Towards the Observation of Signal over Background in Future Experiments

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Abstract

We propose a method to estimate the probability of new physics discovery in future high energy physics experiments. Physics simulation gives both the average numbers $< N_b >$ of background and $< N_s >$ of signal events. We find that the proper definition of the significance for $< N_b >, \ < N_s > \ \gg 1$ is $S_{12} = \sqrt{< N_s > + < N_b >} - \sqrt{< N_b >}$ in comparison with often used significances $S_1 = \dfrac{< N_s >}{\sqrt{< N_b >}}$ and $S_2 = \dfrac{< N_s >}{\sqrt{< N_s > + < N_b >}}$. We also propose a method for taking into account the systematical errors related to nonexact knowledge of background and signal cross sections. An account of such systematics is very essential in the search for supersymmetry at LHC.

1 Introduction

One of the common goals in the forthcoming experiments is the search for new phenomena. In the forthcoming high energy physics experiments (LHC, TEV22, NLC, ...) the main goal is the search for physics beyond the Standard Model (supersymmetry, Z'-, W'-bosons, ...) and the Higgs boson discovery as a final confirmation of the Standard Model. In estimation of the discovery potential of the future experiments (to be specific in this paper we shall use

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as an example CMS experiment at LHC [1]) the background cross section is calculated and for the given integrated luminosity L the average number of background events is $< N_b >= \sigma_b \cdot L$. Suppose the existence of a new physics leads to the nonzero signal cross section σ_s with the same signature as for the background cross section that results in the prediction of the additional average number of signal events $< N_s >= \sigma_s \cdot L$ for the integrated luminosity L.

The total average number of the events is $< N_{ev} >=< N_s > + < N_b >= (\sigma_s + \sigma_b) \cdot L$. So, as a result of new physics existence, we expect an excess of the average number of events. In real experiments the probability of the realization of n events is described by Poisson distribution [2]

$$f(n, < n >) = \frac{< n >^n}{n!} e^{-<n>}. \tag{1}$$

Here $< n >$ is the average number of events.

Remember that the Poisson distribution $f(n, < n >)$ gives [3] the probability of finding exactly n events in the given interval of (e.g. space and time) when the events occur independently of one another and of x at an average rate of $< n >$ per the given interval. For the Poisson distribution the variance σ^2 equals to $< n >$. So, to estimate the probability of the new physics discovery we have to compare the Poisson statistics with $< n >=< N_b >$ and $< n >=< N_b > + < N_s >$. Usually, high energy physicists use the following "significances" for testing the possibility to discover new physics in an experiment:

(a) "significance" $S_1 = \dfrac{N_s}{\sqrt{N_b}}$ [4],

(b) "significance" $S_2 = \dfrac{N_s}{\sqrt{N_s + N_b}}$ [5, 6].

A conventional claim is that for S_1 (S_2) ≥ 5 we shall discover new physics (here, of course, the systematical errors are ignored). For $N_b \gg N_s$ the significances S_1 and S_2 coincide (the search for Higgs boson through the $H \rightarrow \gamma\gamma$ signature). For the case when $N_s \sim N_b$, S_1 and S_2 differ. Therefore, a natural question arises: what is the correct definition for the significance S_1, S_2 or anything else ?

It should be noted that there is a crucial difference between "future" experiment and the "real" experiment. In the "real" experiment the total number of events N_{ev} is a given number (already has been measured) and we compare it with $< N_b >$ when we test the validity of the standard physics. So, the number of possible signal events is determined as $N_s = N_{ev} - < N_b >$ and it is compared with the average number of background events $< N_b >$. The fluctuation of the background is $\sigma_{fb} = \sqrt{N_b}$, therefore, we come to the S_1 significance as the measure of the distinction from the standard physics. In the conditions of the "future" experiment when we want to search for new physics, we know only the average number of the background events and the average number of the signal events, so we have to compare the Poisson distributions $P(n, < N_b >)$ and $P(n, < N_b > + < N_s >)$ to determine the probability to find new physics in the future experiment.

In this paper we estimate the probability to discover new physics in future experiments. We show that for $< N_s >, \ < N_b > \gg 1$ the proper determination of the significance is $S = \sqrt{< N_b > + < N_b >} - \sqrt{< N_b >}$. We also suggest a method which takes into account systematic errors related to nonexact knowledge of the signal and background cross sections.

The organization of the paper is the following. In the next section we give a method for the determination of the probability to find new physics in the future experiment and calculate the probability to discover new physics for the given $(< N_b >, \ < N_s >)$ numbers of background and signal events under the assumption that there are no systematic errors. In section 3 we estimate the influence of the systematics related to the nonexact knowledge of the signal and background cross sections on the probability to discover new physics in future experiments. Section 4 contains the concluding remarks.

2 An analysis of statistical fluctuations

Suppose that for some future experiment we know the average number of the background and signal event $< N_b >, \ < N_s >$. As it has been mentioned in the Introduction, the probability of realization of n events in an experiment is given by the Poisson distribution

$$P(n, < n >) = \frac{< n >^n}{n!} e^{-< n >}, \tag{2}$$

where $< n >=< N_b >$ for the case of the absence of new physics and $< n >=< N_b > + < N_s >$ for the case when new physics exists. So, to determine the probability to discover new physics in future experiment, we have to compare the Poisson distributions with $< n >=< N_b >$ (standard physics) and $< n >=< N_b > + < N_s >$ (new physics).

Consider, at first, the case when $< N_b > \quad \gg 1$, $< N_s > \quad \gg 1$. In this case the Poisson distributions approach the Gaussian distributions

$$P_G(n, \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(n-\mu)^2}{2\sigma^2}}, \tag{3}$$

with $\mu = \sigma^2$ and $\mu =< N_b >$ or $\mu =< N_b > + < N_s >$. Here n is a real number.

The Gaussian distribution describes the probability density to realize n events in the future experiment provided the average number of events $< n >$ is a given number. In Fig.1 we show two Gaussian distributions P_G with $< n >=< N_b >= 53$ and $< n >=< N_b > + < N_s > = 104$ ([6], Table.13, cut 6). As is clear from Fig.1 the common area for these two curves (the first curve shows the "standard physics" events distribution and the second one gives the "new physics" events distribution) is the probability that "new physics" can be described by the "standard physics". In other words, suppose we know for sure that new physics takes place and the probability density of the events realization is described by curve II ($f_2(x)$). The probability κ that the "standard physics" (curve I ($f_1(x)$)) can imitate new physics (i.e. the probability that we measure "new physics" but we think that it is described by the "standard physics") is described by common area of curve I and II.

Numerically, we find that

$$\begin{aligned}
\kappa &= \frac{1}{\sqrt{2\pi}\sigma_2}\int_{-\infty}^{\sigma_1\sigma_2} exp[-\frac{(x-\sigma_2^2)^2}{2\sigma_2^2}]dx + \frac{1}{\sqrt{2\pi}\sigma_1}\int_{\sigma_1\sigma_2}^{\infty} exp[-\frac{(x-\sigma_1^2)^2}{2\sigma_1^2}]dx \\
&= \frac{1}{\sqrt{2\pi}}[\int_{-\infty}^{\sigma_1-\sigma_2} exp[-\frac{y^2}{2}]dy + \int_{\sigma_2-\sigma_1}^{\infty} exp[-\frac{y^2}{2}]dy] \\
&= 1 - erf(\frac{\sigma_2-\sigma_1}{\sqrt{2}}).
\end{aligned} \tag{4}$$

Here $\sigma_1 = \sqrt{N_b}$ and $\sigma_2 = \sqrt{N_b + N_s}$.

As follows from formula (4) the role of the significance S plays

$$S_{12} = \sigma_2 - \sigma_1 = \sqrt{N_b + N_s} - \sqrt{N_b}. \qquad (5)$$

Note that in refs.[7] the following criterion of the signal discovery has been used. The signal was assumed to be observable if $(1-\epsilon)\cdot 100\%$ upper confidence level for the background event rate is equal to $(1-\epsilon)\cdot 100\%$ lower confidence level for background plus signal ($\epsilon = 0.01 - 0.05$). The corresponding significance is similar to our significance S_{12}. The difference is that in our approach the probability density κ that new physics is described by standard physics is equal to 2ϵ.

It means that for $S_{12} = 1, 2, 3, 4, 5, 6$ the probability κ is correspondingly $\kappa = 0.31, 0.041, 0.0027, 6.3 \cdot 10^{-5}, 5.7 \cdot 10^{-7}, 2.0 \cdot 10^{-7}$ in accordance with a general picture. As it has been mentioned in the Introduction two definitions of the significance are mainly used in the literature: $S_1 = \dfrac{N_s}{\sqrt{N_b}}$ [4] and $S_2 = \dfrac{N_s}{\sqrt{N_s + N_b}}$ [5]. The significance S_{12} is expressed in terms of the significances S_1 and S_2 as $S_{12} = \dfrac{S_1 S_2}{S_1 + S_2}$.

For $N_b \gg N_s$ (the search for Higgs boson through $H \to \gamma\gamma$ decay mode) we find that

$$S_{12} \approx 0.5\ S_1 \approx 0.5\ S_2. \qquad (6)$$

It means that for $S_1 = 5$ (according to a common convention the 5σ confidence level means a new physics discovery) the real significance is $S_{12} = 2.5$, that corresponds to $\kappa = 1.2\%$.

For the case $N_s = kN_b$, $S_{12} = k_{12} S_2$, where for $k = 0.5, 1, 4, 10$ the value of k_{12} is $k_{12} = 0.55, 0.59, 0.69, 0.77$. For not too high values of $< N_b >$ and $< N_b + N_s >$, we have to compare the Poisson distributions directly. Again for the Poisson distribution $P(n, < n >)$ with the area of definition for nonnegative integers we can define $P(x, < n >)$ for real x as

$$\tilde{P}(x, < n >) = \begin{cases} 0, & x \le 0, \\ P([x], < n >), & x > 0. \end{cases} \qquad (7)$$

It is evident that

$$\int_{-\infty}^{\infty} \tilde{P}(x, < n >) dx = 1. \qquad (8)$$

So, the generalization of the previous determination of κ in our case is straightforward, namely, κ is nothing but the common area of the curves described by $\tilde{P}(x, < N_b >)$ (curve I) and $\tilde{P}(x, < N_b > + < N_s >)$ (curve II) (see, Fig.2).

One can find that [1]

$$\kappa = \alpha + \beta,$$

$$\alpha = \sum_{n=0}^{n_0} \frac{(< N_b > + < N_s >)^n}{n!} e^{-(<N_b>+<N_s>)} = \frac{\Gamma(n_0+1, < N_s > + < N_b >)}{\Gamma(n_0+1)},$$

$$\beta = \sum_{n=n_0+1}^{\infty} \frac{(< N_b >)^n}{n!} e^{-<N_b>} = 1 - \frac{\Gamma(n_0+1, < N_b >)}{\Gamma(n_0+1)},$$

$$n_0 = [\frac{< N_s >}{ln(1+\frac{<N_s>}{<N_b>})}].$$

Numerical results are presented in Tables 1-6.

As it follows from these Tables for finite values of $< N_s >$ and $< N_b >$ the deviation from asymptotic formula (4) is essential. For instance, for $N_s = 5$, $N_b = 1$ $(S_1 = 5)$ $\kappa = 14\%$. For $N_s = N_b = 25$ $(S_1 = 5)$ $\kappa = 3.9\%$, whereas asymptotically for $N_s \gg 1$ we find $\kappa = 1.2\%$. Similar situation takes place for $N_s \sim N_b$.

3 An account of systematic errors related to nonexact knowledge of background and signal cross sections

In the previous section we determined the statistical error κ (the probability that "new physics" is described by "standard physics"). In this section we investigate the influence of the systematical errors related to a nonexact knowledge of the background and signal cross sections on the probability κ not to confuse a new physics with the old one.

Denote the Born background and signal cross sections as σ_b^0 and σ_s^0. An account of one loop corrections leads to $\sigma_b^0 \to \sigma_b^0(1+\delta_{1b})$ and $\sigma_s^0 \to \sigma_s^0(1+\delta_{1s})$, where typically δ_{1b} and δ_{1s} are O(0.5).

[1] We are indented to Igor Semeniouk for the help in the derivation of these formulae

Two loop corrections at present are not known. So, we can assume that the uncertainty related with nonexact knowledge of cross sections is around δ_{1b} and δ_{1s} correspondingly. In other words, we assume that the exact cross sections lie in the intervals $(\sigma^0_b, \sigma^0_b(1+2\delta_{1b}))$ and $(\sigma^0_s, \sigma^0_s(1+2\delta_{1s}))$. The average number of background and signal events lie in the intervals

$$(< N^0_b >, < N^0_b > (1+2\delta_{1b})) \tag{9}$$

and

$$(< N^0_s >, < N^0_s > (1+2\delta_{1s})), \tag{10}$$

where $< N^0_b >= \sigma^0_b \cdot L$, $< N^0_s >= \sigma^0_s \cdot L$.

To determine the probability that the new physics is described by the old one, we again have to compare two Poisson distributions with and without new physics but in distinction from Section 2 we have to compare the Poisson distributions in which the average numbers lie in some intervals. So, a priori the only thing we know is that the average numbers of background and signal events lie in the intervals (9) and (10), but we do not know the exact values of $< N_b >$ and $< N_s >$. To determine the probability that the new physics is described by the old, consider the worst case when we think that new physics is described by the minimal number of average events

$$< N^{min}_b >=< N^0_b > + < N^0_s > . \tag{11}$$

Due to the fact that we do not know the exact value of the background cross section, consider the worst case when the average number of background events is equal to $< N^0_b > (1+2\delta_{1b})$. So, we have to compare the Poisson distributions with $< n >=< N^0_b > + < N^0_s >=$
$< N^0_b > (1+2\delta_{1b}) + (< N^0_s > -2\delta_{1b} < N^0_b >)$ and $< n >=< N^0_b > (1+2\delta_{1b})$. Using the result of the previous Section, we find that for case $< N^0_b > \gg 1$, $< N^0_s > \gg 1$ the effective significance is

$$S_{12s} = \sqrt{< N^0_b > + < N^0_s >} - \sqrt{< N^0_b > (1+2\delta_{1b})}. \tag{12}$$

For the limiting case $\delta_{1b} \to 0$, we reproduce formula (5). For not too high values of $< N^0_b >$ and $< N^0_s >$, we have to use the results of the previous section (Tables 1-6).

As an example consider the case when $\delta_{1b} = 0.5$, $< N_s >= 100$, $< N_b >= 50$ (typical situation for sleptons search). In this case we find that

$$
\begin{aligned}
S_1 &= \frac{< N_s >}{\sqrt{< N_b >}} = 14.1, \\
S_2 &= \frac{< N_s >}{\sqrt{< N_s > + < N_b >}} = 8.2 \\
S_{12} &= \sqrt{< N_b > + < N_s >} - \sqrt{< N_b >} = 5.2, \\
S_{12s} &= \sqrt{< N_b > + < N_s >} - \sqrt{2 < N_b >} = 2.25.
\end{aligned}
$$

The difference between CMS adopted significance $S_2 = 8.2$ (that corresponds to the probability $\kappa = 0.206 \cdot 10^{-6}$) and the significance $S_{12s} = 2.25$ taking into account systematics related to nonexact knowledge of background cross section is factor 3.6 The direct comparison of the Poisson distributions with $< N_b > (1 + 2\delta_{1b}) = 100$ and $< N_b > (1 + 2\delta_{1b}) + < N_{s,eff} >$ ($< N_{s,eff} >=< N_s > -2\delta_{1b} < N_b >= 50$) gives $\kappa_s = 0.0245$.

Another example is with $< N_s >= 28$, $< N_b >= 8$ and $\delta_{1b} = 0.5$. For such example we have $S_1 = 9.9$, $S_2 = 4.7$, $S_{12} = 3.2$, $S_{12s} = 2.0$, $\kappa_s = 0.045$.

So, we see that an account of the systematics related to nonexact knowledge of background cross sections is very essential and it decreases the LHC SUSY discovery potential.

4 Conclusions

In this paper we determined the probability to discover the new physics in the future experiments when the average number of background $< N_b >$ and signal events $< N_s >$ is known. We have found that in this case for $< N_s > \gg 1$ and $< N_b > \gg 1$ the role of significance plays
$S_{12} = \sqrt{< N_b > + < N_s >} - \sqrt{< N_b >}$
in comparison with often used expressions for the significances $S_1 = \frac{< N_s >}{\sqrt{< N_b >}}$ and $S_2 = \frac{< N_s >}{\sqrt{< N_s > + < N_b >}}$.

For $< N_s > \ll < N_b >$ we have found that $S_{12} = 0.5 S_1 = 0.5 S_2$. For not too high values of $< N_s >$ and $< N_b >$, when the deviations from the Gaussian distributions are essential, our results are presented in Tables 1-6. We also proposed a method for taking into account systematical errors related to the nonexact knowledge of background and signal events. An account of such kind of systematics is very essential in the search for supersymmetry and leads to an essential decrease in the probability to discover the new physics in the future experiments.

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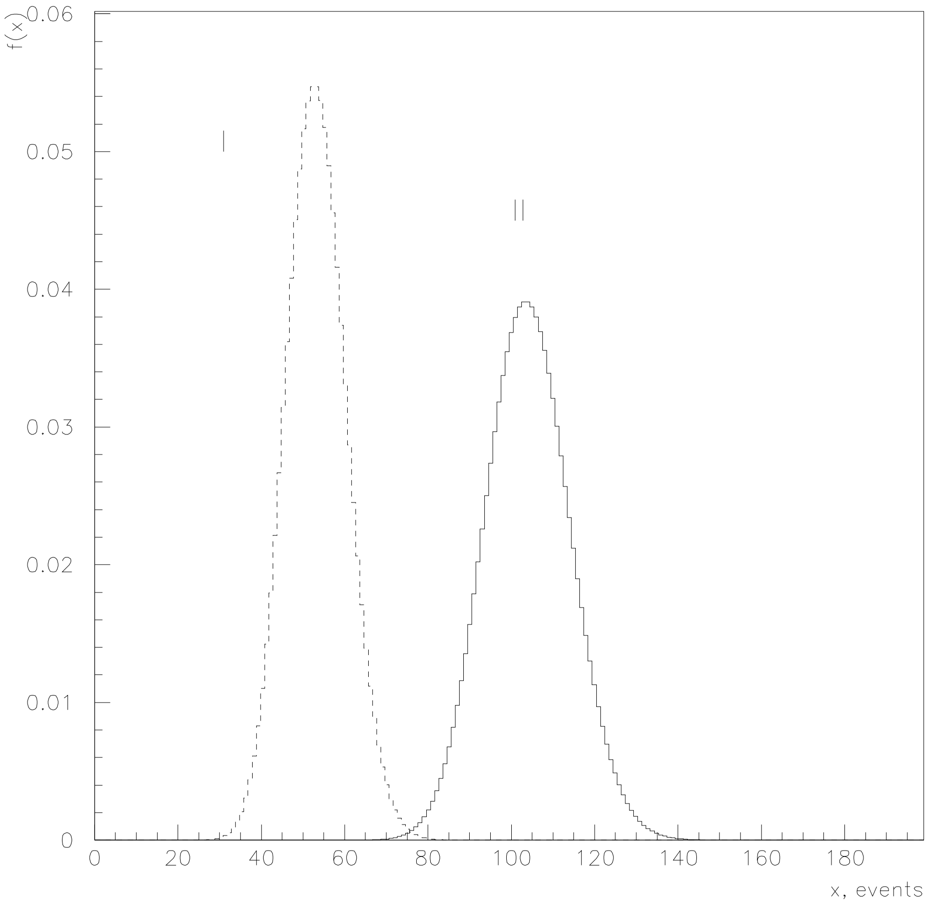

Figure 1: The probability density functions $f_{1,2}(x) \equiv P_G(x, \mu_{1,2}, \sigma^2)$ for $\mu_1 =< N_b >= 53$ and $\mu_2 =< N_b > + < N_s >= 104$.

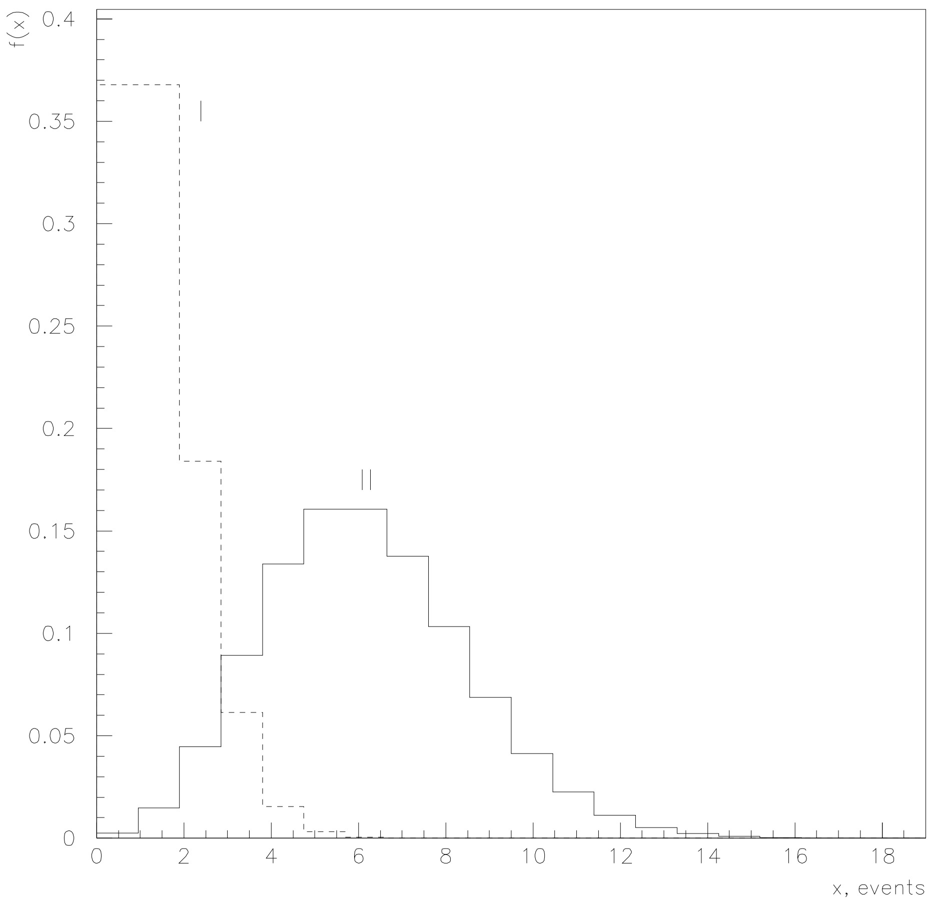

Figure 2: The probability density functions $f_{1,2}(x) \equiv \tilde{P}(x, \mu_{1,2})$ for $\mu_1 = < N_b > = 1$ and $\mu_2 =< N_b > + < N_s >= 6$.

Table 1: The dependence of κ on $< N_s >$ and $< N_b >$ for $S_1 = 5$

$< N_s >$	$< N_b >$	κ
5	1	0.1423
10	4	0.0828
15	9	0.0564
20	16	0.0448
25	25	0.0383
30	36	0.0333
35	49	0.0303
40	64	0.0278
45	81	0.0260
50	100	0.0245
55	121	0.0234
60	144	0.0224
65	169	0.0216
70	196	0.0209
75	225	0.0203
80	256	0.0198
85	289	0.0193
90	324	0.0189
95	361	0.0185
100	400	0.0182
150	900	0.0162
500	10^4	0.0136
5000	10^6	0.0125

Table 2: The dependence of κ on $< N_s >$ and $< N_b >$ for $S_2 \approx 5$.

$< N_s >$	$< N_b >$	κ
26	1	$0.154 \cdot 10^{-4}$
29	4	$0.142 \cdot 10^{-3}$
33	9	$0.440 \cdot 10^{-3}$
37	16	$0.993 \cdot 10^{-3}$
41	25	$0.172 \cdot 10^{-2}$
45	36	$0.262 \cdot 10^{-2}$
50	49	$0.314 \cdot 10^{-2}$
55	64	$0.357 \cdot 10^{-2}$
100	300	$0.735 \cdot 10^{-2}$
150	750	$0.894 \cdot 10^{-2}$

Table 3: $< N_s > = \dfrac{1}{5} \cdot < N_b >$. The dependence of κ on $< N_s >$ and $< N_b >$.

$< N_s >$	$< N_b >$	κ
50	250	0.131
100	500	0.033
150	750	$0.83 \cdot 10^{-2}$
200	1000	$0.24 \cdot 10^{-2}$
250	1250	$0.61 \cdot 10^{-3}$
300	1500	$0.22 \cdot 10^{-3}$
350	1750	$0.50 \cdot 10^{-4}$
400	2000	$0.10 \cdot 10^{-4}$

Table 4: $< N_s >= \frac{1}{10} \cdot < N_b >$. The dependence of κ on $< N_s >$ and $< N_b >$.

$< N_s >$	$< N_b >$	κ
50	500	0.274
100	1000	0.123
150	1500	0.057
200	2000	0.029
250	2500	0.014
300	3000	$0.75 \cdot 10^{-2}$
350	3500	$0.36 \cdot 10^{-2}$
400	4000	$0.20 \cdot 10^{-2}$
450	4500	$0.10 \cdot 10^{-2}$
500	5000	$0.50 \cdot 10^{-3}$

Table 5: $< N_s > = < N_b >$. The dependence of κ on $< N_s >$ and $< N_b >$.

$< N_s >$	$< N_b >$	κ
2.	2.	0.562
4.	4.	0.406
6.	6.	0.308
8.	8.	0.241
10.	10.	0.187
12.	12.	0.150
14.	14.	0.119
16.	16.	0.098
18.	18.	0.079
20.	20.	0.064
24.	24.	0.043
28.	28.	0.027
32.	32.	0.018
36.	36.	0.014
40.	40.	$0.84 \cdot 10^{-2}$
50.	50.	$0.33 \cdot 10^{-2}$
60.	60.	$0.13 \cdot 10^{-2}$
70.	70.	$0.47 \cdot 10^{-3}$
80.	80.	$0.16 \cdot 10^{-3}$
100.	100.	$0.30 \cdot 10^{-4}$

Table 6: $< N_s > = 2 \cdot < N_b >$. The dependence of κ on $< N_s >$ and $< N_b >$.

$< N_s >$	$< N_b >$	κ
2.	1.	0.464
4.	2.	0.295
6.	3.	0.199
8.	4.	0.143
10.	5.	0.101
12.	6.	0.074
14.	7.	0.050
16.	8.	0.038
18.	9.	0.027
20.	10.	0.020
24.	12.	0.011
28.	14.	$0.60 \cdot 10^{-2}$
32.	16.	$0.35 \cdot 10^{-2}$
36.	18.	$0.19 \cdot 10^{-2}$
40.	20.	$0.85 \cdot 10^{-3}$
50.	25.	$0.27 \cdot 10^{-3}$
60.	30.	$0.40 \cdot 10^{-4}$