular terms in the perturbation series around static classical solution [7]. The dimensionless coordinate $\xi(x,t)$ behaves like a real covariant scalar field under the Poincare group transformations, so any c-number function of ξ will also be a covariant scalar. This feature is the starting point for the Lorentz-covariant description of 2-phase field theories with nontrivial boundaries for configurations, which are static in the c.m.s. of the bag in the leading approximation.

Firstly we consider the surface terms for the scalar field, described by the Lagrangian density

$$\mathcal{L}=\tfrac{1}{2}\ \partial_\mu\varphi\partial^\mu\varphi-U(\varphi) \ , \tag{6}$$

which is stationary in the c.m.s. The corresponding action in terms of the invariant c.m.s. variable $\varphi(\xi)$ is obtained by means of evaluating of matrix element between the states with fixed energy-momentum and using the expansion $1 = \int d\xi \ |\xi\rangle\langle\xi|$ [8]. One finds

$$S[\varphi] = \int \frac{d\xi}{M} \ \left\{ \frac{M^2}{4\hbar^2} \left([\varphi_+ - \varphi]^2 + [\varphi_- - \varphi]^2 \right) - U(\varphi) \right\} \ , \tag{7}$$

where $\varphi_\pm = \varphi(\xi \pm i\hbar)$. The variation of this action with respect to the field φ gives

$$\delta S = \int \frac{d\xi}{M} \ \delta\varphi \ \left\{ \frac{M^2}{\hbar^2} [2\varphi - \varphi_+ - \varphi_-] - U'(\varphi) \right\} +$$

$$+ \frac{M}{2\hbar^2} \ \int d\xi \ \left\{ \delta\varphi_+ [\varphi_+ - \varphi] + \delta\varphi_- [\varphi_- - \varphi] + \delta\varphi [\varphi_+ + \varphi_- - 2\varphi] \right\} \ . \tag{8}$$

The first term in (8) is nothing else, but the correct field equation of motion $\Box\varphi + U'(\varphi) = 0$, written in terms of the invariant field, while the second one is the surface term $\delta S_\sigma[\varphi]$. Consider now the scalar field ϕ defined in a region with a boundary, e.g. the segment $[-\infty, \xi_0]$. Due to the analyticity of $\phi(z)$ in the strip $|\mathrm{Im} \ z| \leq \hbar$, what is shown in [8] to be crucial for the fields to have a smooth classical limit, we can express the surface term as a contour integral over the imaginary segment in the complex plane

$$\delta S_\sigma[\phi] = \frac{M}{2\hbar^2} \ \Big(\int\limits_{\xi_0}^{\xi_0 + i\hbar} dz \ \delta\phi \, [\phi - \phi_-] + \int\limits_{\xi_0}^{\xi_0 - i\hbar} dz \ \delta\phi \, [\phi - \phi_+] \Big) \ . \tag{9}$$

In the classical limit $\hbar \to 0$ one finds $\delta S_\sigma[\phi] \to -M\phi'(\xi_0)\delta\phi$, i.e. the surface term, which arises from the variation of the kinetic term of the classical action $S_{cl} = -\frac{1}{2} \ \int^{x_0} dx \ \phi'^2(x)$, provided $\xi \to Mx$.

3 Boundary conditions for the chiral bag

Now we apply the results obtained above to the hybrid bag model. The covariant fermion field is given by [5]

$$\psi(x,t) = A \ \chi(\xi(x,t)) \ , \tag{10}$$

where $A = \exp(\alpha\theta/2)$, $\theta = \mathrm{arcth} P/H$ — the total rapidity operator, and $\alpha = \gamma^0\gamma^1$ is a Dirac matrix. The additional factor A provides the transformation of the laboratory field $\psi(x,t)$ according to the spinor representation under the Lorentz transformations. Therefore $\chi(\xi)$ is the invariant fermion field in the c.m.s. of the bag. The variation of the kinetic part of the fermionic action $i\bar{\psi}\hat{\partial}\psi$ in terms of the invariant variables $\chi(\xi)$ with respect to χ, provided by the validity of the Dirac equation of motion inside the bag, which is now the segment $[-\xi_0, \xi_0]$, yields the fermionic surface term, which can also be presented as a sum of contour integrals over the boundary segments [4]. Therefore the surface action, which has to be included in the total action of boson and fermion field, should be taken in the form $S_{on} = S_{on}(\xi_0) + S_{on}(-\xi_0)$ (taking into account the chiral invariance, the renormalizability and the correct limit $\hbar \to 0$), where

$$S_{on}(\xi) = \frac{1}{2i\hbar}\Big\{ \int\limits_{\xi}^{\xi+i\hbar} dz\ e^{-i\lambda(\phi+\phi_-)/2}\tilde{\chi}_R\chi_L - \int\limits_{\xi}^{\xi-i\hbar} dz\ e^{i\lambda(\phi+\phi_+)/2}\tilde{\chi}_L\chi_R\Big\} , \quad (11)$$

where $\phi_\pm = \phi(z \pm i\hbar)$, $\gamma_5 = \gamma^1\gamma^0$, $\chi_{R,L}$ are the right- and left-hand chiral fermions, $\gamma_5\chi_{R,L} = \pm\chi_{R,L}$, λ is the constant of the chiral coupling and the factor $1/2i\hbar$ provides the correct classical limit. The tilded fermion fields $\tilde{\chi}$ denote the analytical continuation of Dirac conjugated fields $\bar{\chi}$ from the real axis in the complex $\xi-$space.

The variation of the total action of the bag (1), which contains the fermion, the boson and the fermion-boson surface parts, provided by the validity of the equations of motion in the interior as well as exterior sector, yields the following boundary conditions

$$(\gamma^0 - \gamma^1)\chi_- = ie^{-i\lambda(\phi+\phi_-)/2}(1-\gamma_5)\chi \quad (12)$$

for fermions, and

$$2M(\phi - \phi_-) = \hbar\lambda\left[e^{i\lambda(\phi+\phi_-)/2}\,(\tilde{\chi}_L\chi_R)_- - e^{-i\lambda(\phi+\phi_-)/2}\tilde{\chi}_R\chi_L\right] \quad (13)$$

for bosons on the segment $z \in [\xi_0, \xi_0 + i\hbar]$. The conditions on the segment $z \in [\xi_0, \xi_0 - i\hbar]$ can be obtained via the Schwartz reflection [4]. In order to include the contribution of the sea quarks as well as valence ones, one must take the vacuum expectations of bilinear fermionic forms in the r.h.s. of (13).

The eqs. (12,13) are the chirally invariant nonlocal boundary conditions which connect the fields from inside and outside the bag on the complex segments $[\xi_0, \xi_0 \pm i\hbar]$. Due to the uniqueness of the analytical continuation the relations (12,13), which were originally determined on the finite segments $[\xi_0, \xi_0 \pm i\hbar]$ in the complex ξ-plane, should now be extended to the whole area of the field analyticity, what imposes strong constraints on the possible dynamics of fields inside as well as outside the bag.

To demonstrate the role of these constraints, let the boson self-interaction be $U(\phi)$ and $\phi(\xi)$ be the odd topological soliton, which to the leading order in $\hbar$ obeys the equation $-\phi'' + U'(\phi) = 0$ $(x = \xi/M)$. The generic fermion dynamics is determined by the potential $V(A, B)$, where $A = \bar{\chi}\chi$, $B = -i\bar{\chi}\gamma_5\chi$. Then we find from the boundary conditions (12,13) the following connection between fermionic and bosonic potential for the given energy E of a fermionic mode inside the bag

$$V(A,B) = U\left(\frac{1}{\lambda}\ \arctan\frac{A}{B}\right) + E\sqrt{C} - \frac{\lambda^2}{8}C\ , \tag{14}$$

where $C = A^2 + B^2$ is actually the Thirring interaction of fermionic currents $(\bar{\chi}\gamma^\mu\chi)^2$. Note, that this non-linear chirally invariant connection cannot be obtained in any finite order of the perturbative expansion in powers of coupling constant λ. Nevertheless, (14) reflects the situation correctly, what can be easily checked via (1+1)D bosonization [9], what yields $V(\bar{\chi}, \chi) \to U(\phi) + \text{Const}$. In other words, in terms of bosonization the dynamics of fermions described by (14) and the bosonic self-interaction $U(\phi)$ are equivalent.

4 Fermion charge of the bag

Now we show the role of such analytical connection between the exterior and interior regions calculating the total fermion number of the bag in terms of the invariant fields. It is well known that in a hybrid chiral bag the total fermion number is shared between the interior fermions and the topological soliton of the meson theory, where it corresponds to the topological charge [1]. In the standard approach for the bag with a static boundary this fact arises as the result of the special regularization of the secondary quantized fermionic current $j^\mu(x) = \bar{\psi}(x)\gamma^\mu\psi(x)$ and the local boundary conditions [1].

The rate of the charge leakage through the surface appears to be proportional to the time derivative of the scalar field ϕ on the surface. Let us consider now the total fermion number in terms of invariant fields. The vector current j^μ reads

$$j^\mu = \frac{1}{2M}\left[\bar{\chi}(\gamma^\mu - \epsilon^{\mu\nu}\gamma_\nu)\chi_+ K_+ + \bar{\chi}(\gamma^\mu + \epsilon^{\mu\nu}\gamma_\nu)\chi_- K_-\right] , \qquad (15)$$

where $K_\pm = H \pm P$. Note, that this current is a well-defined operator due to the imaginary shifts of the arguments. The calculation of the total fermion number $Q_F = \int dx\ j^0(x)$ in terms of invariant fields is described in detail in [4]. The final result is

$$Q_F[\chi] = \int \frac{d\xi}{2M}\ \left[\bar{\chi}(\gamma^0 + \gamma^1)\chi_+ + \bar{\chi}(\gamma^0 - \gamma^1)\chi_-\right] . \qquad (16)$$

As in (13), the average of the bilinear fermionic forms over corresponding vacuum state is implied.

For a 2-phase system under consideration the integration in (18) has to be performed over the fermionic region, i. e. over the segment $[-\xi_0, \xi_0]$. Using the boundary conditions (13), analytically continued to the interior sector of the bag, we immediately find the total fermion number in terms of the boson field only

$$Q_F = \frac{1}{i\hbar\lambda}\int_{-\xi_0}^{+\xi_0} d\xi\ (\phi_+ - \phi_-) . \qquad (17)$$

The limit $\hbar \to 0$ gives

$$Q_F = \frac{1}{i\hbar\lambda}\int_{-\xi_0}^{+\xi_0} d\xi\ 2i\hbar\ \partial_\xi \phi = \frac{2}{\lambda}\ \left[\phi(\xi_0) - \phi(-\xi_0)\right] , \qquad (18)$$

what coincides with the expression for the vector anomaly of the standard approach, obtained via special regularization of the fermionic current [1]. Let us emphasize, that in this approach the vector anomaly appears to be the purely quantum mechanical effect caused by the finite-difference structure of the dynamics of invariant fields and the boundary conditions, rather than by second quantization.

To conclude, we have studied the generic structure of surface terms and boundary conditions for the hybrid chiral quark bag models in (1+1)D within a covariant treatment of the center-of-mass motion. It is shown that the correct account of the relativistic covariance for the field theories in the closed space leads to rather strong constraints on the dynamics compared to the standard approach. These constraints are trivial in the case of the exactly equivalent (in terms of bosonization) theories, such as the sin-Gordon and Thirring models, due to validity of the Cheshire Cat principle [1], but in a more general situation might lead to strong limitations on the possible field dynamics in 2-phase field theories with a boundary.

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