Nonequilibrium Dynamics in Quantum Field Theory*

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1 Introduction

The nonperturbative dynamics of quantum fields out of and far from equilibrium has received recently an increasing interest in the contexts of cosmology, of the QCD phase transition, and in solid state physics. As an introduction to the field we will present three typical examples:

1.1 Field emission in quark-gluon plasma

In a heavy ion collision quarks and antiquarks moving in different directions or getting separated within jets will stretch chromoelectric field lines which in turn will lead to the production of $q\bar{q}$ pairs by the Schwinger mechanism. Cooper et al. [1] considered this situation within the framework of quantum field theory, and in a classical formulation using the Boltzmann-Vlasov equation. We will use here their formulae - which are written in light cone coordinates - without too detailed an explanation. The back-reaction to the electric field is given by Gauss' law which here reads

$$\frac{1}{\tau}\frac{dE}{d\tau} = j_\eta \ . \tag{1}$$

*presented by J. Baacke

In the classical description the current is computed from the distribution function which satisfies the Boltzmann-Vlasov equation

$$\begin{aligned} \frac{\partial f}{\partial \tau} + eE\frac{\partial f}{\partial p} &= \pm\left[1 \pm 2f(p,\tau)\right] \\ &\quad \times \ln\left[1 \pm \exp\left(-\frac{\pi(m^2+p_\perp^2)}{e|E(\tau)|}\right)\right]\delta(p_\eta) \end{aligned} \tag{2}$$

as

$$j_\eta = 2e\int \frac{dp_\eta}{2\pi\tau p_\tau} p_\eta f(p_\eta,\tau) + \dots \,. \tag{3}$$

The r.h.s. of the Boltzmann-Vlasov equation is Schwingers formula for pair creation. In the quantum field theoretical description one computes the current as an expectation value

$$j_\eta = \frac{e}{2}\left\langle\left[\bar{\Psi},\gamma^\eta\Psi\right]\right\rangle = \frac{-2e}{\tau^2}\sum_s\int\frac{d^3k}{(2\pi)^3 2\omega_{k0}}(k_\perp^2+m^2)\lambda_s|f_{ks}^+|^2 \tag{4}$$

in a quantum state which is characterized by quantum modes satisfying the (reduced) Dirac equation

$$\left(\frac{d^2}{d\tau^2} + \omega_k^2 - i\lambda_s E\right) f_{ks}^+(\tau) = 0\,. \tag{5}$$

The authors find an essential agreement between the two approaches, it has to be pointed out, though, that Schwingers formula is a quantum field theoretical one. The close relation between transport equations and the nonperturbative quantum evolution, as apparent here, has not been formulated in a general way, yet.

1.2 Disordered chiral condensates (DCC)

In QCD chiral symmetry is spontaneously broken, it is assumed to be restored above the phase transition at $T \simeq 170$ MeV. In this range of energies the interactions of pions can be described by an effective theory, a linear $O(4) \simeq SU(2)\times SU(2)$ sigma model

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\pi^a\partial^\mu\pi^a\right) \\ &\quad -\frac{\lambda}{2}\left(\sigma^2+\pi^a\pi^a-f_\pi\right)^2 + f_\pi m_\pi^2\sigma \end{aligned} \tag{6}$$

with Mexican hat potential at $T = 0$. After cooling down, in a heavy ion collision, from temperatures above 170 MeV the symmetry breaking direction (σ) will in general be different from that of the one in the outside vacuum. This means that a coherent state with different isospin direction is formed after such a collision; such a disordered chiral condensate (DCC) [2] results in an unusual charge distribution in final state, that may be observable in experiment.

The evolution of such a condensate can be investigated using methods of nonequilibrium quantum field theory [3]. Realistic computations of spatially inhomogeneous condensates are still to be developed [4].

1.3 Inflation

The theory of inflation in the early universe [5] is characterized by a plethora of different models, there is old, new, chaotic inflation, and many other ones. A common problem consists in describing the reheating after the slow-roll-down period, i.e., in finding a mechanism by which the energy of oscillating inflaton field is transferred into thermal energy of the various quantum fields. In early formulations the relaxation of the inflaton field was described by a friction term via

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \Gamma\dot{\phi} + V'(\phi) = 0 \ . \tag{7}$$

Early attempts of a microscopic description of particle production [6, 7] were based on parametric resonance. Copious particle production was predicted, but these formulations neglected the back reaction of the quantum fields onto the classical oscillation.

More recently reheating has been discussed in the framework of nonequilibrium dynamics [8] at one loop or large N; one finds that dissipation is nonlocal in time, i.e., non-Markovian. This consequence of quantum back-reaction has been found in numerical and analytical studies. Furthermore, even when the oscillation of the inflation field relaxes completely, the created quantum ensemble is a non-thermal one; what is lacking is a mechanism which transports the low momentum excitations to higher momentum. The creation of a nonthermal ensemble, as already present in the simple parametric resonance models, has been called *preheating*.

2 Nonperturbative evolution of homogenous condensates

The basic equations of quantum field theory out of equilibrium, the Schwinger-Keldysh [9, 10] or Closed-Time-Path formalism, have been formulated long ago. We will not discuss this formalism here, as the equations for the evolution of condensates given below take a quite intuitive form.

As an example that has been studied extensively we consider scalar O(N) model

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi^i\partial^\mu\phi^i - \frac{1}{2}m^2\phi^i\phi^i - \frac{\lambda}{4N}(\phi^i\phi^i)^2 \tag{8}$$

where $\phi^i, i = 1, .., N$ are N real scalar fields. Such a Lagrangian could possibly describe the post-inflationary quantum evolution of the preheating period, for which the inclusion of the cosmological expansion is not essential [1].

The nonequilibrium system is characterized by a time dependent expectation value (condensate, classical field, inflaton ..) $\phi^N = \phi(t)$, chosen to lie in the direction of the N'th component, and by quantum fluctuations $\phi^1(\mathbf{x}, t) = \phi^2(\mathbf{x}, t) = \ldots = \phi^{N-1}(\mathbf{x}, t) = \psi(\mathbf{x}, t)$, so that

$$\phi^i(\mathbf{x}, t) = \delta^i_N\sqrt{N}\phi(t) + \psi(\mathbf{x}, t)\ . \tag{9}$$

In the large-N limit the Lagrangian reduces to

$$\mathcal{L} = N\left(\mathcal{L}_\phi + \mathcal{L}_\psi + \mathcal{L}_I\right)\ , \tag{10}$$

with

$$\begin{aligned}
\mathcal{L}_\phi &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4}\phi^4\ , \\
\mathcal{L}_\psi &= \frac{1}{2}\partial_\mu\psi\partial^\mu\psi - \frac{1}{2}m^2\psi^2 + \frac{\lambda}{4}(\psi^2)^2\ , \\
\mathcal{L}_{\mathrm{I}} &= -\frac{\lambda}{2}\psi^2\phi^2\ ,
\end{aligned} \tag{11}$$

[1] The inclusion of the cosmological expansion is discussed in the contribution presented by C. Pätzold at this conference.

where ψ^2 denotes $\sum \psi^i \psi^i / N$. In view of the spatial homogeneity of the classical field $\phi(t)$ it is appropriate to decompose the quantum fluctuations into momentum eigenmodes via

$$\psi(\mathbf{x},t) = \int \frac{d^3k}{(2\pi)^3 2\omega_{k0}} \left[a_{\mathbf{k}} U_k(t) e^{i\mathbf{k}\mathbf{x}} + a_{\mathbf{k}}^\dagger U_k^*(t) e^{-i\mathbf{k}\mathbf{x}} \right] \tag{12}$$

with $\omega_{k0} = \sqrt{m_0^2 + k^2}$. The "initial mass" m_0 is given by the second derivative of the classical potential at the initial value of the field $\phi(0)$, it is not the physical mass. The creation and annihilation operators depend on m_0, and so does the "initial vacuum" state annihilated by the $a_{\mathbf{k}}$, or the initial density matrix built on this initial Fock space. From this Lagrangian we derive the field equations for the field ϕ as

$$\left(\frac{d^2}{dt^2} + m^2 + \lambda \left\{ \phi^2(t) + \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix} \mathcal{F}(t,T) \right\} \right) \phi(t) = 0 \; . \tag{13}$$

$\mathcal{F}(t,T) = \langle \psi^2(\mathbf{x},t) \rangle$ is the *fluctuation integral* determined by the initial density matrix

$$\mathcal{F}(t,T) = \int \frac{d^3k}{(2\pi)^3 2\omega_{k0}} \mathcal{N}_0(k) |U_k(t)|^2 \; , \tag{14}$$

where $\omega_{k0} = \sqrt{m_0^2 + k^2}$, and $\mathcal{N}_0$ is the initial particle number. The quantum modes satisfy

$$\left(\frac{d^2}{dt^2} + k^2 + m^2 + \lambda \left\{ \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix} \phi^2(t) + \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} \mathcal{F}(t,T) \right\} \right) U_k(t) = 0 \; . \tag{15}$$

with initial conditions:

$$U_k(0) = 1 \;\; ; \;\; \dot{U}_k(0) = -i\omega_{k0} \; . \tag{16}$$

The factors given in the upper entry of the square brackets refers to the large-N approximation as introduced here; the middle entry refers to the one-loop approximation and the lower entry to the Hartree approximation for N_1. Note that the one loop equation contain the quantum back reaction to the classical field only, while the other approximations contain also the back reaction to the quantum modes themselves. These approximations include a resummation of tadpole graphs. Note that in the large-N case the evolution of the classical field and the one of the quantum fluctuations are both determined by the same effective mass

$$\mathcal{M}^2(t) = m^2 + \lambda(\phi^2(t) + \mathcal{F}(t)) \tag{17}$$

Along with the field equations it is in general useful, and in cosmology necessary, to consider as well the energy-momentum tensor

$$T_{\mu\nu} = -g_{\mu\nu}\left[\frac{1}{2}\partial_\alpha\Phi\partial^\alpha\Phi - \frac{m^2}{2}\Phi^2 - \frac{\lambda}{4}\Phi^4\right] + \partial_\mu\Phi\partial_\nu\Phi \;, \tag{18}$$

which for spatial homogenous systems is determined by the energy density

$$\begin{aligned}\mathcal{E} = \frac{1}{2}\dot{\phi}^2(t) + \frac{1}{2}m^2\phi^2(t) + \frac{\lambda}{4}\phi^4(t) - \frac{\lambda}{4}\mathcal{F}^2(t) \\ + \int\frac{d^3k}{(2\pi)^3 2\omega_{k0}}\mathcal{N}_0(k)\frac{1}{2}\left\{|\dot{U}_k(t)|^2 + \omega_k^2(t)|U_k(t)|^2\right\}\end{aligned} \tag{19}$$

and the pressure [2]

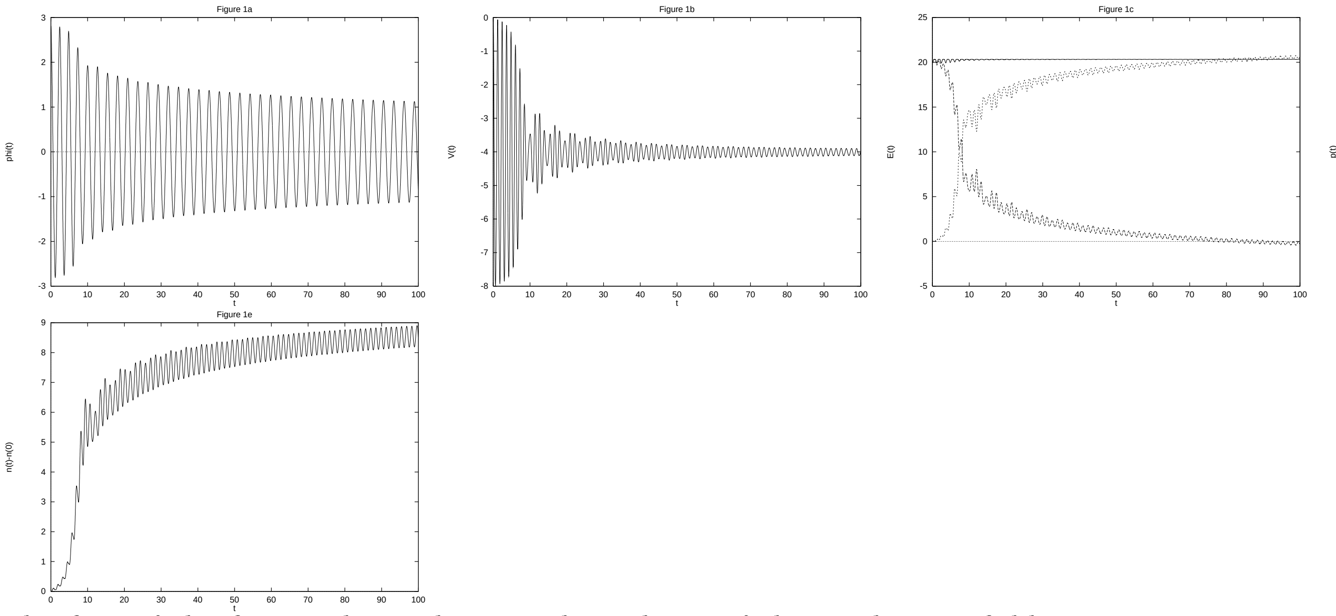
$$\begin{aligned}p &= \phi^2(t) - \mathcal{E} + A\frac{d^2}{dt^2}\left[\phi^2(t) + \mathcal{F}(t,T)\right] \\ &\quad + \int\frac{d^3k}{(2\pi)^3 2\omega_{k0}}\mathcal{N}_0(k)\left(\omega_{k0}^2 + \frac{k^2}{3}\right)|U_k(t)|^2 \;.\end{aligned} \tag{20}$$

Conservation of the energy density represents a useful check for the numerics, the ratio $p/\mathcal{E}$ determines whether the generated quantum fluctuations form a relativistic or a nonrelativistic ensemble. The energy momentum tensor is quartically divergent, its covariant renormalization is essential for applications on cosmology.

The quantum evolution equations we have written down here are not yet suitable for numerical computation, the fluctuation integral in the equation of motion, and the fluctuation contributions to energy and pressure are divergent, the equations have to be renormalized. Before discussing this aspect, we would like to present a typical result of a numerical computation of the

[2] The term proportional to A is a counter term necessary for renormalization.

renormalized equations [11]. We have chosen results for the large-N case at temperature 0, with $\phi(0) = 2.8$, and $\lambda = 1$.



The first of the figures shows the time dependence of the condensate field $\phi(t)$, the amplitude of oscillation decreases slowly, not exponentially. The second figure shows the evolution of the quantity $\mathcal{V}(t) = \mathcal{M}^2(t) - m_0^2$ which is seen to become constant asymptotically. The third figure shows the decreasing classical energy, the increasing energy of the quantum fluctuations, and the constant total energy. The fourth figure shows the pressure which is seen to approach a value of about one third of the energy, the produced quantum ensemble is ultrarelativistic, albeit nonthermal. The last figure shows the number density of produced particles. Such results are typical for the large-N evolution, in the one-loop case the amplitude of $\phi(t)$ decreases usually but becomes asymptotically constant.

3 Regularization and renormalization

While in a perturbative framework renormalization is discussed on the basis of suitably regularized Feynman graphs, in the present formalism one has to consider the ultraviolet behaviour of the mode functions entering the fluctuation integral. It is still true that the leading divergences can be obtained from the perturbative expansion of the mode functions $U_k(t)$ with respect to orders in the potential or "vertex operator" [3].

$$V(t) = \lambda(\phi(t) - \phi(0)) = m^2(t) - m_0^2 \tag{21}$$

We start [12] with the ansatz

$$U_k(t) = e^{-i\omega_{k0}t}(1 + h_k(t)) \tag{22}$$

in which the zeroth order has been taken out and $h_k(t)$ contains all higher orders. We expand further in orders of V(t) as

$$h_k(t) = \sum_{n=1}^{\infty} h_k^{(n)}(t) \; . \tag{23}$$

We also introduce the partial sums

$$h_k^{\overline{(n)}}(t) = \sum_{l=n}^{\infty} h_k^{(l)}(t) \; , \tag{24}$$

so that

$$h_k(t) \equiv h_k^{\overline{(1)}}(t) = h_k^{(1)} + h_k^{\overline{(2)}}(t) \; . \tag{25}$$

The function $h_k(t)$ satisfies the integral equation

$$h_k(t) = \frac{i}{2\omega_{k0}} \int_0^t dt' (e^{2i\omega_{k0}(t-t')} - 1) V(t')(1 + h_k(t')) \; . \tag{26}$$

So the first order contribution is given by

$$h_k^{(1)} = \frac{i}{2\omega_{k0}} \int_0^t dt' (e^{2i\omega_{k0}(t-t')} - 1) V(t') \; . \tag{27}$$

High momentum behaviour of the function $h_k^{(1)}$ can be obtained using integrations by parts

$$\begin{aligned} h_k^{(1)}(t) &= \frac{-i}{2\omega_{k0}} \int_0^t dt' V(t') \\ &+ \sum_{l=0}^{n} \left(\frac{-i}{2\omega_{k0}}\right)^{l+2} \left(V^{(l)}(t) - e^{2i\omega_{k0}t} V^{(l)}(0)\right) \\ &- \left(\frac{-i}{2\omega_{k0}}\right)^{n+2} \int_0^t dt' e^{2i\omega_{k0}(t-t')} V^{(n+1)}(t') \; , \end{aligned} \tag{28}$$

[3] We consider the one-loop equations in this section.

Inserting this, for n=2, into the fluctuation integral one finds

$$\begin{aligned}
\mathcal{F}(t) &= \int \frac{d^3k}{(2\pi)^3 2\omega_{k0}} \left(1 + 2\mathrm{Re}h_k(t) + |h_k(t)|^2\right) && (29)\\
&= \int \frac{d^3k}{(2\pi)^3 2\omega_{k0}} \left(1 - \frac{V(t)}{2\omega_{k0}^2} + \frac{V(0)}{2\omega_{k0}^2}\cos(2\omega_{k0}t) + \right.\\
&\quad \frac{\dot V(0)}{4\omega_{k0}^3}\sin(2\omega_{k0}t) + \frac{\ddot V(t)}{8\omega_{k0}^3} - \frac{\ddot V(0)}{8\omega_{k0}^3}\cos(2\omega_{k0}t) && (30)\\
&\quad \left. - \frac{1}{8\omega_{k0}^4}\mathcal{C}(\dddot V, t) + 2\mathrm{Re}h_k^{(2)} + |h_k|^2\right) .
\end{aligned}$$

with

$$\mathcal{C}(f,t) = \int_0^t dt' f(t') \cos(2\omega_{k0}t') , \tag{31}$$

The divergent parts of this integral come from the first terms in the parenthesis, $1 - V(t)/(2\omega_{k0}^2)$. They are obviously local in $\phi(t)$ and can be absorbed, therefore, into local counter terms corresponding to mass and coupling constant renormalization[4]. The three-dimensional integrals can be regularized dimensionally or à la Pauli-Villars, in both cases they are equal to the expected 4-dimensional Feynman integrals. As these terms can be simply omitted from the fluctuation integral, which is computed using the functions $h_k(t)$ one has in a simple way separated the divergent parts from the numerical computation of the finite ones. One finds for the coefficients of the counter terms the standard expressions

$$\begin{aligned}
\delta m^2 &= \frac{\lambda m^2}{32\pi^2}(L+1) , && (32)\\
\delta\lambda &= \frac{3\lambda^2}{32\pi^2} L , && (33)\\
L &= \frac{2}{\epsilon} + \ln\frac{4\pi\mu^2}{m^2} - \gamma
\end{aligned}$$

4 The initial state

The renormalized fluctuation integral is not well-behaved at $t = 0$. This is due to the terms proportional to $V(0), \dot V(0)$, and $\ddot V(0)$, which behave as

[4]Note that the fluctuation integral appears with a factor $\phi(t)$ in the equation of motion and acts like an effective mass term.

$1/t, \ln t$, and $t \ln t$, respectively. Now with our choice of initial mass $V(0) = 0$ and if we choose $\dot{\phi}(0) = 0$ then $\dot{V}(0)$ vanishes as well. However even then we find similar singularities in the pressure, proportional to $\ddot{V}(0)$ which is certainly discontinuous due to the equation of motion. In cosmology one finds that $\dot{V}(0)$ is in general nonzero, so one should certainly care about these terms.

Their origin can be traced back to discontinuities of the external field, they are also known as Stueckelberg singularities [13]. Physically our initial conditions imply that we start with a system which corresponds to a coherent state (the condensate) on a Bogoliubov-transformed vacuum state ($m_0 \neq m_{phys}$), m_0 being determined by the curvature of the classical potential at $\phi(0)$. If we choosed another initial mass this would correspond to a jump of the external field at $t = 0$, and $V(0)$ would be nonzero, leading to the most prominent $1/t$ singularity. The different initial states would be related again by a Bogoliubov transformation. This means that one could get rid of the singularity by applying the inverse Bogoliubov transformation.

This observation is the clue for solving the problem of the less prominent initial singularities: the initial state has to be modified by *another* Bogoliubov transformation

$$\tilde{a}_{\mathbf{k}} = \cosh\beta(k) a_{\mathbf{k}} + e^{i\delta(k)} \sinh\beta(k) a^{\dagger}_{-\mathbf{k}} \; , \tag{34}$$

as a consequence the mode functions are replaced by

$$F_k(t) = \cosh(\gamma_k) U_k(t) + e^{i\delta_k} \sinh(\gamma_k) U_k^*(t) \; . \tag{35}$$

The explicit form of this transformation, which leads to the cancellation of all singular terms, is [14]

$$\tan(\delta_k) = 2\omega_{k0} \frac{\dot{V}(0)}{\ddot{V}(0)} \; , \tag{36}$$

$$\tanh(2\gamma_k) = \frac{1}{4\omega_{k0}^3} \left(\dot{V}^2(0) + \frac{\ddot{V}^2(t)}{4\omega_{k0}^2} \right)^{1/2} \; . \tag{37}$$

Removing Stueckelberg singularities by Bogoliubov transformations has also been considered recently by Maslov and Shvedov [15].

5 Late-time behaviour

Looking back at the numerical results presented above it is obvious that the amplitude of the classical oscillation decreases very slowly, and it is hard to guess what it does at even larger times, whether - and how - it tends to zero, or whether it settles to a constant value. Furthermore, one may not trust a purely numerical computation if extended over really large times, as numerical errors may pile up. It is therefore satisfying that that the asymptotic behaviour has recently been analyzed analytically for the large-N and for the one-loop equations.

The late-time behaviour of the one-loop equations has been analyzed by de Vega and Salgado [16]. They considered the asymptotic behaviour of a small-amplitude oscillation (which can be assumed to be reached if the amplitude is decreasing). The equations then become perturbative. These perturbative equations are characterized by the occurence of secular terms; a renormalization group equation relating equivalent initial conditions at different times reveals that for large times the amplitude becomes constant, the asymptotic limit being reached slowly, as $t^{-2/3}$. So there is no complete relaxation.

An *anomalous relaxation* was found by Boyanovsky et al. [17] for the asymptotic behaviour of the large-N equations. These authors used a multiple-time-scale analysis in order to derive some results even analytically. The effective mass $\mathcal{M}^2(t)$ (Eq. (16)) becomes constant asymptotically, and satisfies a sum rule

$$\mathcal{M}^2_\infty = \lim_{t\to\infty} \mathcal{M}^2(t) = m_0^2(1+\eta_0^2/2) \tag{38}$$

where η_0 is the initial amplitude rescaled as $\eta_0 = \sqrt{\lambda/2}\phi(0)/m_0$. Our numerical results presented above satisy this sum rule very well, see [11] for details. This amplitude also determines the asymptotic ratio of pressure and energy

$$p_\infty = \frac{\mathcal{E}}{3(1+2/\eta_0^2)} \; . \tag{39}$$

Finally, the amplitude exhibits an ‘ anomalous’ power instead of exponential decrease as

$$|\eta(\tau)| \propto \tau^{-.27} \; , \tag{40}$$

the power $-.27$ being determined empirically.

6 Conclusions and outlook

The study of quantum fields out of equilibrium has disclosed a new area of nonperturbative semiclassical behaviour. New types of evolution of quantum systems have emerged, displaying non-Markovian long-memory kernels that result in the absence of relaxation or in anomalous, nonexponential forms of relaxation. The late-time behaviour is understood on an almost analytical level, with sum rules and universal scaling [16, 17]. The full nonperturbative quantum evolution has, however, not yet led to an understanding of thermalization; presumably higher order $1/N_C$ corrections would have to be studied, but this seems at present outside of the reach of even large computers.

The renormalization of the nonperturbative evolution equations has been fully developed. We have derived the covariantly renormalized equations for scalar [12], fermion [18] and nonabelian gauge fields [19] using the technique presented in section 3. Covariant regularization is essential for a consistent formulation of the evolution in FRW universes [20].

Besides divergences that can be taken care of via renormalization, one finds singularities in the time variable, which are related to the initial state. We have given recently an explicit construction [14] which solves this problem.

There are many interesting and physically important aspects of quantum field theory out of equilibrium that have to be tackled and solved in the near future, among them the consideration of spatially inhomogeneous background fields [4], the problem of thermalization and the application to gauge theories at high temperature.

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