Instantons and Scalar Matter in a Closed Volume

A. A. Abrikosov, Jr.*

117 259, Moscow, ITEP.

Abstract

The leading correction to the action of a small pseudoparticle interacting with scalar matter in a closed volume is calculated. We discover suppression of instantons which is proportional to the density of matter.

Introduction

Among the unsettled questions of quantum chromodynamics is that about the role of instantons [1, 2]. After the cooling of the first enthusiasm it was practically left till better times. Phenomenological studies, [3], indicate that instantons could be extremely interesting for physics. However instanton models are missing something important: instantons do not provide confinement and there is no consistent way to cut the divergence of the instanton density. The key may lie in physics of large scale gauge fields that are responsible for confinement.

In principle instantons could tell in the range between the asymptotic freedom and confinement scales. The problem is whether they are discernible or completely blurred by the background. This makes interesting to include pseudoparticles into models of confinement. The simplest of those is the famous MIT bag, [4], which secures quarks in a closed volume. The only

*E-mail: PERSIK@VXITEP.ITEP.RU

manifestation of confinement is that matter and gauge fields are subject to boundary conditions.

I investigate interaction of pseudoparticles with matter in a closed volume. This continues my previous work, [5]. For scalar quarks the problem may be solved exactly[1], boundary terms being under control. The confined matter suppresses instantons similarly to the unbounded quark plasma, [6]. This may indicate that the same happens with fermions.

1 Interaction of instantons with the bag.

1.1 Instantons in the bag model.

Instantons look most natural in the framework of the 4-dimensional euclidean field theory. We shall follow the tradition. Let us consider a hadron N. The state $|N\rangle$ is characterized by the quark contents, spin, parity *etc.* Euclidean generating functional for gauge fields and matter in a spherical bag is:

$$Z_N(\beta,\, R_B) = \langle N,\, R_B| \int [d\bar{\phi}\, d\phi\, dA] \exp - \int_0^\beta dx_4 \int_B d^3x\, (L + B^0)|N,\, R_B\rangle. \tag{1}$$

The functional integral over $[d\bar{\phi}\, d\phi\, dA]$ keeps account of gauge fields and matter described by the QCD-Lagrangian L. We shall disregard quantum fluctuations of the bag. In order to define it's radius R_B the energy as usual must be minimized at the end.

Following the analogy with quantum statistical physics we shall assume that fields are periodic in x_4, the inverse temperature $\beta \to \infty$ being the period. Conditions at the surface of the bag will be specified later.

The generating functional (1) is directly related to the free energy $F = -\beta^{-1} \ln Z$. At zero temperature it is defined by the mass of the hadron,

$$M_N = \lim_{\beta\to\infty} \frac{1}{\beta} \min_{R_B} |\ln Z_N(\beta,\, R_B)|\,. \tag{2}$$

To avoid standard problems of instanton physics let us focus on small instantons in the dilute gas approximation. It is convenient to characterize

[1] Boundary conditions break chiral invariance and greatly complicate the problem for fermions.

the density of instantons in the bag d_N in terms of the additional action, δS_N:

$$d_N(\rho,\, x,\, R_B) = d_0(\rho)\, \exp -\delta S_N(\rho,\, x,\, R_B). \tag{3}$$

Here ρ is the pseudoparticle radius and d_0 is the density in the infinite space.

The function δS_N is of crucial importance. It is well known that the integral density of pseudoparticles $D_0 = \int \rho^{-5}\, d\rho\, d_0(\rho)$ diverges. A growth of δS_N could cut the divergence and make the model quantitatively consistent.

Suppose that this is the case. Then the statistical sum of the dilute instanton gas in the hadron is easy to calculate:

$$Z_N^I(R_B) = Z_N^0(R_B) \left\{ \exp\left[\int_0^\beta dx_4 \int_B \frac{d^3x\, d\rho}{\rho^5}\, d_N(\rho,\, x,\, R_B) \right] \right\}^2 ; \tag{4}$$

(the square of the exponential takes into account anti-instantons).

Comparing this with the definition (1) of Z_N we see that the contribution of instantons may be included into the bag constant:

$$B^{eff} = B^0 - 2 \int \frac{d\rho}{\rho^5}\, d_N(\rho,\, x,\, R_B). \tag{5}$$

We see that B^{eff} is smaller than B^0 by the total density of topological fluctuations. The outward pressure of pseudoparticles $B^{eff} - B^0$ depends on structure and varies from hadron to hadron.

1.2 Additional action of an instanton in the bag.

Now we shall discuss the additional action δS_N of a pseudoparticle in the bag. The correction to the action δS_N may be split into summands δS according to their origin:

$$\delta S_N = \delta S_{cl} + \delta S_d + \delta S_q + \delta S_C. \tag{6}$$

- The first addend $\delta S_{cl} = S_{cl}\big|_B - S_{cl}\big|_0$ allows for deformations of the instanton. This takes into account interactions of the classical field with the walls. An estimate for δS_{cl} is given by the part of the action of the original instanton gained outside the bag. For a small pseudoparticle located at the center of the bag this gives:

$$\delta S_{cl} \sim \int_{x>R_B} d^4x\, \frac{F_{\mu\nu}^2\big|_0}{4} \approx \frac{3\rho^4}{R_B^4} S_{cl}, \qquad S_{cl} = \frac{8\pi^2}{g^2}. \tag{7}$$

- The second piece $\delta S_d = B\,\delta V_4$ is associated with the swelling δV_4 of the bag around a pseudoparticle. It must be of the same order:

$$\delta S_d = B^0 \delta V_4 \sim \delta S_{cl} \sim \frac{\rho^4}{R_B^4} S_{cl}. \tag{8}$$

  Note that the collective effect of the instanton gas may be significant even if each of individual δV_4's is small.

- The third term describes interaction of the pseudoparticle with quarks in the closed volume:

$$\begin{aligned} \delta S_q \quad = \quad & - \ln \frac{\langle N| \int [d\bar{\phi}\, d\phi\, d\delta A]\ \exp - \int_0^\beta dx_4 \int_B d^3x\, L^I |N\rangle_B}{\langle 0| \int [d\bar{\phi}\, d\phi\, d\delta A] \exp - \int_0^\beta dx_4 \int_B d^3x\, L^I |0\rangle_B} \\ & + \ln \frac{\langle N| \int [d\bar{\phi}\, d\phi\, dA]\ \exp - \int_0^\beta dx_4 \int_B d^3x\, L^0\, |N\rangle_B}{\langle 0| \int [d\bar{\phi}\, d\phi\, dA] \exp - \int_0^\beta dx_4 \int_B d^3x\, L^0 |0\rangle_B}. \end{aligned} \tag{9}$$

  The notations $\langle N\,|\ldots|\,N\rangle_B$ and $\langle 0\,|\ldots|\,0\rangle_B$ stand for bag with and without quarks respectively. We shall show that δS_q is of the first order in ρ^2/R_B^2 and plays the dominant role for small instantons.

- The last is the Casimir correction δS_C coming from the quantization of gauge fields in the closed volume. Certainly it could stop the growth of $g^2(\rho)$ at $\rho \sim R_B$ and cut off the divergence of the density. However for small pseudoparticles the Casimir correction appears mainly because of deformations of the quantum modes. The latter are of the same value as those of the classical field. Thus one could expect that it is of the same order,

$$\delta S_C \sim \frac{\rho^4}{R_B^4} S_{cl}, \quad (?). \tag{10}$$

The analysis indicates that for small pseudoparticles interaction with matter is more important than deformations. The question of the Casimir correction may require additional investigation.

2 Interaction with scalar quarks.

Let us return to the bag model. Let the quark Hamiltonian and states be $\hat{H}$, $|\alpha,\, a,\, f\rangle_B$. The multi-index α includes the principal quantum number, angular momentum *etc.*, while a and f stand for colour and flavour. The wave functions $\phi^{\alpha,\,a,\,f}(\vec{x})$ satisfy lineary boundary conditions. Quarks are created and destroyed by operators $\hat{a}^{\dagger}_{\alpha,\,a,\,f}$, $\hat{a}_{\alpha,\,a,\,f}$ with usual commutation relations:

$$\left[\hat{a}_{\alpha,\,a,\,f},\;\hat{a}^{\dagger}_{\beta,\,b,\,g}\right] = \delta_{ab}\,\delta_{\alpha\beta}\,\delta_{fg}. \tag{11}$$

The role of the vacuum in the model belongs to the empty bag $|0\rangle_B$. A hadron $|N\rangle_B$ is produced by simultaneous action of quark operators. Baryons are characterized by colour singlet vertices $N^{\alpha\beta\gamma\cdots}$ (we suppress colour and flavour indices):

$$|N\rangle_B = N^{\alpha\beta\gamma\cdots}\,\hat{a}^{\dagger}_{\alpha}\,\hat{a}^{\dagger}_{\beta}\,\hat{a}^{\dagger}_{\gamma}\ldots|0\rangle_B\,; \qquad \langle N|_B = N^{*}_{\alpha\beta\gamma\ldots}\,\langle 0|_B\,\hat{a}_{\alpha}\,\hat{a}_{\beta}\,\hat{a}_{\gamma}\ldots. \tag{12}$$

It is possible to define the Heisenberg creation and annihilation operators of quarks in the coordinate representation:

$$\hat{\phi}^{\dagger}_{a,\,f}(\vec{x},\,t) = \sum_{\alpha} e^{\hat{H}t}\,\hat{a}^{\dagger}_{\alpha,\,a,\,f}\,e^{-\hat{H}t}\,\phi^{*}_{\alpha,\;f};$$

$$\hat{\phi}^{a,\,f}(\vec{x},\,t) = \sum_{\alpha} e^{\hat{H}t}\,\hat{a}_{\alpha,\,a,\,f}\,e^{-\hat{H}t}\,\phi^{\alpha,\;f}; \tag{13}$$

where $\phi^{\alpha,\,f}(\vec{x})$ and $\phi^{*}_{\alpha,\,f}(\vec{x})$ are wave functions of the corresponding states. The propagation function of quarks in the bag is defined as:

$$\Delta_f(\vec{x},t;\;\vec{y},\,t') = \langle 0|\,\mathbf{T}\,\hat{\phi}^{f}(\vec{x},\,t)\,\hat{\phi}^{\dagger}_{f}(\vec{y},\,t')\,|0\rangle_B\,; \tag{14}$$

where $\mathbf{T}$ stands for time ordering.

It does not take long to see that δS_q, (9), may be expressed in terms of matrix elements of the propagator with and without instanton.

$$\delta S_q = -\ln\frac{N^{*}_{\alpha\beta\gamma\ldots}\,\Delta^{\alpha}_{\alpha'}(\beta)^{I}\,\Delta^{\beta}_{\beta'}(\beta)^{I}\,\Delta^{\gamma}_{\gamma'}(\beta)^{I}\ldots N^{\alpha'\beta'\gamma'\ldots}}{N^{*}_{\alpha\beta\gamma\ldots}\,\Delta^{\alpha}_{\alpha'}(\beta)^{0}\,\Delta^{\beta}_{\beta'}(\beta)^{0}\,\Delta^{\gamma}_{\gamma'}(\beta)^{0}\ldots N^{\alpha'\beta'\gamma'\ldots}}. \tag{15}$$

The matrix element $\Delta^{\alpha}_{\alpha'}(\beta)$ is defined as follows:

$$\Delta^{\alpha,\,a}_{\beta,\,b}(\beta) = \langle 0|\,\hat{a}_{\alpha,\,a}(\beta)\,a^{\dagger}_{\beta,\,b}(0)\,|0\rangle_B = \int_B d^3x\,d^3y\,\phi^{*}_{\beta,\,b}(\vec{x})\Delta(\vec{x},\,\beta;\,\vec{y},\,0)\,\phi^{\alpha,\,a}(\vec{y}). \tag{16}$$

Expression (15) simplifies if corrections are small. Expansion of the matrix element in ρ^2/R_B^2 has the form (up to irrelevant terms):

$$\Delta_{\alpha',\,a'}^{\alpha,\,a}(\beta)^I = \Delta_{\alpha',\,a'}^{\alpha,\,a}(\beta)^0 \left(1 + \frac{\rho^2}{R_B^2}\epsilon_{\alpha,\,a}(\beta,\, R_B)\right) + \ldots; \tag{17}$$

the nonperturbed propagator being diagonal, $\Delta_{\alpha',\,a'}^{\alpha,\,a}(\beta)^0 \propto \delta_{\alpha'}^{\alpha}\,\delta_{a'}^{a}$.

In order to calculate the sum in (15) we recall that the instanton is embedded into a $SU(2)$ subgroup of the gauge group and only two of ϵ's are not zero. The vertex N is colour singlet, *i. e.* $N \propto \varepsilon^{abc\cdots}$ and summation gives:

$$\delta S_q = -\frac{2}{N_c}\sum_{\alpha=1}^{N_c}\epsilon_\alpha \frac{\rho^2}{R_B^2}. \tag{18}$$

Thus δS_q may be expressed in terms of the instanton correction to the quark propagator. An interesting feature is that $\delta S_q \propto N_c^{-1}$ and matter does not influence instantons in the large-N_c limit.

3 Scalar correlator in the instanton field.

Let ϕ be a massless scalar field in fundamental representation of the $SU(2)$ gauge group. The action of the matter and the gauge field is:

$$S = -\int_{B_4} d^4x \left[\phi^\dagger \nabla_\mu^2 \phi + \frac{1}{4g^2}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \epsilon^{abc}A_\mu^b A_\nu^c)^2\right]. \tag{19}$$

Vector potential is denoted by A_μ^a with $\mu = 1, \ldots, 4$ and $a = 1,\, 2,\, 3$ being the Lorentz and group indices respectively. The covariant derivative in fundamental representation is $\nabla_\mu = \partial_\mu - \frac{i\tau^a}{2}A_\mu^a$ where τ^a are Pauli matrices. We shall impose the linear Dirichlet boundary condition, *i. e.* put ϕ to zero at the boundary:

$$\phi(x) = 0 \quad \text{for} \quad x \in \partial B_3. \tag{20}$$

The object under study will be the correlation function[2].

$$\Delta(x,\, y) = \frac{\delta}{\delta J^\dagger(x)}\frac{\delta}{\delta J(y)}\bigg|_{J^\dagger = J = 0} \frac{1}{V_4}\ln \int DA_\mu^a\, D\phi\, D\phi^\dagger \exp(-S + J^\dagger\phi + \phi^\dagger J); \tag{21}$$

[2]Surprisingly in a limited space it does not automatically coincide with the propagator.

The classical field A^a_μ is an instanton of a radius ρ in singular gauge:

$$A^a_\mu(x) = \frac{2\rho^2 \bar\eta^a_{\mu\nu}(x-z_0)_\nu}{(x-z_0)^2((x-z_0)^2+\rho^2)} = -\bar\eta^a_{\mu\nu}\partial_\nu \ln \Pi(x-z_0). \qquad (22)$$

Here $\Pi(x-z_0) = 1 + \rho^2/(x-z_0)^2$ (z_0 is the instanton center) and $\bar\eta^a_{\mu\nu}$ is the 'tHooft's $\bar\eta$-symbol. At the main order in ρ^2/R_B^2 we may neglect deformations of the instanton.

Instanton field can not be taken into account perturbatively. Fortunately for the chosen boundary conditions the problem can be solved exactly.

It is convenient to make use of the two sets of 2×2 matrices, (latin indices stand for the "spatial" dimensions):

$$\tau_\mu = (\tau_i, i), \qquad \tau^\dagger_\mu = (\tau_i, -i). \qquad (23)$$

Those are closely related to the η-symbols:

$$\tau^\dagger_\mu \tau_\nu = \delta_{\mu\nu} + i\eta^a_{\mu\nu}\tau_a; \qquad \tau_\mu \tau^\dagger_\nu = \delta_{\mu\nu} + i\bar\eta^a_{\mu\nu}\tau_a. \qquad (24)$$

Convolutions of τ-matrices with 4-vectors are denoted by hats:

$$\hat x = x_\mu \tau_\mu; \qquad \hat x^\dagger = x_\mu \tau^\dagger_\mu; \qquad (25)$$

The covariant d'Alembert operator in the instanton field, Eq. (22), can be converted to the form:

$$\nabla^2_\mu = \Pi^{\frac12}(x)\,\hat\partial \frac{1}{\Pi(x)}\,\hat\partial^\dagger \Pi^{\frac12}(x) = DD^\dagger; \qquad (26)$$

where we have introduced linear differential operators D and $D^\dagger$:

$$D(x) = \Pi^{\frac12}(x)\hat\partial \frac{1}{\Pi^{\frac12}(x)}; \quad D^\dagger(x) = \frac{1}{\Pi^{\frac12}(x)}\hat\partial^\dagger \Pi^{\frac12}(x). \qquad (27)$$

With the help of the representation (26) we shall immediately obtain for the correlator (21):

$$\Delta(x,\,y) = -\frac{\delta}{\delta J^\dagger(x)}\frac{\delta}{\delta J(y)} Z\Bigg|_{J^\dagger = J = 0} = -\int_0^\beta dz_4 \int_{B_3} d^3z\, D^{\dagger\,-1}(x,\,z)\, D^{-1}(z,\,y). \qquad (28)$$

The integral operators D^{-1} and $D^{\dagger\,-1}$ are defined as follows (mind the order):

$$D^{\dagger\,-1}(x,\,y)\,\overleftarrow{D}^{\dagger}=\overrightarrow{D}\,D^{-1}(x,\,y)=\mathcal{I}(x,\,y); \tag{29}$$

where $\mathcal{I}(x,\,y)$ is the unity integral operator which fulfills the Dirichlet boundary conditions, (20). It is not a problem to check that,

$$D^{-1}(x,\,y)=\Pi^{\frac{1}{2}}(x)\;\overrightarrow{\hat{\partial}}^{\dagger}\;\frac{1}{\partial^2}(x,\,y)\Big|_B\;\Pi^{-\frac{1}{2}}(y); \tag{30a}$$

$$D^{\dagger\,-1}(x,\,y)=\Pi^{-\frac{1}{2}}(x)\;\frac{1}{\partial^2}(x,\,y)\Big|_B\;\overleftarrow{\hat{\partial}}\,\Pi^{\frac{1}{2}}(y). \tag{30b}$$

Substituting these formulae into the equation (28) we obtain the general expression for the scalar correlator:

$$\Delta^I(x,\,y)\Big|_B=-\frac{1}{\sqrt{\Pi(x)\Pi(y)}}\int_0^\beta dz_4\int_{B_3}d^3z\;\frac{1}{\partial^2}(x,\,z)\Big|_B\;\hat{\partial}\Pi(z)\;\hat{\partial}^\dagger\frac{1}{\partial^2}(z,\,y)\Big|_B\,. \tag{31}$$

Note that since the free Green function $\partial^{-2}|_B$ satisfies the boundary condition the correlator (31) also is zero at the boundary.

Suppose that x, y are far from the instanton. Retaining only the leading diagonal terms we obtain after some algebra:

$$\Delta(x,\,y)|_B=-\;\frac{1}{\partial^2}(x,\,y)\Big|_B-2\pi^2\rho^2\;\frac{1}{\partial^2}(x,\,z_0)\Big|_B\;\frac{1}{\partial^2}(z_0,\,y)\Big|_B+\ldots \tag{32}$$

The same expression would describe scattering by a δ-potential. Hence interaction with a small pseudoparticle can be modelled by adding to the action the point-like vertex:

$$S_{\text{matter}}\Rightarrow S_{\text{matter}}(z_0)=-\int_{B_4}d^4x\;\left[\phi^\dagger(x)\,\partial^2\phi(x)-2\pi^2\rho^2\;|\phi(x)|^2\;\delta^4(x-z_0)\right]; \tag{33}$$

At the lowest order instanton feels only the density of matter near the center.

4 Matrix elements in the spherical bag.

Correlation function can be expressed in terms of 3-dimensional harmonics. Let $\phi_n(\vec{x})$, $\phi_n(\vec{y})^\dagger$ be eigenfunctions of the $3d$-Laplace operator ∂_i^2 that fulfill the boundary condition (20):

$$-\overrightarrow{\partial}_i^2\phi_n(\vec{x}) = E_n^2\,\phi_n(\vec{x}); \qquad -\phi_n(\vec{y})^\dagger\ \overleftarrow{\partial}_i^2 = E_n^2\,\phi_n(\vec{y})^\dagger. \tag{34}$$

The scalar propagator in the $3+1$-cylinder without pseudoparticles is:

$$-\frac{1}{\partial^2}(x,\,y)\Big|_{B_3} = \sum_{n=1}^{\infty}\frac{\phi_n(\vec{x})\,\phi_n(\vec{y})^\dagger}{2E_n}\exp -E_n|x_4-y_4|. \tag{35}$$

Substituting this into (32) we obtain for the diagonal matrix element, (16):

$$\Big\langle n,\,\beta,\,b\,\Big|\Delta^I(z_0,\,\beta)\Big|\,n,\,\alpha,\,a\Big\rangle = \frac{\exp -E_n\beta}{2E_n}\left(1-\frac{\pi^2\rho^2}{E_n}\,|\phi_n(\vec{z}_0)|^2\right)\delta^\alpha_\beta\delta^a_b. \tag{36}$$

This formula contains all the information about interaction of the instanton with scalar quarks in the leading approximation. With the help of (18) we obtain for a "baryon" with N_c scalar quarks:

$$\delta S_q = \frac{2\pi^2\rho^2}{N_c}\sum_{\alpha=1}^{N_c}\frac{|\phi_\alpha(\vec{z}_0)|^2}{E_\alpha}. \tag{37}$$

The sign of δS_q corresponds to the suppression of pseudoparticles by matter.

Symmetrical wave functions that satisfy the boundary condition (20) are

$$\varphi_n(\vec{z}) = \frac{1}{\sqrt{2\pi R_B}|\vec{z}|}\sin\pi n\frac{|\vec{z}|}{R_B}, \qquad E_n = \frac{\pi n}{R_B}. \tag{38}$$

Inserting these into the formula (36) we get for diagonal matrix elements,

$$\Big\langle n,\,\beta,\,b\,\Big|\Delta^I(z^0,\,\beta)\Big|\,n,\,\alpha,\,a\Big\rangle_{\rm sph} = \frac{e^{-E_n\beta}}{2E_n}\left(1-\frac{\rho^2}{2n(\vec{z}_0)^2}\sin^2\pi n\frac{|\vec{z}_0|}{R_B}\right)\,\delta^\alpha_\beta\delta^a_b. \tag{39}$$

The correction is of the order ρ^2/R_B^2 and reaches the maximum at $\vec{z}_0=0$:

$$\delta S_q \le \lim_{z\to 0}\frac{2}{N_c}\sum_{\alpha=1}^{N_c}\frac{\rho^2}{2n_\alpha z^2}\sin^2\pi n_\alpha\frac{z}{R_B} = \frac{\pi^2}{N_c}\sum_{\alpha=1}^{N_c}n_\alpha\frac{\rho^2}{R_B^2}. \tag{40}$$

This justifies discarding the higher powers of ρ^2/R_B^2. The question is what values of ρ are allowed? It is natural to claim that the correction should not exceed the main term. Introducing the average value $\bar{n} = N_c^{-1} \sum_{\alpha=1}^{N_c} n_\alpha$ we find:

$$\frac{\rho^2}{R_B^2} \le \frac{1}{\pi^2 \bar{n}} \approx 0.1\, \bar{n}^{-1} \quad \text{and} \quad \frac{\rho}{R_B} \le \frac{1}{\pi}\sqrt{\frac{1}{\bar{n}}} \approx 0.32\, \bar{n}^{-\frac{1}{2}}, \tag{41}$$

This is in accord with the previous assumption that $\rho^2/R_B^2 \ll 1$.

Conclusion.

The analysis proves that scalar matter suppresses pseudoparticles. The leading effect depends only on the density and the energy of matter. The same suppression takes place in the quark plasma, [6]. It is highly probable that the present result is universal and is not specific to the bag model.

The interaction may be modelled by a point-like term in the Lagrangian. Instantons are swept towards the boundary where the quark density is less. Thus the model indicates spatial separation of the internal asymptotically free, intermediate instanton and external confinement domains.

The derivation was mathematically exact and limitations come from physical assumptions. Neglecting deformations of the instanton and of the bag is allowed unless (see Eqs. 7, 40), $\delta S_{cl} \ge \delta S_q$. This leads to the condition

$$\frac{\rho^2}{R_B^2} \le g^2 \ll 1, \tag{42}$$

that is more restrictive than (41). Our work is a step towards and may offer a hint at what happens in the more realistic case of spinor quarks.

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