The ρ-meson Wave Functions in QCD Sum Rules Approach with Nonlocal Condensates

A. P. Bakulev[†]
S. V. Mikhailov[‡]

Joint Institute for Nuclear Research, Bogoliubov Lab. of Theoretical Physics, 141980, Moscow Region, Dubna, Russia

Abstract

We apply the nonlocal condensate formalism to construct generalized sum rules (including $O(\alpha_s)$-radiative corrections) for π-, ρ- and ρ'-meson wave functions of twist 2. Besides, we predict the lepton decay constant $f_{\rho'}$ and estimate the mass of ρ'-meson. For all these mesons we obtain the first 10 moments of longitudinal wave functions, suggest the models for them and discuss their properties. We consider the peculiarities of the QCD sum rules with nonlocal condensates for transverse ρ- and b_1-meson wave functions. These results are compared with those found by Ball and Braun and by Chernyak and Zhitnitsky.

1. Introduction. An important problem in the theory of strong interactions is to calculate hadronic wave functions $\varphi_\pi(x)$, $\varphi_\rho(x)$, ..., $\varphi_N(x_1, x_2, x_3)$, *etc.* from the first principles of QCD. These phenomenological distributions of partons on the fraction xP of a hadron momentum P are a natural result of "factorization theorems" applied to hard exclusive processes [1, 2, 3, 4]. They accumulate all the necessary information about non-perturbative long-distance dynamics of partons in hadrons.

[†]E-mail: bakulev@thsun1.jinr.dubna.su
[‡]E-mail: mikhs@thsun1.jinr.dubna.su

In the standard QCD sum rule (SR) calculation of light meson wave functions (WF's), first introduced by Chernyak and Zhitnitsky (C&Z) [5] and recently re-estimated by Ball and Braun (B&B) for the ρ-meson [6], it is assumed that the correlation length Λ of vacuum fluctuations is large compared to a typical hadronic scale $\sim 1/m_\rho$. Thus, one can replace the original nonlocal objects like $M(z^2) = \langle\bar{q}(0)E(0,z)q(z)\rangle$ [1] by constant quantities of $\langle\bar{q}(0)q(0)\rangle$-type. Based on this hypothesis, the well-known QCD SR approach [7] has been applied in [5] to calculate the first two moments $\langle\xi^N\rangle \equiv \int_0^1 \varphi(x)(2x-1)^N dx$ with $N = 2,\ 4$ for WF's of light mesons. And just from these moments the whole WF's have been reconstructed which are now referred to as C&Z WF's.

Now it is known that hadronic WF's are rather sensitive to the widths of the function $M(z^2)$ and of other nonlocal condensates [8, 9, 10] and the crucial parameter $\Lambda\cdot m_\rho \sim 1$. Therefore, one should use nonlocal condensates (NLC's) like $M(z^2)$ whose forms reflect the complicated structure of QCD vacuum. Certainly, these objects can subsequently be expanded over the local condensates $\langle\bar{q}(0)q(0)\rangle$, $\langle\bar{q}(0)\nabla^2 q(0)\rangle$, *etc.* (here $\nabla_\mu = \partial_\mu - ig\hat{A}_\mu$), and one can come back to the standard SR by truncating this series. Our strategy is to avoid an expansion of that sort because we thus lose an important physical property of non-perturbative vacuum – the possibility of vacuum quarks and gluons to flow through vacuum **with non-zero virtuality** $k^2 \neq 0$. Indeed, the average virtuality of vacuum quarks $\langle k^2\rangle = \lambda_q^2$ is not small whereas the standard approach inevitably suggests that k is exact zero. The value of λ_q^2 can be extracted from the QCD SR analysis [11] and is connected with the condensate of the next dimension $d = 5$

$$\lambda_q^2 \equiv \frac{\langle\bar{q}\nabla^2 q\rangle}{\langle\bar{q}q\rangle} = \frac{\langle\bar{q}\left(ig\sigma_{\mu\nu}G^{\mu\nu}\right)q\rangle}{2\langle\bar{q}q\rangle} = 0.4 \pm 0.1\,\mathrm{GeV}^2, \qquad (1)$$

that is of an order of the typical hadronic scale, $m_\rho^2 \approx 0.6$ GeV^2. An estimate for λ_q^2 in the framework of the instanton liquid model [12] yields a similar number, and the lattice QCD calculations [13] give $\lambda_q^2 = 0.55 \pm 0.05\,\mathrm{GeV}^2$, a similar estimate $\lambda_q^2 = 0.5 \pm 0.05\,\mathrm{GeV}^2$ has been obtained in [14].

Careful inspection of consequences of that approach to QCD SR for pion WF [8, 9, 15] has revealed that the introduction of the correlation length

[1] Here $E(0,z) = P\exp(i\int_0^z dt_\mu A_\mu^a(t)\tau_a)$ is the Schwinger phase factor required for gauge invariance.

$\Lambda \sim 1/\lambda_q$ into condensate distributions produces much smaller values for the first moments of pion WF than C&Z values. As a result, the pion WF is strongly different in shape from the C&Z and approaches the asymptotic WF $\varphi^{as}(x) \equiv 6x(1-x)$ [4], *i.e.* , $\varphi_\pi(x) \approx \varphi^{as}(x)$ [2]. Later, this WF was confirmed by independent consideration of the QCD SR directly for the function $\varphi_\pi(x)$ based on the non-diagonal correlator and on the advanced smooth distribution function for the quark nonlocal condensate [18, 19]. The key element of both the ways was to take into account the main physical reason – vacuum correlation length $\Lambda \sim 1/\lambda_q$ is of an order of $1/m_\rho$.

Our goal here is to show that in the case of QCD SR for the ρ-meson channel the situation is similar to the pion one. All the predictions of the standard QCD SR (the C&Z ones for the longitudinal case [5] and B&B ones for the transverse case [6] – we call them the Local QCD SR) for moments $\langle\xi^N\rangle_\rho$ with $N > 2$ could not be considered as reliable; as we show in our analysis, even the value of $\langle\xi^2\rangle_\rho$ is indefinite. We apply the NLC formalism to calculate the diagonal correlators for ρ-meson currents, introduced in [5], and construct generalized SR including $O(\alpha_s)$-radiative corrections to obtain WF's of twist 2. The first ten moments of longitudinal WF's of ρ- and ρ'-mesons are estimated (Table 1) and the models for them (see Figs.2) are suggested. Also, we predict the lepton decay constant $f_{\rho'}$ and estimate the mass of ρ'-meson. We discuss the strong dependence of results for moments of transverse WF's in ρ-channel on the model of nonlocal gluon condensate contribution. The calculation technique is the same as in Refs. [8, 9]; therefore, the corresponding details are omitted below, but we shall through the text keep the connection with the well-understood pion case.

2. Generalized sum rules for the π- and ρ-meson channels vs the standard version. For the helicity-zero charged vector meson $M_{|\lambda|=0} = M_L$, $M_L = \rho,\ \rho' \ldots$, the leading-twist WF is defined as

$$\langle 0 \mid \bar{d}(z)\gamma_\mu u(0) \mid M_L(p)\rangle\Big|_{z^2=0} = f^L_{M_L} p_\mu \int_0^1 dx e^{ix(zp)}\ \varphi^L_{M_L}(x) \qquad (2)$$

(In accordance with the B&B definition, in this case $\varepsilon^{\lambda=0}_\mu(p) \simeq p_\mu/m_{M_L}$ as $p_z \to \infty$, where $\varepsilon_\mu(p)$ is the polarization vector); and for axial mesons

[2]Note here other independent evidences that the pion WF is close to its asymptotic form: lattice calculations of the corresponding $\langle\xi^2\rangle_\pi$ in [16]; QCD SR estimate of the magnitude of $\varphi_\pi(x)$ at the midpoint $x = 1/2$ in [17].

$(M_A = \pi, A_1 \ldots)$ as

$$\langle 0 \mid \bar{d}(z)\gamma_\mu\gamma_5 u(0) \mid M_A(p)\rangle\Big|_{z^2=0} = i f_{M_A} p_\mu \int_0^1 dx e^{ix(zp)}\ \varphi_{M_A}(x). \tag{3}$$

The information about $\varphi^L(x)$ can be obtained from the correlator $I_{(n0)}(q^2)$ of vector currents $V_{(n)}(y)$, see *e.g.* [5]:

$$\begin{aligned} i \textstyle\int dy e^{iqy} \langle 0|T\left\{V^+_{(0)}(y)V_{(n)}(0)\right\}|0\rangle &= i^n\,(zq)^{n+2}\, I_{(n0)}, \\ V_{(n)}(y) &\equiv \bar{d}(y)\hat{z}\,(z\nabla)^n\, u(y)\Big|_{z^2=0}; \end{aligned} \tag{4}$$

and the information about $\varphi_{M_A}(x)$, from an analogous formula with the substitution $V \to A$ and $A_{(n)}(y) \equiv \bar{d}(y)\hat{z}\gamma_5\,(z\nabla)^n\,u(y)$. The corresponding formula for the unit helicity state, $M_\perp = M_{|\lambda|=1}$, $M_\perp = \rho,\ b_1 \ldots$, is as follows

$$\begin{aligned} &\langle 0 \mid \bar{d}(z)\sigma_{\mu\nu}u(0) \mid M_\perp(p)\rangle\Big|_{z^2=0} \\ &= i f^T_{M_\perp}\left(\varepsilon^\lambda_\mu(p)p_\nu - \varepsilon^\lambda_\nu(p)p_\mu\right)\int_0^1 dx e^{ix(zp)}\ \varphi^T_{M_\perp}(x). \end{aligned} \tag{5}$$

Moments of these functions $\varphi^T(x)$ are extracted from the correlator $J_{(n0)}(q^2)$ of tensor currents $T^\mu_{(n)}(y)$ [5, 6]

$$\begin{aligned} i \textstyle\int dy e^{iqy} \langle 0|T\left\{T^{\mu+}_{(0)}(y)T^\mu_{(n)}(0)\right\}|0\rangle &= -2i^n\,(zq)^{n+2}\, J_{(n0)}, \\ T^\mu_{(n)}(y) &\equiv \bar{d}(y)\sigma^{\mu\alpha}z_\alpha\,(z\nabla)^n\,u(y). \end{aligned} \tag{6}$$

This correlator $J_{(n0)}(q^2)$ contains the contribution from states with different parity (see the analysis in [6]). Therefore, the contamination from b_1-meson $\left(J^{PC} = 1^{+-}\right)$ in the phenomenological part of SR is mandatory.

The theoretical "condensate" parts for both the correlators, $I_{(n0)}$ in (4) and $J_{(n0)}$ in (6), contain the same 6 universal elements as for the pion case. An exception concerning transverse SR's (see (8)) will be discussed below. The diagram origins of these elements $\Delta\Phi_\Gamma\,(x; M^2)$, where M^2 is the Borel parameter, are described in detail in [8, 9]. By the direct SR formulation for WF, one immediately constructs a "daughter SR" for any functional of $\varphi_M(x)$. Let us write down the final SR's including WF's of ρ-meson and next resonances ρ' and b_1 into the phenomenological parts (the corresponding SR for the pion (axial) channel can be derived from SR (7) by substitution $\rho \to \pi$, $\rho' \to A_1$, $m_\rho \to 0$, $s_0^L \to s_0^A$ and $\Delta\Phi_S(x; M^2) \to -\Delta\Phi_S(x; M^2)$):

$$\begin{aligned}
&\left(f_\rho^L\right)^2 \varphi_\rho^L(x) e^{-m_\rho^2/M^2} + \left(f_{\rho'}^L\right)^2 \varphi_{\rho'}^L(x) e^{-m_{\rho'}^2/M^2} = \\
&\quad \int_0^{s_0^L} \rho_L^{pert}(x; s) e^{-s/M^2} ds + \Delta\Phi_G(x; M^2) - \Delta\Phi_S(x; M^2) + \\
&\quad + \Delta\Phi_V(x; M^2) + \Delta\Phi_{T_1}(x; M^2) + \Delta\Phi_{T_2}(x; M^2) + \Delta\Phi_{T_3}(x; M^2);
\end{aligned} \tag{7}$$

$$\begin{aligned}
&\left(f_\rho^T\right)^2 \varphi_\rho^T(x) e^{-m_\rho^2/M^2} + \left(f_{b_1}^T\right)^2 \varphi_{b_1}^T(x) e^{-m_{b_1}^2/M^2} = \\
&\quad \int_0^{s_0^T} \rho_T^{pert}(x; s) e^{-s/M^2} ds + \Delta\Phi_G(x; M^2) - \Delta\Phi'_G(x; M^2) + \\
&\quad + \Delta\Phi_V(x; M^2) + \Delta\Phi_{T_1}(x; M^2) + \Delta\Phi_{T_2}(x; M^2) - \Delta\Phi_{T_3}(x; M^2),
\end{aligned} \tag{8}$$

where $s_0^{L,T}$ are the effective continuum thresholds in the ρ-meson L and T cases, respectively. Perturbative spectral densities $\rho_{L,T}^{pert}(x; s)$ are presented in an order of $O(\alpha_s)$ in [8, 9] for the L case and in [6] for the T case (see Appendix B). Radiative corrections reach 10 % of $\rho_{L,T}^{pert}(s \sim 1 \text{ GeV}^2)$. Contributions $\Delta\Phi_\Gamma(x; M^2)$ depend on a specific form of NLC's $M(z^2)$, ..., *etc.* To construct SR for WF's, it is useful to parametrize these NLC behaviors by the "distribution functions" [8, 9, 18, 19] *a'la* α-representation of propagators, *e.g.* , $f_S(\nu)$ for the scalar condensate $M(z^2)$ [3]

$$M\left(z^2\right) = \langle \bar{q}(0) q(0) \rangle \int_0^\infty e^{\nu z^2/4} f_S(\nu)\, d\nu, \tag{9}$$

$$\text{where } \int_0^\infty f_S(\nu)\, d\nu = 1, \ \int_0^\infty \nu f_S(\nu) d\nu = \frac{\lambda_q^2}{2}.$$

The function $f_S(\nu)$ and other similar functions $f_\Gamma(\nu)$ describe distributions of vacuum fields in virtuality ν for every type of NLC. They completely determine the r.h.s. of SR's in (7)–(8). For the standard (constant) condensates $\langle G(0)G(0) \rangle$ and $\langle q(0)q(0) \rangle$ these distributions are trivial in form, *e.g.* , $f_S(\nu) = \delta(\nu)$ (Appendix A). To include the condensates of the next ($\sim \lambda_q^2$) and higher dimensions into consideration , one should add the contributions to $f_S(\nu)$ proportional to the derivatives of δ-functions, $\delta(\nu)'$, $\delta(\nu)''$, ... It is clear that since there is no QCD vacuum theory, merely models of real distributions can be suggested for these distribution functions. However, for the

[3] In deriving these sum rules we can always make a Wick rotation, i.e., we assume that all coordinates are Euclidean, $z^2 < 0$.

purpose of QCD SR's for moments $\langle\xi^N\rangle$ we need an approximate information about the $f_\Gamma(\nu)$ behaviour. Therefore, we here apply the simplest ansatz [8, 9], like $f_S(\nu)=\delta(\nu-\lambda_q^2/2)$, to take into account only the main effect, the non-zero average virtuality $\langle k^2\rangle$ of vacuum fields. This form of $f_S(\nu)$ can be regarded as a result of resummation of an infinite subset of the above-mentioned contributions $\sim(\lambda_q^2/2)^n\ \delta(\nu)^{\{n\}}$ connected with the single scale λ_q^2 [8, 9]. The corresponding expressions for $\Delta\Phi_{G,S,V,T_i}(x;M^2)$ are collected in Appendix A.

Now let us take the limits $\lambda_q^2\to 0,\quad \Delta\Phi_\Gamma(x,M^2)\to\Delta\varphi_\Gamma(x,M^2)$ and $\rho_L^{pert}(x,s)\to\rho_L^{Born}(x,s)\equiv 3x\bar{x}/(2\pi^2)$ for SR in eq.(7) to return to the standard approach (see these reduced elements in Appendix A). We try to inspect the subtle points and the range of validity of C&Z SR. These authors extracted $\langle\xi^N\rangle$ exactly in the same way as the f_ρ value. However, the nonperturbative terms in their sum rule (the ρ'-contribution is omitted for simplicity) have a completely different N-dependence compared to the perturbative one and, *a priori,* it is not clear whether a straightforward use of the "$N=0$ technology" can be justified for higher N (we consider here only the ρ-meson L-case; the same arguments apply also to the pion case, see criticism in [9, 10, 15])

$$\left(f_\rho^L\right)^2\langle\xi^N\rangle_\rho^L e^{-m_\rho^2/M^2}+\frac{3}{4\pi^2}\frac{M^2}{(N+1)(N+3)}e^{-s_N/M^2}=$$
$$=\frac{3}{4\pi^2}\frac{M^2}{(N+1)(N+3)}+\frac{\langle(\alpha_s/\pi)GG\rangle}{12M^2}+\frac{16}{81}\pi(4N-7)\frac{\langle\sqrt{\alpha_s}\bar{q}q\rangle^2}{M^4}. \quad (10)$$

The scale determining the magnitude of all hadronic parameters including s_N (the "continuum threshold" [7]) is eventually settled by the ratios of condensate contributions to the perturbative term. If the condensate contributions in the C&Z sum rule (10) had the same N-behavior as the perturbative term, the N-dependence of $\langle\xi^N\rangle$ would be determined by the overall factor $3/(N+1)(N+3)$ and the resulting WF $\varphi(x)$ would coincide with the "asymptotic" form $\varphi^{as}(x)$.

However, the ratios of the $\langle\bar{q}q\rangle$- and $\langle GG\rangle$-corrections to the perturbative term in eq.(10) are growing functions of N. This reduces the predictable power of the local QCD SR's with the growth of N. To reveal consequences of this effect more clearly, let us consider the so-called SR fidelity windows, *i.e.* , regions of the Borel parameter M^2 where one should obtain reliable SR predictions. In accordance with the QCD SR practice [7] (see also very clear discussion in [20]), these fidelity windows are determined by two conditions: the lower bound M_-^2 by demanding that the relative value of $\langle GG \rangle$- and $\langle \bar{q}q \rangle$-contributions to OPE series should not be larger than 30%, the upper one M_+^2 by requiring that a relative contribution of higher states in the phenomenological part of SR should not be larger than 30%. Suggesting independence of the threshold of N ($s_N \approx s_0 \approx 1.5$ GeV^2), we have in the case of $N = 0$: $M_-^2 = 0.4$ GeV^2, $M_+^2 = 1.34$ GeV^2. But for $N = 2$ we have $M_-^2 = 0.73$ GeV^2, $M_+^2 = 1.34$ GeV^2, and for $N = 4$ – even $M_-^2 = 1.5$ GeV^2, $M_+^2 = 1.34$ GeV^2. That is, the fidelity window shrinks to an empty set in the last case. C&Z had to suggest that $s_2 \approx 1.9$ GeV^2 and $s_4 \approx 2.2$ GeV^2 to extend the fidelity windows and to obtain any stability with respect to M^2. The criticism of such a strange type of a spectral model see in [9].

In our opinion, there is no need to propose such an exotic spectral model because the reason for this "exploding" behaviour of Local SR is quite evident, namely, a completely different dependence on N of the perturbative (the first term in the r.h.s. of Eq.(10)) contribution and of condensate ones. And the origin of this difference was explained in a series of papers [9, 10, 15]; this is due to the Taylor expansion of initial nonlocal objects like $\langle \bar{q}(0)E(0,z)q(z) \rangle$ in powers of z. The first constant term of this expansion, $\langle \bar{q}q \rangle$, produces an $(N)^0$-dependent term in SR (10); the next term, an $(N)^1$-dependent, and so on. On the contrary, the NLC terms $\Delta\Phi_\Gamma(x; M^2)$ in (7) and (8) lead to the moments $\langle \xi^N \rangle$ which well decay with N growing; so a physically motivated N-independent continuum threshold s_0^L naturally appears in the SR processing.

3. The moments and the models of wave functions. Before analyzing the results of processing of SR (7) and (8) for the moments $\langle \xi^N \rangle_M$, let us consider the peculiarity of the QCD SR structure represented in the r.h.s. of Eqs. (7-8). Unlike the π-meson case, the contribution of the numerically most significant "four-quark condensate" $\Delta\Phi_S\,(x, M^2)$ [8] is equal to zero (for the T case, see also the sign of $\Delta\Phi_{T3}$) or even has the opposite sign (for the L case). For this reason, the role of the vacuum interaction for the ρ-meson is weaker than for the pion. As a consequence of such an SR structure the nonlocal effects partially compensate themselves. Therefore, the extracted values of $\langle \xi^2 \rangle_M$ in the framework of NLC SR don't drastically differ from the results of B&B obtained in the standard way [4]. However, the sensitivity and stability of NLC SR are much better than for the standard one, compare the accuracy for the first moments in Table 1. This allows us to estimate the first ten moments in the L case for ρ- and ρ'-mesons. For clarity see Fig.1, where the curves for $\langle \xi^2 \rangle_\rho^L$ are represented as functions of the Borel parameter M^2 in the range of stability window for two types of QCD SR's: the solid line is determined from NLC SR (7) with $s_0^L = 2.4$ GeV^2, the short-dashed line – from NLC SR without ρ'-meson contribution ($s_0^L = 1.5$ GeV^2), and the dashed one, from B&B SR [6] ($s_0^L = 1.5$ GeV^2). As one can see, in the local SR case there is no stability at all. We have determined the fol-

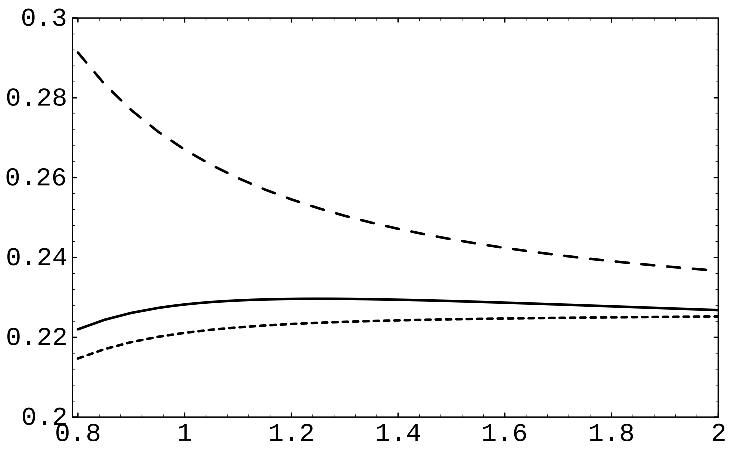

Fig. 1: Curves $\langle \xi^2 \rangle_\rho^L$ as functions of Borel parameter M^2

lowing values for practically N-independent continuum thresholds: $s_0^L \approx 2.4$ GeV^2. Fidelity windows for the L case are: 0.6 GeV$^2 \leq M^2 \leq 2.0$ GeV^2 for all $N = 0, 2, \ldots, 10$. The ranges of stability within these fidelity windows almost coincide with these windows, starting for all N at $M^2 \approx 0.8$ GeV^2.

For transverse SR (8), in its theoretical side, we face the problem that an important part of nonlocal gluon contribution $\Delta\Phi'_G(x, M^2)$ is not yet esti-

[4]Note here that the results of the T case in an original CZ work should not be taken as a pattern for a standard SR, there is an error in the sign of the quark condensate contribution, see [21, 6]

mated. We have suggested a naive model, where the local result $\Delta\varphi'_G\left(x, M^2\right) \to \Delta\Phi'_G\left(x, M^2\right)$ (see Appendix (A.6)). This simulation eliminates end-point $(x = 0, 1)$ effects due to the influence of the vacuum gluon nonlocality, which is inspired by the analysis in [22] and our experience in the nonlocal quark case (see Appendix A). For the zeroth moment SR ($N = 0$) any model is not too important, they give a gluonic contribution close to one in the local SR, and we obtain the best stability with $s_0^T \approx 2.3$ GeV^2 and for 0.75 GeV$^2 \le M^2 \le 1.9$ GeV^2. But for the next moment the results are very sensitive to the model of gluon contribution. Indeed, these results (see Table 1) drastically change if we leave the local expression $\Delta\varphi'_G\left(x, M^2\right)$ unchanged: $\langle\xi^2\rangle^T_\rho = 0.231(8)$ and $\langle\xi^2\rangle^T_{b_1} = 0.220(8)$). So, we realize that our modelling is of great importance for the results discussed and we can't give a reliable prediction for $\langle\xi^{2N}\rangle^T_{\rho,b_1}$ $(N = 1, \ldots)$ without essential calculational efforts.

We also re-estimate the first ten moments for the pion case and confirm the main previous results of nonlocal approach [9]. In this axial channel we include the pion and A_1-meson into the phenomenological part of SR and obtain: $s_0^A \approx 2.3$ GeV^2, and the ranges of stability are 0.75 GeV$^2 \le M^2 \le 1.9$ GeV^2.

Another evidence of the efficiency of NLC SR is the estimate of the ρ'-meson mass. First, the ρ'-resonance with tabular mass $m_{\rho'} = 1465$ MeV was inserted into SR (7) to improve the stability. But at the second step $m_{\rho'}$ was estimated from our SR, it appears to be rather close to the experimental value [23]

$$m_{\rho'}^{theor} = 1496 \pm 37 \text{ MeV} , \qquad m_{\rho'}^{exp} = 1465 \pm 22 \text{ MeV}. \tag{11}$$

Possible models of WF's corresponding to the moments in Table 1 have the form

$$\varphi_\pi^{mod}(x, \mu^2) = \varphi^{as}(x)\, 6.739\, \left(\varphi^{as}(x)\right)^{1.305} \left(x^2 + (1-x)^2\right)^{4.01}, \tag{12}$$

$$\varphi_\rho^{L,mod}(x, \mu^2) = \varphi^{as}(x) \left(1 + 0.077 \cdot C_2^{3/2}(\xi) - 0.077 \cdot C_4^{3/2}(\xi)\right), \tag{13}$$

$$\varphi_{\rho'}^{L,mod}(x, \mu^2) = \varphi^{as}(x) \left(1 + 0.075 \cdot C_2^{3/2}(\xi) + 0.4 \cdot C_4^{3/2}(\xi) - 0.04 \cdot C_6^{3/2}(\xi)\right), \tag{14}$$

where $\xi \equiv 1 - 2x$, $C_n^\nu(\xi)$ are the Gegenbauer polynomials and $\mu^2 \simeq 1$ GeV^2 corresponds to an average value of M^2. The important problem of sensitivity of the results to the condensate parameters in SR's will be addressed to in a separate paper. Here we only note that the main qualitative conclusions concerning the shapes of $\varphi^L_{\rho,\rho'}$ and φ_π do not change under accessible variation of λ_q^2.

Table 1. The moments $\langle\xi^N\rangle_M(\mu^2)$ at $\mu^2 \sim 1$ GeV[25] (errors are depicted in brackets following a standard manner)

Type of SR	f_M $\left(\mathrm{GeV}^2\right)$	$N=2$	$N=4$	$N=6$	$N=8$	$N=10$
Asympt. WF	1	0.2	0.086	0.047	0.030	0.021
NLC : π	0.131(2)	0.25(1)	0.110(7)	0.054(3)	0.031(2)	0.0217(7)
C&Z : π	0.131	0.40	0.24	–	–	–
NLC : ρ^L	0.201(5)	0.227(7)	0.095(5)	0.051(4)	0.030(2)	0.020(5)
B&B : ρ^L	0.205(10)	0.26(4)	–	–	–	–
C&Z : ρ^L	0.194	0.26	0.15	–	–	–
NLC : ρ'^L	0.175(10)	0.226(10)	0.145(7)	0.106(5)	0.082(4)	0.064(4)
NLC : ρ^T	0.169(5)	0.325(10)	?	?	?	?
B&B : ρ^T	0.160(10)	0.27(4)	–	–	–	–
C&Z : ρ^T	0.200	0.15	≤ 0.06	–	–	–
NLC : b_1^T	0.181(5)	0.090(5)	?	?	?	?
B&B : b_1^T	0.180-0.170	–	–	–	–	–

To check the reliability of these models, let us estimate the functional $I[\varphi_M] = \int_0^1 \frac{\varphi_M(x)}{x} dx$ that is often used in calculations of different form

[5]C&Z give all moments normalized to the normalization point $\mu_0 = 500$ MeV; here we present these moments normalized to the normalization point $\mu = 1$ GeV.

factors. This integral naturally appears in the perturbative QCD approach to the electromagnetic form factor F_π [1, 2], to the evaluation of $\gamma\gamma^* \to \pi^0$ [24] and also to the QCD description of heavy meson semileptonic decays [25]. It is clear that $I[\varphi_M]$ is a new independent (of moments $\langle\xi^N\rangle_M$) quantity. Besides, the values of $I[\varphi_M]$ allow us to better discriminate between different models for the same $\varphi_M(x)$. The $I[\varphi_M]$ can be obtained in two different ways: (i) from QCD SR adapted to the quantity $I[\varphi^L]$ by integration with weight $1/x$ and (ii) by direct integration of the WF models (12)–(14).

Let us first consider the results for the pion case. The range of stability here is as wide as 0.6 GeV$^2 \le M^2 \le 1.9$ GeV^2 and the quality of stability is very high. As it is seen from Table 2 (columns 3 and 4), our estimates for $I[\varphi_\pi]$, obtained in both the mentioned ways, are consistent with each other within 10% accuracy. Moreover, the estimation for $I[\varphi_\pi]$ from SR, is quite close to the one obtained independently by using the nondiagonal NLC QCD SR in [19] (column 5). So, we may conclude that the nonlocal approach leads to self-consistent results which are not far from the estimate of $I[\varphi_\pi] \approx 2.4$ in [24] [6] (see column 6) and can be reliably discriminated from the C&Z ones.

Table 2. Evaluations of the functional $I[\varphi_M]$ in different approaches

	WF (asymp)	SR (here)	WF (here)	WF ([19])	SR ([24])	WF (B&B)	WF (C&Z)
$I[\varphi_\pi]$	3	2.75 ± 0.05	3.03	2.8	$2.4(\le 2.9)$	—	5.0
$I[\varphi_\rho^L]$	3	3.1 ± 0.1	3.0	—	—	3.54	4.38
$I[\varphi_{\rho'}^L]$	3	4.7 ± 0.2	4.3	—	—	—	—

A similar situation holds also for the ρ_L-results, the stability is also of high quality in the range 0.5 GeV$^2 \le M^2 \le 2.0$ GeV^2. Agreement of both kinds of the results for $I[\varphi^L]$ is rather good for the ρ_L-case (the discrepancy is smaller than 5%) and worse for the $\rho_L^{'}$-case. The models (12)–(13) confirm the property of NLC SR's that WF of a meson in the "ground state" approaches the asymptotic WF (due to nonlocality effects), proposed by Radyushkin

[6]The authors of [24] state that the value $I \approx 2.4$ may be underestimated not more than by 20%

[15]. The curve $\varphi^L_{\rho,mod}(x)$ (see the left fig.2) is not far from the asymptotic WF curve and looks close to the B&B model WF. Note, nevertheless, that the latter visual closeness seems rather crude and does not allow quantitative conclusions, *e.g.* , NLC SR provides the values of moments $\langle\xi^2\rangle_\rho$, $I[\varphi^L]$ in 15% smaller than in the B&B case. One may expect that WF's of resonances would oscillate by analogy with the pion resonance π' case [18, 19]. Indeed, the shape of WF $\varphi^{L,mod}_{\rho'}(x)$ (see the right fig.2) for resonance looks similar to WF of π'.

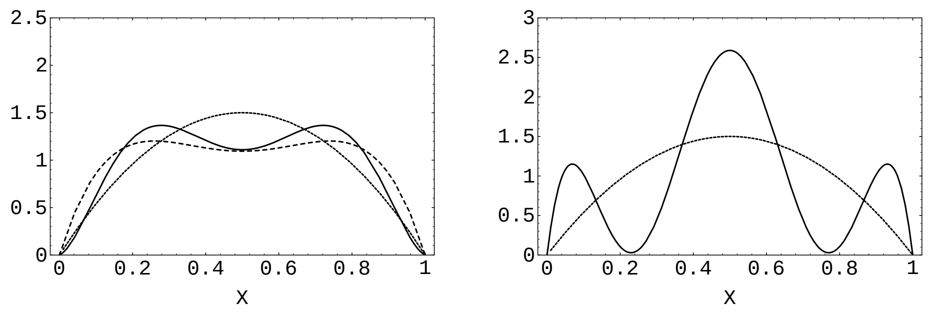

Fig. 2: Longitudinal wave function of the ρ-meson (the left picture) and of the ρ'-meson (the right picture): solid line – from NL QCD SR, dotted line – asymptotic WF, dashed line – C&Z WF (on the left picture).

4. Conclusion. Our basic interest in the present paper is to explore the well-working method of NLC QCD SR in analyzing WF's in the ρ-meson vector and tensor channels. As has been noted in the previous papers [8, 9, 10, 15, 18, 19], just in problems of "nonlocal" characteristics of hadrons, such as wave functions, form factors, *etc.* , one should use the formalism of nonlocal condensates. Let us summarize the main results of this paper.

1. The generalized sum rules for WF's of the ρ-meson and related resonances with nonlocal condensates are constructed. Using the simplest ansatz [9] for nonlocal quark condensates we obtain new estimates for the first ten moments of the ρ-meson and its resonance WF's. It should be emphasized that analogous evaluation within the standard QCD SR approach is impossible.

2. We suggest the models for (see Fig.2) longitudinal WF's of π-, ρ- and ρ'-mesons and verify their self-consistency. The form of the obtained longitudinal ρ-meson WF is not far from the asymptotic WF (this conclusion noticeably differs from the results of the Local QCD SR [5]).

3. As a by-product, we predict the lepton decay constant $f_{\rho'} = 0.175 \pm 0.005$ GeV^2 and estimate the mass of the ρ'-meson, $m_{\rho'} = 1496 \pm 37$ MeV, which is now under experimental investigation [23].

4. For π- and ρ^L-channels we perform processing of SR directly for the integral $I[\varphi] = \displaystyle\int_0^1 \frac{\varphi_M(x)}{x} dx$ and obtain high-stable predictions for these quantities. Their values unambiguously favor WF's close to the asymptotic form rather than to the C&Z-one. Moreover, the result for π-meson is also consistent with the model WF obtained in a special variant of NLC QCD SR's (in the non-diagonal QCD SR approach [18, 19] which directly connects π-WF only with nonlocal condensates) and with analysis of the $\gamma\gamma^* \to \pi^0$ transition form factor [24].

So we can conclude that the nonlocal QCD SR approach is self-consistent and gives the results of high stability. To complete the calculation for transverse ρ-meson WF, we need an exact result for the nonlocal gluon contribution to SR. An open problem of this approach is the determination of well-established models of distribution functions $f_\Gamma(\nu)$ from the theory of nonperturbative QCD vacuum.

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Appendix A. Expressions for nonlocal contributions to SR. The contributions of different NLC's $\Delta\Phi_\Gamma(x, M^2)$ corresponding to the set of the simplest ansatzes $f_\Gamma(\nu)$ (discussed in [8, 9]) are shown below; the limit of these expressions to the standard (local) contributions $\varphi_\Gamma(x,M^2)$ – $\lambda_q^2 \to 0$, $\Delta\Phi_\Gamma(x,M^2) \to \Delta\varphi_\Gamma(x,M^2)$ are also written for comparison. Here and in what follows $\Delta \equiv \lambda_q^2/(2M^2)$, $\bar{\Delta} \equiv 1-\Delta$, and we suggest that $\Delta < 1$:

$$\begin{aligned}
\Delta\Phi_S\left(x,M^2\right) &= \frac{A_S}{M^4}\frac{18}{\bar{\Delta}\Delta^2}\left\{\theta\left(\bar{x} > \Delta > x\right)\bar{x}\left[x + (\Delta - x)\ln\left(\bar{x}\right)\right] + (\bar{x}\to x) + \right. \\
&\qquad \left. +\theta\left(\Delta > x > \bar{\Delta}\right)\left[\bar{\Delta} + (\Delta - 2\bar{x}x)\ln(\Delta)\right]\right\}, \quad \text{(A.1)} \\
\Delta\varphi_S\left(x,M^2\right) &= \frac{A_S}{M^4}9\left(\delta(x) + \delta(\bar{x})\right); \\
\Delta\Phi_V\left(x,M^2\right) &= \frac{A_S}{M^4}\left(x\delta'\left(\bar{x} - \Delta\right) + (\bar{x}\to x)\right), \qquad \text{(A.2)} \\
\Delta\varphi_V\left(x,M^2\right) &= \frac{A_S}{M^4}\left(x\delta'\left(\bar{x}\right) + \bar{x}\delta'\left(x\right)\right);
\end{aligned}$$

$$\begin{aligned}
\Delta\Phi_{T_1}\left(x,M^2\right) &= -\frac{3A_S}{M^4}\left\{\theta(2\Delta > x > \Delta)\theta(x > 3\Delta - 1)\frac{\bar{x}}{\bar{\Delta}}\right. \\
&\qquad +\theta(1 > 2\Delta)\left(1 - 2\Delta\right)\cdot \\
&\qquad \left.\cdot\frac{\left[\delta(x - 2\Delta) - \delta(x - \Delta)\right]}{\Delta}\left[\frac{3x}{\Delta} - 6 - \frac{1+\bar{x}}{\bar{\Delta}}\right]\right\} \\
&\quad + (\bar{x}\to x), \qquad \text{(A.3)} \\
\Delta\varphi_{T_1}\left(x,M^2\right) &= \frac{3A_S}{M^4}\left(\delta'\left(\bar{x}\right) + \delta'\left(x\right)\right); \\
\Delta\Phi_{T_2}\left(x,M^2\right) &= -\frac{4A_S}{M^4}\bar{x}\left\{\theta(2\Delta > x > \Delta)\theta(x > 3\Delta - 1)\frac{1 + 2x - 4\Delta}{\bar{\Delta}\Delta^2}\right. \\
&\quad \left. -\theta(1 > 2\Delta)\frac{\delta(x - 2\Delta)}{\Delta}\right\} + (\bar{x}\to x), \qquad \text{(A.4)} \\
\Delta\varphi_{T_2}\left(x,M^2\right) &= -\frac{2A_S}{M^4}\left(x\delta'\left(\bar{x}\right) + \bar{x}\delta'\left(x\right)\right); \\
\Delta\Phi_{T_3}\left(x,M^2\right) &= \frac{3A_S\bar{x}}{M^4\bar{\Delta}\Delta}\left\{\theta(2\Delta > x > \Delta)\theta(x > 3\Delta - 1)\left[2 - \frac{\bar{x}}{\bar{\Delta}} - \frac{\Delta}{\bar{\Delta}}\right]\right\} \\
&\quad + (\bar{x}\to x), \qquad \text{(A.5)} \\
\Delta\varphi_{T_3}\left(x,M^2\right) &= \frac{3A_S}{M^4}\left(\delta\left(\bar{x}\right) + \delta\left(x\right)\right); \\
\Delta\Phi_G\left(x,M^2\right) &= \frac{\langle\alpha_s GG\rangle}{24\pi M^2}\left(\delta\left(x - \Delta\right) + (\bar{x}\to x)\right)
\end{aligned}$$

$$
\begin{aligned}
\Delta\varphi_G\left(x, M^2\right) &= \frac{\langle\alpha_s GG\rangle}{24\pi M^2}\left(\delta\left(\bar{x}\right)+\delta\left(x\right)\right);\\
\Delta\Phi'_G\left(x, M^2\right) &= \frac{\langle\alpha_s GG\rangle}{6\pi M^2}\frac{\theta\left(\Delta<x\right)\theta\left(x<1-\Delta\right)}{1-2\Delta}\\
\Delta\varphi'_G\left(x, M^2\right) &= \frac{\langle\alpha_s GG\rangle}{6\pi M^2}.
\end{aligned}
\tag{A.6}
$$

Here $A_S = \dfrac{8\pi}{81}\langle\sqrt{\alpha_s}\bar{q}(0)q(0)\rangle^2$, for quark and gluon condensates we use the standard estimates $\langle\sqrt{\alpha_s}\bar{q}(0)q(0)\rangle \approx (-0.238\ \mathrm{GeV})^3$, $\dfrac{\langle\alpha_s GG\rangle}{12\pi} \approx 0.001\ \mathrm{GeV}^4$ [7] and $\lambda_q^2 = \dfrac{\langle\bar{q}\left(ig\sigma_{\mu\nu}G^{\mu\nu}\right)q\rangle}{2\langle\bar{q}q\rangle} = 0.4 \pm 0.1\ \mathrm{GeV}^2$, normalized at $\mu^2 \approx 1\ \mathrm{GeV}^2$.

Appendix B. Expressions for perturbative spectral densities. First, $\rho(x,s)_L^{pert}$ in an order of $O(\alpha_s)$ was calculated in [8, 9], but there was omitted the trivial term $2\ln\left[\dfrac{s}{\mu^2}\right]$ that follows from the $\ln^2\left[\dfrac{-q^2}{\mu^2}\right]$-term in correlators (4)–(6); here we restore it. The corresponding term for the T case $\rho(x,s)_T^{pert}$ has recently been presented in [6]. We have recalculated it and confirmed this result.

$$
\rho_L^{pert}(x,s) = \frac{3}{2\pi^2}x\bar{x}\left\{1+\frac{\alpha_s(\mu^2)C_F}{4\pi}\left(2\ln\left[\frac{s}{\mu^2}\right]+5-\frac{\pi^2}{3}+\ln^2(\bar{x}/x)\right)\right\},
\tag{B.1}
$$

$$
\begin{aligned}
\rho_T^{pert}(x,s) &= \frac{3}{2\pi^2}x\bar{x}\left\{1+\frac{\alpha_s(\mu^2)C_F}{4\pi}\right.\\
&\quad\left.\times\left(2\ln\left[\frac{s}{\mu^2}\right]+6-\frac{\pi^2}{3}+\ln^2(\bar{x}/x)+\ln(x\bar{x})\right)\right\}.
\end{aligned}
\tag{B.2}
$$

Here $\mu^2 \sim 1\ \mathrm{GeV}^2$ corresponds to the average value of the Borel parameter M^2 in the stability window; $\alpha_s\left(1\mathrm{GeV}^2\right) \approx 0.52$.

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